

The π in Stokes' 6π Is a Solid Angle

— What the Drag Keeps and Loses When the Dimension Moves — [Paper 36]

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— *From thinking to syncing.* —

ABSTRACT

In the viscous drag on a sphere, $F = 6\pi\mu Ua$, a π sits with a perfectly innocent air. This paper turns the corpus's own test on it (Paper 31: when the dimension d is moved, does that π move?). **NO NEW FLUID MECHANICS IS CLAIMED. First, Stokes flow was solved in d dimensions from scratch.** Taking a superposition of a Stokeslet and a potential dipole, the residual of the Stokes equation is identically 0, so is the incompressibility condition, and the no-slip condition is met, checked for $d = 3, \dots, 7$ (Section 2). Second, and this is the core. The drag has a closed form:

$$F = \frac{d(d-2)}{d-1} \text{Vol}(S^{d-1}) \mu U a^{d-2}.$$

At $d = 3, 4, 5, 6, 7$ the coefficients came out as $3/2, 8/3, 15/4, 24/5, 35/6$, and all five agree with this single expression (computed here) (Section 3). Third, the π of 6π is therefore a solid angle. At $d = 3$ one has $\frac{3}{2} \times 4\pi = 6\pi$, and that 4π is $\text{Vol}(S^2)$ itself. Move d and this π moves as $\pi^{d/2}/\Gamma(d/2)$ — by the criterion of Paper 31, it is a solid angle and not a period (Section 4). Fourth, the “6” does not survive. It is merely the value of $\frac{d(d-2)}{d-1}$ times 4 at $d = 3$, and one dimension up it becomes $8/3$ times $2\pi^2$ (Section 4). Fifth, the coefficient carries $(d-2)$, which vanishes exactly at $d = 2$. Stokes' paradox is thus written into the formula itself. Indeed at $d = 2$ the only solution meeting the no-slip condition is the fluid at rest everywhere, and the uniform stream at infinity is lost (constructed here) (Section 5).

Scope of this paper: NO NEW FLUID MECHANICS IS CLAIMED. Stokes flow, the d -dimensional Stokeslet and Stokes' paradox (Stokes 1851) are all standard. What is added is turning the corpus's dimension test on 6π , deriving the closed form independently and checking it in five dimensions, writing the address of the π as a solid angle and of the “6” as not one, and noting that the paradox appears as a zero of the coefficient at $(d-2)$.

1. Notation and position

Let μ be the viscosity, U the uniform stream at infinity, a the radius of the sphere and d the spatial dimension.

$\text{Vol}(S^{d-1}) = 2\pi^{d/2}/\Gamma(d/2)$ is the area of the unit sphere in $\mathbb{R}^d$. The Stokes equations are $\nabla p = \mu \nabla^2 u$ and $\nabla \cdot u = 0$.

Position: Paper 31 used “does it move with d ?” to fix the address of a π , and Paper 130 showed that the 4π of Biot–Savart moves with the dimension as $(d-2)\text{Vol}(S^{d-1})$. This paper applies both in a place the corpus has never touched: viscous flow.

2. Solving Stokes flow in d dimensions

Construction 2.1: Write the solution matching U at infinity as a superposition of a Stokeslet G_{i1} and a potential dipole D_{i1} :

$$u_i = U(\delta_{i1} + A G_{i1} + B D_{i1}), \quad p = C \mu U x_1 r^{-d},$$

$$G_{i1} = \delta_{i1} r^{2-d} + (d-2) x_i x_1 r^{-d}, \quad D_{i1} = \delta_{i1} r^{-d} - d x_i x_1 r^{-d-2}.$$

Verification 2.2: For each of $d = 3, 4, 5, 6, 7$,

$\nabla \cdot u$	identically 0
$\nabla p - \mu \nabla^2 u$	identically 0 (which fixes C against A)
$u _{r=a}$ (at the pole and at the equator)	0 (which fixes A and B)

(QED)

Observation 2.3

The $(d-2)$ was not put in by hand. It comes from the fundamental solution of the Laplace equation being r^{2-d} , which the Stokeslet inherits. It is the same $(d-2)$ as in the $(d-2)\text{Vol}(S^{d-1})$ of Paper 130.

3. The core: a closed form for the drag

Verification 3.1: Integrating the traction over the sphere,

d	drag F	$\text{Vol}(S^{d-1})$	$F/(\mu U a^{d-2})$	ratio
3	$6\pi\mu U a$	4π	6π	$3/2$
4	$\frac{16}{3}\pi^2\mu U a^2$	$2\pi^2$	$\frac{16}{3}\pi^2$	$8/3$
5	$10\pi^2\mu U a^3$	$\frac{8}{3}\pi^2$	$10\pi^2$	$15/4$
6	$\frac{24}{5}\pi^3\mu U a^4$	π^3	$\frac{24}{5}\pi^3$	$24/5$
7	$\frac{56}{9}\pi^3\mu U a^5$	$\frac{16}{15}\pi^3$	$\frac{56}{9}\pi^3$	$35/6$

All five of the ratios on the right agree with

$$F = \frac{d(d-2)}{d-1} \text{Vol}(S^{d-1}) \mu U a^{d-2}$$

while $d/2$, $(d-1)/2$ and $d(d-2)/(2(d-1))$ each fail on some of the five; only the expression above fits them all (checked here). (QED)

Observation 3.2

Putting $d = 3$ gives $\frac{3-1}{2} \times 4\pi = 6\pi$, reproducing the textbook value.

4. The π is a solid angle; the “6” is not

Observation 4.1

Paper 31's criterion is “does it move with d ?” The π here sits inside $\text{Vol}(S^{d-1}) = 2\pi^{d/2}/\Gamma(d/2)$ and moves with d right down to its exponent. It is therefore a solid-angle π , not the period π of a series like Ramanujan's.

Observation 4.2

The “6”, by contrast, does not cross dimensions. It is the product $\frac{3-1}{2} \times 4$, which holds only at $d = 3$. At $d = 4$ it becomes $\frac{4-2}{3} \times 2\pi^2$, that is $\frac{16}{3}\pi^2$.

Observation 4.3

So 6π must not be handled as one lump. The π is geometry, a fact; the 6 is a number that appears only after $d = 3$ has been substituted — this is the distinction for which the corpus has said, since Paper 43, “leave it as a letter”.

5. $d = 2$: the paradox is a zero of the coefficient

Observation 5.1

The coefficient $\frac{d(d-2)}{d-1}$ is exactly 0 at $d = 2$. That same $(d - 2)$ stood in front of $x_i x_1$ in the Stokeslet of Section 2. The drag formula itself thus announces, as a zero, that something breaks at $d = 2$.

Verification 5.2: At $d = 2$ one has $r^{2-d} = r^0 = 1$, so the Stokeslet does not decay. Imposing no-slip on what is left,

condition at the pole	$AU - BU/a^2 + U = 0$
condition at the equator	$AU + BU/a^2 + U = 0$
the only common solution	$A = -1, B = 0$
its far field	$u \rightarrow 0$ (where it must be U)

(QED)

Observation 5.3

The only solution satisfying no-slip is the fluid at rest everywhere. The uniform stream at infinity is lost. This is Stokes' paradox (Stokes 1851), and the paper exhibits it rather than asserting it.

Observation 5.4

The true two-dimensional Stokeslet is logarithmic, and a logarithm cannot be made to vanish at infinity. It is the same obstruction seen from another angle.

6. The census

Observation 6.1

quantity	when d moves	address
the π inside 6π	moves as $\pi^{d/2}/\Gamma(d/2)$	solid angle (geometry, a fact)
the coefficient $\frac{d(d-2)}{d-1}$	moves as a rational function	the boundary-value solution
the “6”	does not survive	a number obtained after setting $d = 3$
existence of a solution	lost at $d = 2$	dimension-selective
the power $d - 2$ of a	moves with the dimension	dimensional analysis

Observation 6.2

What selects a dimension is not the value of the drag but whether the problem has a solution at all (in the sense of Paper 114, this row alone is dimension-selective).

7. Conclusion

1. Stokes flow was solved in d dimensions from scratch (Ver. 2.2): the residual is identically 0.
2. The drag is $F = \frac{d(d-2)}{d-1} \text{Vol}(S^{d-1})\mu U a^{d-2}$ (Ver. 3.1): five dimensions agree.
3. The π of 6π is a solid angle (Obs. 4.1): $\frac{3}{2} \times 4\pi$ at $d = 3$.
4. The “6” does not cross dimensions (Obs. 4.2).
5. At $d = 2$ the paradox appears as a zero of the coefficient (Obs. 5.1, Ver. 5.2).
6. What is dimension-selective is the existence of a solution, not the value (Obs. 6.2).

Closing: Inside the short symbol 6π , the π and the 6 do not hold the same rank. The π is geometry brought in by a sphere; the 6 is a record of our living at $d = 3$. And one dimension down, the drag does not take a different value: the question itself loses its answer.

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