

Adding Noise Can Improve Detection — But “More Is Better” Is Not the Rule

— Zero noise gives detection rate **0**, $\sigma = 0.50$ gives correlation **0.687458**, $\sigma = 4$ gives **0.127152** — The optimum is not at either end but in the middle — [Paper 353]

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September 2026

— *From thinking to syncing.* —

Abstract

One is taught that noise is to be reduced. This paper asks whether reducing it always improves matters—the answer is that in a system with a threshold it does not. No new mathematical theorem and no new law is claimed. First, against a threshold of 1.0, a signal of amplitude 0.7 can never cross it alone (Section 2). Second, this is the core of the paper. At zero noise the detection rate is **0.0000** and the correlation is **0.000000** (Section 3). Third, adding noise raises the correlation to a maximum of **0.687458** at $\sigma = 0.50$ (Section 3). Fourth, adding more lowers it, down to **0.127152** at $\sigma = 4$ (Section 3). Fifth, raise the amplitude above the threshold and detection needs no noise at all (Section 4). Sixth, the separator is whether there is a threshold, not any property of the noise (Section 5).

Scope of this paper (scope note): No new mathematical theorem and no new law is claimed——stochastic resonance, threshold detectors, and signal-to-noise ratio are all standard. We do not build probability theory——the analytic theory of stochastic resonance (Kramers rate, linear response theory) is not entered. It is named and no more. We perform no experiment——no measurement in a sensory organ or an electronic circuit is made. The numbers are on a model stated explicitly. Kinds of noise are not treated——only white Gaussian noise is used; coloured and non-Gaussian noise are not the subject. We do not generalise——“noise is useful” is not claimed as a general law. It fails in a linear system without a threshold. We build no optimisation theory——the optimal σ is found by scanning, not solved analytically. Relation to earlier papers: Paper 300 showed whether two things share a root is decidable——by that test, “noise” and “interference” have distinct roots. Paper 352 showed a transition temperature moving with the measurement’s time scale——there too where the threshold is placed fixed the number. Paper 313 showed only the fifth sense to have a threshold——the shape of presence-or-absence of a threshold as the axis of classification is the same. Paper 182 showed one word naming several things——“noise” has the same shape. What is added is showing numerically that the detection rate at zero noise is exactly 0, giving the correlation at ten points so the peak and its interior location are visible, showing as a control that raising the amplitude removes the need for noise, and putting the separator on presence of a threshold.

1. Notation and the Question

Setting: Feed a detector with threshold $\theta = 1.0$ a periodic signal of amplitude $A = 0.7$ plus white Gaussian noise of standard deviation σ .

$$x(t) = A \sin(2\pi t) + \xi(t), \quad \xi \sim \mathcal{N}(0, \sigma^2)$$

A detection occurs when $x(t) > \theta$.

Observation 1.1

$A = 0.7 < \theta = 1.0$. The signal reaches at most 0.7——without noise it is, in principle, never detected.

Observation 1.2

Noise pushes the signal both up and down. Only the upward pushes cross the threshold——and pushing across is easiest where the signal is already high.

Observation 1.3

So detections correlate with the phase of the signal. The noise is disordered, but its combination with the threshold makes order—that correlation is the index used here.

Question: Is less noise always better?

2. At Zero Noise, Nothing Is Detected

Observation 2.1

The detection rate at $\sigma = 0$ is **0.0000**. The correlation is **0.000000** too—the cleanest condition is the condition in which nothing at all is seen.

Observation 2.2

This is exact, not approximate. Since $0.7 < 1.0$, with zero noise the inequality never holds—the probability is not small; the event is empty.

Observation 2.3

Here the intuition “reduce the noise” breaks. In ordinary design noise is the enemy—for a signal beneath the threshold, noise is the only thing that can carry it across.

3. The Correlation Is a Hill

Calculation 3.1: Detection rate and correlation against noise (computed in this paper).

Noise σ	Detection rate	Correlation with the signal
0.00	0.0000	0.000000
0.10	0.0001	0.244618
0.20	0.0090	0.585053
0.30	0.0330	0.651566
0.40	0.0594	0.687057
0.50	0.0803	0.687458
0.70	0.1296	0.654218
1.00	0.1824	0.533744
2.00	0.3192	0.253630
4.00	0.4027	0.127152

Observation 3.2

This is the core of the paper. The correlation peaks at **0.687458** for $\sigma = 0.50$ —not at either end but in the middle: 0 at $\sigma = 0$ and 0.127152 at $\sigma = 4$.

Observation 3.3

Detection rate and correlation are different things. The rate rises monotonically with σ , reaching 0.4027 at $\sigma = 4$ —while the correlation has fallen. Detecting a great deal is worthless if it is unrelated to the signal.

Observation 3.4

That distinction is the content of “improve”. “Detect often” and “represent the signal faithfully” are not the same—what noise helps is the latter, and only in a middling amount.

Observation 3.5

Near the optimum the curve is flat. 0.687057 at $\sigma = 0.40$ and 0.687458 at $\sigma = 0.50$ —a difference of only 0.000401. The location of the optimum is not sharp.

Observation 3.6

The climb from $\sigma = 0.10$ to 0.20, by contrast, is steep. From 0.244618 to 0.585053, a factor of 2.39—there is a region where a little noise does a great deal.

4. When Noise Becomes Unnecessary

Observation 4.1

Raise the amplitude above the threshold and the story changes. With $A > \theta$ the threshold is crossed over part of the cycle even at zero noise—in the scan of this paper, at 17 of 64 points.

Observation 4.2

This is the control. Noise helps only when the signal does not reach the threshold—when it does, adding noise merely makes things worse.

Observation 4.3

So the shape of the claim needs care. Not “noise is useful” but “for a signal beneath the threshold, the right amount of noise is useful”—drop the condition and it becomes false.

Observation 4.4

It does not happen in a linear system. Without a threshold the output is proportional to the input and noise simply adds—nonlinearity is a necessary condition for this phenomenon.

Observation 4.5

To be stated honestly——this paper does not show that stochastic resonance occurs in real systems. What is shown is only that the correlation forms a hill on a model stated explicitly——biological measurements are left to the literature.

5. The Separator: Is There a Threshold?

Observation 5.1

The separator——whether the system has a threshold; not any property of the noise.

	Detection at zero noise	Optimal noise	Adding noise
Threshold, $A < \theta$	0.0000	$\sigma = 0.50$	helps (up to the middle)
Threshold, $A > \theta$	possible	$\sigma = 0$	hurts
No threshold (linear)	possible	$\sigma = 0$	hurts

Observation 5.2

Only one of the three rows is the exception. “Reduce the noise” is right in the other two——the general rule fails only when one condition is met.

Observation 5.3

The separator can be moved. Lower the threshold and $A > \theta$, and noise becomes unnecessary——so “add noise” is a substitute for “lower the threshold”. If it can be lowered, lower it.

Observation 5.4

Sometimes it cannot. In a detector whose threshold is physically fixed, the noise is the easier thing to move—that is where stochastic resonance is used.

Observation 5.5

To be stated honestly—0.687458 and $\sigma = 0.50$ are values on a model with $\theta = 1.0$ and $A = 0.7$ stated explicitly. Another ratio gives other numbers—what does not move is only that the optimum lies in the middle rather than at an end.

Observation 5.6

One last thing—before a hill-shaped function, “more is better” and “less is better” are both wrong. Whether the optimum sits at an end or in the middle is a question of monotonicity—assume monotonicity and head for an end, and one arrives on the far side of the hill.

6. The Census and the Close

1. $A = 0.7 < \theta = 1.0$ (Observation 1.1): the signal alone does not reach.
2. Detection rate 0.0000 at $\sigma = 0$ (Calculation 3.1): the event is empty.
3. Correlation 0.687458 at $\sigma = 0.50$ (Calculation 3.1): the maximum.
4. 0.127152 at $\sigma = 4$ (Calculation 3.1): too much falls off.
5. Detection rate rises monotonically to 0.4027 (Observation 3.3): a different thing.
6. 0.40 and 0.50 differ by 0.000401 (Observation 3.5): the optimum is not sharp.
7. $0.10 \rightarrow 0.20$ is a factor of 2.39 (Observation 3.6): the climb is steep.
8. $A > \theta$ crosses at 17 of 64 points (Observation 4.1): no noise needed.
9. It does not happen in a linear system (Observation 4.4): nonlinearity is necessary.
10. Lower the threshold if you can (Observation 5.3): noise is the substitute.

Closing: One is taught that noise is to be reduced—and in many settings that is right. But in a system with a threshold it can fail—against a threshold of 1.0, a signal of amplitude 0.7 is never detected without noise. At $\sigma = 0$ the detection rate is 0.0000 and the correlation 0.000000—not because the probability is small but because the

event is empty. The cleanest condition is the condition in which nothing at all is seen. Adding noise raises the correlation—0.244618 at $\sigma = 0.10$, 0.585053 at 0.20 (a factor of 2.39), and a maximum of **0.687458** at $\sigma = 0.50$. Adding more lowers it, down to 0.127152 at $\sigma = 4$. The optimum is not at either end but in the middle. But detecting and representing are different things—the detection rate rises monotonically with σ and reaches 0.4027 at $\sigma = 4$, by which point the correlation has fallen. Detecting a great deal is worthless if it is unrelated to the signal. What noise helps is the latter, and only in a middling amount. Drop the condition and the claim becomes false—raise the amplitude above the threshold and it is crossed at 17 of 64 points even at zero noise, and adding noise there merely makes things worse. So what can be said is not “noise is useful” but “for a signal beneath the threshold, the right amount of noise is useful”. Without a threshold the output is proportional to the input, and nonlinearity is a necessary condition for the phenomenon. One thing separates them—whether the system has a threshold; not any property of the noise. And the separator can be moved—lower the threshold and $A > \theta$ and noise becomes unnecessary, so “add noise” is a substitute for “lower the threshold”. If it can be lowered, lower it; only where the threshold is physically fixed is the noise the easier thing to move. To be said honestly—this paper does not show that stochastic resonance occurs in real systems; what is shown is only that the correlation forms a hill on a model stated explicitly. The 0.687458 and $\sigma = 0.50$ hold for $\theta = 1.0$ and $A = 0.7$, and what does not move is only that the optimum lies in the middle. One last thing—before a hill-shaped function, “more is better” and “less is better” are both wrong. Assume monotonicity and head for an end, and one arrives on the far side of the hill.

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