

Standing and Sitting on a Swing Are Two Mechanisms

— Standing pumps at **twice** the natural frequency and sitting at **once** — One is parametric excitation and the other is forced oscillation, and a single word “pumping” names two different things — [Paper 351]

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— *From thinking to syncing.* —

Abstract

There are two ways to pump a swing. This paper asks whether the two are the same mechanism—the answer is that they are not. No new mathematical theorem and no new law is claimed. First, a 3 m swing has natural frequency 0.287753 Hz (period 3.4752 s) (Section 2). Second, this is the core of the paper. Standing pumps at 0.575506 Hz, twice the natural frequency (Section 3). Third, sitting pumps at 0.287753 Hz, once (Section 3). Fourth, the former is parametric excitation (changing the pendulum length) and the latter is forced oscillation (applying an external torque) (Section 3). Fifth, raising and lowering the centre of mass by 0.30 m gives 1.171214 per cycle and 4.8569 after ten (Section 4). Sixth, the separator is what is changed — a length, or a force (Section 5).

Scope of this paper (scope note): No new mathematical theorem and no new law is claimed—the simple pendulum, parametric excitation, and Mathieu’s equation are all standard. We do not build mechanics—the rigorous theory of the stability regions of Mathieu’s equation (Arnold tongues) is not entered. It is named and no more. We perform no experiment—no measurement of actual human centre-of-mass motion is made. The numbers are on a model stated explicitly. We do not treat large amplitude—outside the approximation $\sin \theta \approx \theta$ is not the subject. Damping is not the subject—air drag and bearing friction are not in the numbers. The gains are before damping is subtracted. We do not treat the human body—muscle output and joint range are not the subject. Relation to earlier papers: Paper 313 showed the corpus using “synchrony” in four senses—there too the distinction between twice and once did the work. Here one word “pumping” splits in two. Paper 350 showed that dead time eats only the phase—there too the relation between frequency and phase decided. Paper 300 showed whether two things share a root is decidable—by that test, standing and sitting have distinct roots. Paper 182 showed the curse of dimensionality is not one curse—the shape of one word naming several things is the same. What is added is giving the natural frequency as a number, showing the ratio of the two driving frequencies to be exactly 2 to 1, giving the gain against the size of the centre-of-mass motion at four points, and putting the separator on length against force.

1. Notation and the Question

Setting: Take a simple pendulum of length $L = 3.0$ m.

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{g}{L}}, \quad g = 9.80665 \text{ m/s}^2$$

Observation 1.1

Two ways of pumping a swing are known. Standing and flexing the knees, or sitting and leaning the torso back and forth——both are called pumping.

Observation 1.2

Standing changes the length of the pendulum. Bending the knees lowers the centre of mass and straightening raises it——what is changed is a parameter of the system itself.

Observation 1.3

Sitting applies an external moment. Leaning the torso creates a torque about the pivot——the length does not change.

Question: Are the two the same mechanism?

2. The Natural Frequency

Calculation 2.1: Natural frequency of a 3 m pendulum (computed in this paper).

Quantity	Value
Length L	3.0 m
Natural frequency f_0	0.287753 Hz
Natural period T_0	3.4752 s

Observation 2.2

3.4752 seconds matches ordinary experience. It is the time a playground swing takes for one full return—a check that the model has not drifted from reality.

Observation 2.3

Shortening by 0.3 m gives 0.303318 Hz. Up from 0.287753—a shorter pendulum swings faster. That fact builds the mechanism of the next section.

3. Twice and Once

Calculation 3.1: Driving frequency of the two ways of pumping (computed in this paper).

Way	What is changed	Driving frequency	Type
Standing	height of the centre of mass	0.575506 Hz = $2f_0$	parametric excitation
Sitting	lean of the body	0.287753 Hz = $1f_0$	forced oscillation

Observation 3.2

This is the core of the paper. The ratio is exactly 2 to 1— $0.575506/0.287753 = 2.000000$. Not an approximation but an integer ratio coming out of the mechanism.

Observation 3.3

Why twice. Standing, the knees straighten each time the pendulum passes the lowest point—and there are two lowest points per full return, so the flexing enters twice per period.

Observation 3.4

Why once. Sitting, the torso leans with the direction of travel—and the direction repeats only once per full return, so it enters once per period.

Observation 3.5

This difference changes the resonance condition. Forced oscillation grows when the driving frequency matches the natural one—parametric excitation grows at twice. The resonance point itself differs.

Observation 3.6

Distinct roots by the criterion of Paper 300. Not only do the numbers not coincide; the shape of the equation differs—forced oscillation has an inhomogeneous term, parametric excitation has a time-dependent coefficient. One word “pumping” names two different things.

4. Centre-of-Mass Motion and Gain

Calculation 4.1: Amplitude gain per cycle when standing and moving the centre of mass by dL (computed in this paper).

dL [m]	dL/L	Gain per cycle	After ten cycles
0.05	0.0167	1.025531	1.2867
0.10	0.0333	1.052168	1.6628
0.20	0.0667	1.109034	2.8148
0.30	0.1000	1.171214	4.8569

Observation 4.2

Small per cycle, large when stacked. At $dL = 0.30$ m, 1.171214 per cycle looks like very little, but ten cycles give 4.8569—because it grows exponentially.

Observation 4.3

Ten cycles is 34.752 seconds. In half a minute the amplitude multiplies several times over—which matches how quickly a child gets high on a swing.

Observation 4.4

Without moving the centre of mass the gain is 1.000000. At $dL = 0$, exactly one—unless a parameter is changed, parametric excitation does nothing at all.

Observation 4.5

To be stated honestly—these gains have not had damping subtracted. In practice air drag and bearing friction pull the other way—whether the gain beats the damping is what decides whether one can pump at all. That is why dL has a lower bound.

5. The Separator: Change a Length, or Apply a Force?

Observation 5.1

The separator—whether a parameter of the system is changed, or a force is applied from outside.

	What changes	Resonant frequency	Starts from zero
Parametric excitation	a parameter	$2f_0$	no
Forced oscillation	an external force	f_0	yes

Observation 5.2

The rightmost column matters most in practice. Standing and pumping on a completely motionless swing will, in principle, not start it—parametric excitation is a mechanism that multiplies an amplitude already present, and there is nothing to multiply at zero.

Observation 5.3

Sitting can start from zero. An external force is not proportional to the amplitude, so it acts even at rest—which is why the first stroke is sensibly taken sitting.

Observation 5.4

The separator can be moved. Even standing, shifting the centre of mass sideways introduces a force component—a real person probably mixes both. The two pure types are a distinction on the model's side.

Observation 5.5

To be stated honestly—0.287753 Hz and 1.171214 are values on a model with $L = 3$ m stated explicitly. Another swing gives other numbers—what does not move is only that the driving ratio is 2 to 1 and that the resonance points differ.

Observation 5.6

One last thing—using the same apparatus for the same purpose does not make the mechanism the same. One purpose — getting high on a swing — is achieved by two physics—agreement of purpose is not agreement of mechanism.

6. The Census and the Close

1. $f_0 = 0.287753$ Hz (period 3.4752 s) (Calculation 2.1): length 3 m.
2. Standing at 0.575506 Hz (Calculation 3.1): twice the natural.
3. Sitting at 0.287753 Hz (Calculation 3.1): once.
4. Ratio exactly 2 to 1 (Observation 3.2): not an approximation.
5. Two lowest points per return (Observation 3.3): the origin of the two.
6. $dL = 0.30$ m gives 1.171214 per cycle (Calculation 4.1): 4.8569 after ten.
7. $dL = 0$ gives gain 1.000000 (Observation 4.4): no parameter, nothing happens.
8. Shortening by 0.3 m gives 0.303318 Hz (Observation 2.3): shorter is faster.

9. Parametric excitation does not start from zero (Observation 5.2): forced oscillation does.

10. Distinct roots by Paper 300 (Observation 3.6): the equations differ in shape.

Closing: Two ways of pumping a swing are known—standing and flexing the knees, or sitting and leaning the torso. Both are called pumping. But the two are not the same mechanism—a 3.0 m swing has natural frequency **0.287753 Hz** (period **3.4752 s**). Standing pumps at twice that, **0.575506 Hz**; sitting pumps at once, **0.287753 Hz**. The ratio is exactly 2 to 1 — not an approximation but an integer ratio coming out of the mechanism. The reason for the two comes out by counting—standing, the knees straighten each time the pendulum passes the lowest point, and there are two lowest points per full return, so the flexing enters twice per period. Sitting, the torso leans with the direction of travel, and the direction repeats only once per return. The former is parametric excitation and the latter forced oscillation—forced oscillation grows when the driving frequency matches the natural one, while parametric excitation grows at twice; the resonance point itself differs. By the criterion of Paper 300 the two have distinct roots: forced oscillation has an inhomogeneous term, parametric excitation a time-dependent coefficient. The gain is large when stacked—moving the centre of mass by 0.30 m gives **1.171214** per cycle, which looks like very little, but ten cycles (**34.752 seconds**) give **4.8569**. Without moving the centre of mass the gain is exactly 1.000000: unless a parameter is changed, nothing happens at all. One thing separates them—whether a parameter of the system is changed, or a force is applied from outside. And the difference shows in the most practical place—standing and pumping on a completely motionless swing will, in principle, not start it, because parametric excitation multiplies an amplitude already present and there is nothing to multiply at zero. An external force is not proportional to amplitude, so the first stroke is sensibly taken sitting. To be said honestly—these gains have not had damping subtracted, and whether the gain beats the damping is what decides whether one can pump at all. The 0.287753 Hz and 1.171214 are values on a 3 m model; what does not move is only that the driving ratio is 2 to 1 and that the resonance points differ. One last thing—using the same apparatus for the same purpose does not make the mechanism the same. One purpose — getting high on a swing — is achieved by two physics. Agreement of purpose is not agreement of mechanism.

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