

# The Coefficient of Restitution Is Not a Material Constant — And the Empirical Law Points at Its Own Breaking Speed

— A hundredfold speed multiplies  $e$  by **0.31622777** — The same ball gives **0.6204** dropped from 1 m and **0.3489** from 100 m — Extended to low speed the law predicts  $e > 1$  below **0.6561** m/s, which is a lower bound the model states about itself — [Paper 355]

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— *From thinking to syncing.* —

## Abstract

The coefficient of restitution appears in tables of material properties. This paper asks whether it is a constant—the answer is that it is not, and that the empirical law for its speed dependence names its own limit of validity. No new mathematical theorem and no new law is claimed. First,  $e$  falls roughly as the  $-1/4$  power of impact speed (Section 2). Second, a hundredfold speed multiplies  $e$  by **0.31622777** (Section 2). Third, the same ball gives **0.6204** dropped from 1 m and **0.3489** from 100 m (Section 3). Fourth, this is the core of the paper. Extended to low speed the law predicts  $e > 1$  below **0.6561** m/s (Section 4). Fifth, since  $e > 1$  violates energy conservation, this is a lower bound the model states about itself (Section 4). Sixth, the separator is the exponent; set it to 0 and both the speed dependence and the breakdown vanish together (Section 5).

Scope of this paper (scope note): No new mathematical theorem and no new law is claimed—the speed dependence of restitution, Hertzian contact, and plastic deformation are all standard. We do not build contact mechanics—the derivation of Hertz theory and viscoelastic constitutive laws are not entered. They are named and no more. We make no measurement—no  $e$  of any particular material is measured. The numbers are on an empirical law stated explicitly. We do not derive the exponent—the  $-1/4$  is not derived from first principles. It is posited empirically and only its consequences are traced. Air drag is not treated—a vacuum is assumed when converting drop height to speed. Spin is not treated—oblique impact and tangential restitution are not the subject. Relation to earlier papers: Paper 352 showed a transition temperature moving with the speed of measurement—there too a quantity wearing the face of a constant was conditional. Here the condition is impact speed. Paper 354 showed length ceasing to be the quantity that measures a curve—there too the trouble was on the ruler's side. Paper 300 showed whether two things share a root is decidable—by that test,  $e$  and a genuine material constant such as density have distinct roots. Paper 353 showed a claim turning false when its condition is dropped—here dropping the condition yields the physically impossible prediction  $e > 1$ . What is added is giving the speed dependence as numbers at nine points, showing that two decades of drop height multiply  $e$  by 0.5623, solving for the 0.6561 m/s at which the law predicts  $e = 1$  and checking both sides of it, using an exponent of 0 as a control to confirm the formula is doing the work, and putting the separator on the exponent.

## 1. Notation and the Question

Setting: Posit the speed dependence of restitution empirically in the form

$$e(v) = e_0 \left( \frac{v}{v_0} \right)^{-1/4}, \quad e_0 = 0.90, \quad v_0 = 1.0 \text{ m/s}$$

### Observation 1.1

$e$  appears in tables as a constant. A steel ball is 0.9, a tennis ball 0.7, and so on—listed in the same format as density and elastic modulus.

### Observation 1.2

But  $e$  contains a measurement condition. The value changes with the height it was dropped from—density does not change with drop height.

Observation 1.3

This paper does not clip the value.No cap at  $e = 1$  is imposed——so that where the model breaks can be seen.

Question: Is  $e$  a material constant?

2. The Speed Dependence as Numbers

Calculation 2.1:  $e$  against impact speed (computed in this paper).

| Impact speed [m/s] | $e$    | Height ratio $e^2$ | Physically possible  |
|--------------------|--------|--------------------|----------------------|
| 0.1000             | 1.6005 | 2.561445           | no (model breakdown) |
| 0.5000             | 1.0703 | 1.145513           | no (model breakdown) |
| 0.6561             | 1.0000 | 1.000000           | the boundary         |
| 1.0000             | 0.9000 | 0.810000           | yes                  |
| 2.0000             | 0.7568 | 0.572756           | yes                  |
| 5.0000             | 0.6019 | 0.362243           | yes                  |
| 10.0000            | 0.5061 | 0.256144           | yes                  |
| 50.0000            | 0.3385 | 0.114551           | yes                  |
| 100.0000           | 0.2846 | 0.081000           | yes                  |

Observation 2.2

Faster impacts give lower  $e$ .0.9000 at 1 m/s and 0.5061 at 10 m/s——the direction is constant.

### Observation 2.3

A hundredfold speed multiplies  $e$  by **0.31622777**. Because the power is  $-1/4$ ,  $100^{-1/4} = 10^{-1/2}$ ——down to under a third.

### Observation 2.4

Seen through  $e^2$  the difference is larger still. The bounce-height ratio is  $e^2$ ——0.810000 at 1 m/s and 0.081000 at 100 m/s: exactly one tenth.

### Observation 2.5

Why it falls. Faster impacts lose a greater fraction to plastic deformation and internal friction——a perfectly elastic collision is an approximation that holds only in the slow limit.

## 3. One Ball, Two Heights, Two Values

Calculation 3.1: Impact speed from drop height, and the resulting  $e$  (computed in this paper, vacuum assumed).

| Drop height [m] | Impact speed [m/s] | $e$           |
|-----------------|--------------------|---------------|
| 1               | 4.4287             | <b>0.6204</b> |
| 100             | 44.2869            | <b>0.3489</b> |

### Observation 3.2

The same ball has  $e = 0.6204$  and  $e = 0.3489$ . The ratio is 0.5623——not one thing about the material has changed, and the value falls by close to 44 per cent.

### Observation 3.3

So a table entry needs its conditions. Writing “this ball has  $e = 0.62$ ” does not say which measurement it was—the same shape as  $T_g$  requiring its measurement conditions in Paper 352.

### Observation 3.4

Density has no such need. It does not change with the height you drop it from—whether the condition may be omitted is what separates the character of the two quantities.

## 4. The Law Points at Its Own Breaking Speed

Calculation 4.1: Solve for the speed at which the law predicts  $e = 1$  (computed in this paper).

| Quantity                                            | Value               |
|-----------------------------------------------------|---------------------|
| Speed with $e = 1$ , $v_{\text{break}} = v_0 e_0^4$ | <b>0.656100</b> m/s |
| $e$ at $0.99v_{\text{break}}$                       | 1.002516            |
| $e$ at $1.01v_{\text{break}}$                       | 0.997516            |
| $e$ at $v = 0.1$ m/s                                | <b>1.600451</b>     |

### Observation 4.2

This is the core of the paper. Below 0.6561 m/s this empirical law predicts  $e > 1$ —at  $v = 0.1$  m/s it gives 1.600451, so the ball would rebound to 2.561445 times the height it was dropped from.

### Observation 4.3

That violates energy conservation. A collision does not add energy—so  $e > 1$  is not “a regime not yet measured” but a regime in which the model is wrong.

#### Observation 4.4

The sign flips across the boundary. 1.002516 at 0.99 times and 0.997516 at 1.01 times——straddling 1. The boundary is sharp.

#### Observation 4.5

There is use in this. The lower bound of validity is not supplied from outside but read off the formula itself——the model answers the question of how far it may be extrapolated.

#### Observation 4.6

To be stated honestly——that real slow impacts approach  $e = 1$  is itself correct. Slower collisions are more nearly elastic——what is wrong is only the part above 1; the direction is right. The 0.6561 m/s is a sign that beyond it another formula is needed.

## 5. The Separator: The Exponent

#### Observation 5.1

The separator——whether the exponent on speed is 0 or not.

| Exponent | Speed dependence            | A material constant | Breaking speed |
|----------|-----------------------------|---------------------|----------------|
| 0        | none ( $e = 0.9000$ always) | yes                 | none           |
| -1/4     | yes                         | no                  | 0.6561 m/s     |

#### Observation 5.2

Set the exponent to 0 and two things vanish at once. The speed dependence goes, and so does the  $e > 1$  breakdown——both come out of the same single exponent.

### Observation 5.3

This is the control. At exponent 0,  $e = 0.9000$  at  $v = 50$  and at  $v = 0.1$  alike—confirming that the speed dependence comes from the formula.

### Observation 5.4

The separator can be moved. Soften the exponent to  $-1/8$  and the breaking speed falls; strengthen it to  $-1/2$  and it rises—breadth of validity and strength of speed dependence are two faces of one number.

### Observation 5.5

To be stated honestly—0.6561 m/s and 0.3489 are values on an empirical law with  $e_0 = 0.90$  and exponent  $-1/4$  stated explicitly. Another material gives other numbers—what does not move is only the structure: a power law with speed dependence must cross  $e = 1$  somewhere.

### Observation 5.6

One last thing—when extrapolating an empirical law yields a physically impossible value, that is not a failure but information. The  $e > 1$  tells you how far the formula is valid—looking for the point at which a model predicts the impossible is one way of measuring its range.

## 6. The Census and the Close

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1.  $e$  falls as the  $-1/4$  power (Section 1): **posited empirically.**
2. 0.9000 at 1 m/s, 0.5061 at 10 m/s (Calculation 2.1): **a constant direction.**
3. Hundredfold speed gives 0.31622777 (Observation 2.3):  $100^{-1/4} = 10^{-1/2}$ .
4.  $e^2$  from 0.810000 to 0.081000 (Observation 2.4): **exactly one tenth.**
5. 0.6204 from 1 m, 0.3489 from 100 m (Calculation 3.1): **ratio 0.5623.**
6.  $e = 1$  at 0.656100 m/s (Calculation 4.1): **the model's lower bound.**
7.  $e = 1.600451$  at  $v = 0.1$  m/s (Calculation 4.1): **2.561445 times the height.**
8. 1.002516 and 0.997516 across the boundary (Observation 4.4): **straddling 1.**

9. Exponent 0 gives  $e = 0.9000$  always (Observation 5.3): the control.

10. The direction is right (Observation 4.6): only the part above 1 is wrong.

Closing: The coefficient of restitution appears in tables as a constant—a steel ball is 0.9, a tennis ball 0.7, listed in the same format as density and elastic modulus. But  $e$  contains a measurement condition—posit  $e(v) = e_0(v/v_0)^{-1/4}$  and it falls in a constant direction, 0.9000 at 1 m/s and 0.5061 at 10 m/s, with a hundredfold speed multiplying  $e$  by **0.31622777**. Seen through the bounce-height ratio  $e^2$  the difference is larger still: 0.810000 down to 0.081000, exactly one tenth. One ball gives two values at two heights—converting drop height to speed in vacuum, 1 m gives  $e = 0.6204$  and 100 m gives  $e = 0.3489$  (ratio 0.5623). Not one thing about the material has changed, and the value falls by close to 44 per cent. Density does not change with drop height; whether the condition may be omitted is what separates the character of the two quantities. And the law points at its own breaking speed—extended to low speed without clipping, it predicts  $e > 1$  below **0.656100** m/s. At  $v = 0.1$  m/s it gives  $e = 1.600451$ , so the ball would rebound to 2.561445 times the height it was dropped from. A collision does not add energy, so this is not a regime not yet measured but a regime in which the model is wrong. Across the boundary the values are 1.002516 and 0.997516, straddling 1. The direction, though, is right—slower collisions really are more nearly elastic, and what is wrong is only the part above 1. The 0.6561 m/s is a sign that beyond it another formula is needed. One thing separates them—whether the exponent on speed is 0 or not. Set it to 0 and two things vanish at once—the speed dependence goes, and so does the  $e > 1$  breakdown; both come out of the same single exponent. Soften it to  $-1/8$  and the breaking speed falls, strengthen it to  $-1/2$  and it rises: breadth of validity and strength of speed dependence are two faces of one number. To be said honestly—0.6561 m/s and 0.3489 hold for  $e_0 = 0.90$  and exponent  $-1/4$ , and another material gives other numbers. What does not move is only the structure: a power law with speed dependence must cross  $e = 1$  somewhere. One last thing—when extrapolating an empirical law yields a physically impossible value, that is not a failure but information. The  $e > 1$  tells you how far the formula is valid, and looking for the point at which a model predicts the impossible is one way of measuring its range.

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