

Maximum Power and Maximum Efficiency Are Not the Same Point

— At $R = r$, where the power is greatest, the efficiency is exactly **50** per cent — Efficiency rises monotonically with R to **99.0099** per cent, where the power is only **0.0392** of its maximum — “Optimal” names a different place depending on which is maximised — [Paper 357]

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September 2026

— *From thinking to syncing.* —

Abstract

One is taught that matching is optimal. This paper asks optimal for what—the answer is optimal for power, and not for efficiency. No new mathematical theorem and no new law is claimed. First, with an emf of 12 V and internal resistance $0.5\ \Omega$, the load power peaks at **72.0000 W** at $R = r = 0.5\ \Omega$ (Section 2). Second, this is the core of the paper. The efficiency there is exactly **50** per cent (Section 2). Third, efficiency rises monotonically as R grows (Section 3). Fourth, at $R = 50\ \Omega$ the efficiency is **99.0099** per cent and the power is only **0.0392** of its maximum (Section 3). Fifth, at $R = 10r$ the efficiency is **90.9091** per cent with **0.3306** of maximum power — the practical compromise (Section 4). Sixth, the separator is what is being maximised; nothing on the circuit’s side changes at all (Section 5).

Scope of this paper (scope note): No new mathematical theorem and no new law is claimed—the maximum power transfer theorem, internal resistance, and matching are all standard. We do not build circuit theory—complex impedance matching in AC and distributed-parameter circuits are not entered. They are named and no more. We do not build batteries—the electrochemical origin of internal resistance and its change on discharge are not the subject. r is treated as a constant. Heat is not treated—the temperature rise from internal dissipation and its effect on r are not the subject. We design no equipment—no design guidance for particular supply circuits is given. Nonlinearity is not treated—the load is a pure resistance; constant-power loads are not the subject. Relation to earlier papers: Paper 353 showed an optimum lying in the middle rather than at an end—there there was one hill. Here a hill and a monotone rise exist at the same time, and the optimum splits in two. Paper 356 showed what flows and what works to be different quantities—there too one circuit held two quantities that parted. Paper 300 showed whether two things share a root is decidable—by that test, “maximum power” and “maximum efficiency” have distinct roots. Paper 313 showed one word used in several senses—“optimal” has the same shape. What is added is giving power and efficiency together as numbers at eight points, showing the efficiency at the maximum power point to be exactly 50 per cent, showing the power at the highest-efficiency point to be 0.0392 of maximum, placing $R = 10r$ numerically as the compromise, and putting the separator on what is being maximised.

1. Notation and the Question

Setting: Connect a load R to a source of emf $E = 12.0$ V and internal resistance $r = 0.5$ Ω .

$$I = \frac{E}{R + r}, \quad P_R = I^2 R, \quad \eta = \frac{R}{R + r}$$

Observation 1.1

Lowering R raises the current. But it also raises the internal voltage drop— $P_R = I^2 R$ falls whether R is too small or too large.

Observation 1.2

Efficiency has a different shape. $\eta = R/(R + r)$ is monotone increasing in R —it has no hill.

Observation 1.3

That difference of shape is the whole of this paper. One is a hill, the other a monotone rise—their maxima cannot possibly coincide.

Question: An “optimal load” is optimal for what?

2. Power and Efficiency Side by Side

Calculation 2.1: Current, power and efficiency against load (computed in this paper).

Load R [Ω]	Current [A]	Load power [W]	Efficiency [per cent]
0.10	20.0000	40.0000	16.6667
0.25	16.0000	64.0000	33.3333
0.50	12.0000	72.0000	50.0000
1.00	8.0000	64.0000	66.6667
2.00	4.8000	46.0800	80.0000
5.00	2.1818	23.8017	90.9091
10.00	1.1429	13.0612	95.2381
50.00	0.2376	2.8233	99.0099

Observation 2.2

This is the core of the paper. At $R = 0.50 \Omega$, where the power peaks at 72.0000 W, the efficiency is exactly **50** per cent—not approximately: at $R = r$, $\eta = R/(2R) = 1/2$ exactly.

Observation 2.3

So at maximum power the source consumes half of it itself. The internal resistance dissipates as much as the load—the point called optimal is simultaneously the point at which half is thrown away.

Observation 2.4

The two sides of the hill are not symmetric. $R = 0.25$ and $R = 1.00$ both give 64.0000 W, but the efficiencies are 33.3333 and 66.6667—for the same power, the larger R is always the better choice.

Observation 2.5

That asymmetry is a practical rule. If you leave the matched point, always leave it on the high- R side—the loss in power is the same and the efficiency differs by a factor of two.

3. Efficiency Peaks at the End

Observation 3.1

Efficiency rises monotonically with R . From 16.6667 to 99.0099 per cent—there is no maximum in between.

Observation 3.2

But at 99.0099 per cent the power is only 2.8233 W. Against a maximum of 72.0000 W, a factor of **0.0392**—almost nothing is being extracted.

Observation 3.3

Raising efficiency is the same operation as lowering power. Raising R lowers the current—two goals pulling one knob in opposite directions.

Observation 3.4

In the limit the efficiency is 100 per cent and the power is 0. $R \rightarrow \infty$ gives $\eta \rightarrow 1$ and $P_R \rightarrow 0$ —the degenerate answer that the most efficient use is not to use it at all.

Observation 3.5

Set the internal resistance to zero and the conflict disappears. With $r = 0$ the efficiency is 100.0000 per cent for every R —so the source of the conflict is r itself. This is a check that the formula is doing the work.

4. The Compromise Sits Around Tenfold

Calculation 4.1: Both quantities at $R = 10r$ (computed in this paper).

Point	Efficiency [per cent]	Power (relative to maximum)
$R = r$ (maximum power)	50.0000	1.0000
$R = 10r$	90.9091	0.3306
$R = 100r$	99.0099	0.0392

Observation 4.2

$R = 10r$ raises efficiency from 50 to 90.9091 and drops power to 0.3306. Efficiency by a factor of 1.82, power to a third—whether that trade is worth taking depends on which one is weighted.

Observation 4.3

Another tenfold buys only 8.1 points of efficiency. From 90.9091 to 99.0099—while power falls from 0.3306 to 0.0392, to less than an eighth. Less is gained and more is lost.

Observation 4.4

So in practice something near $R = 10r$ is used. It is the rule of thumb when a battery is the source——matching is reserved for cases in which the power itself is the goal, such as radio reception.

Observation 4.5

To be stated honestly——there is nothing theoretically special about $R = 10r$. The exchange point between two monotone quantities moves with how they are weighted——what is shown here is the numbers of the options, not the answer.

5. The Separator: What Is Being Maximised

Observation 5.1

The separator——the objective being maximised; nothing on the circuit's side changes at all.

Maximised	Optimal R	Efficiency	Power
Load power	$R = r$	50.0000	72.0000 W
Efficiency	R as large as possible	approaching 100	approaching 0

Observation 5.2

Same battery, same internal resistance, same formulae. Only the choice of what to maximise has changed——and with it the optimal R jumps from $0.5\ \Omega$ to infinity.

Observation 5.3

Here is the difference from Paper 353. There a single hill put the optimum in the middle——here a hill and a monotone rise coexist and the optimum splits, so that “optimal” names two things.

Observation 5.4

The separator can be moved. Take the objective $P_R\eta$ (power times efficiency) and the optimum is exactly $R = 2r$, that is $1.0\ \Omega$ — a third point — unless what is being maximised is written down, “optimal” is undetermined.

Observation 5.5

To be stated honestly — $72.0000\ \text{W}$ and 99.0099 per cent are values at $E = 12\ \text{V}$ and $r = 0.5\ \Omega$ stated explicitly. Another source gives other numbers — what does not move is only that the efficiency at the maximum power point is exactly 50 per cent, which depends on neither E nor r .

Observation 5.6

One last thing — the phrase “we optimised it” has no content until the objective function is written down. One circuit holds two optima, and they lie a hundredfold apart in R — before doubting the result of an optimisation, look at what was maximised.

6. The Census and the Close

1. Maximum power $72.0000\ \text{W}$ at $R = r = 0.50\ \Omega$ (Calculation 2.1): the top of the hill.
2. Efficiency there is exactly 50 per cent (Observation 2.2): independent of E and r .
3. $R = 0.25$ and $R = 1.00$ both give $64.0000\ \text{W}$ (Observation 2.4): efficiencies 33.3333 and 66.6667 .
4. Leave on the high side (Observation 2.5): same power lost, twice the efficiency.
5. Efficiency rises monotonically to 99.0099 (Observation 3.1): no interior maximum.
6. Power there is 0.0392 of maximum (Observation 3.2): $2.8233\ \text{W}$.
7. $R \rightarrow \infty$ gives 100 per cent and $0\ \text{W}$ (Observation 3.4): the degenerate answer.
8. $r = 0$ gives 100.0000 always (Observation 3.5): the conflict’s source is r .
9. $R = 10r$ gives 90.9091 and 0.3306 (Calculation 4.1): the practical compromise.
10. Another tenfold: $+8.1$ points, under an eighth the power (Observation 4.3): the exchange rate worsens.

Closing: One is taught that matching is optimal — but until “optimal for what” is written down, the word has no content. With an emf of $12\ \text{V}$ and internal resistance $0.5\ \Omega$, the load power peaks at $72.0000\ \text{W}$ at $R = r = 0.5\ \Omega$

—and the efficiency there is exactly **50** per cent: not approximately, since at $R = r$, $\eta = R/(2R) = 1/2$ exactly. The point called optimal is simultaneously the point at which half is thrown away. Efficiency has a different shape — $\eta = R/(R + r)$ is monotone, with no maximum in between. At $R = 50\ \Omega$ it reaches **99.0099** per cent, where the power is **2.8233** W — only **0.0392** of the maximum. In the limit it gives 100 per cent efficiency at zero power: the degenerate answer that the most efficient use is not to use it at all. The two sides of the hill are not symmetric — $R = 0.25$ and $R = 1.00$ both give **64.0000** W, but the efficiencies are **33.3333** and **66.6667**. So if you leave the matched point, always leave it on the high- R side: the loss in power is the same and the efficiency differs by a factor of two. The compromise sits around tenfold — $R = 10r$ gives **90.9091** per cent efficiency at **0.3306** of maximum power. Another tenfold buys only 8.1 points while power falls to less than an eighth: less is gained and more is lost. There is nothing theoretically special about $R = 10r$; what is shown is the numbers of the options, not the answer. One thing separates them — what is being maximised; nothing on the circuit's side changes at all. Same battery, same internal resistance, same formulae — only the objective changed, and with it the optimal R jumps from $0.5\ \Omega$ to infinity. Set $r = 0$ and the efficiency is 100.0000 per cent for every R , so the source of the conflict is r itself. To be said honestly — **72.0000** W and **99.0099** per cent hold at $E = 12\text{ V}$ and $r = 0.5\ \Omega$, and another source gives other numbers. What does not move is only that the efficiency at the maximum power point is exactly 50 per cent, which depends on neither E nor r . One last thing — the phrase “we optimised it” has no content until the objective function is written down. One circuit holds two optima lying a hundredfold apart in R — and a third, at exactly $R = 2r$, if the objective is power times efficiency. Before doubting the result of an optimisation, look at what was maximised.

References

1. Jacobi, M. H. (1840). *Über die Anwendung der Elektromagnetismus zum Bewegen von Maschinen*. Potsdam.
2. Thévenin, L. (1883). Sur un nouveau théorème d'électricité dynamique. *Comptes Rendus de l'Académie des Sciences*.
3. Desoer, C. A., Kuh, E. S. (1969). *Basic Circuit Theory*. New York: McGraw-Hill.
4. Horowitz, P., Hill, W. (2015). *The Art of Electronics* (3rd ed.). Cambridge: Cambridge University Press.
5. Harrison, M. J. (2013). Physical collapse of the maximum power transfer theorem. *American Journal of Physics*.
6. Yamagishi, Y. (2026). *The Three Hundredth Census: Whether Two Things Share a Root Is Decidable* [Paper 300]. Zenodo. 10.5281/zenodo.22171914
7. Yamagishi, Y. (2026). *This Corpus Has Used “Synchrony” in Four Senses, and Only the Fifth Has a Threshold* [Paper 313]. Zenodo. 10.5281/zenodo.22174953
8. Yamagishi, Y. (2026). *Adding Noise Can Improve Detection — But “More Is Better” Is Not the Rule* [Paper 353]. Zenodo. 10.5281/zenodo.22696835
9. Yamagishi, Y. (2026). *The Current Flows and Does No Work* [Paper 356]. Zenodo. 10.5281/zenodo.22697124

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