

The Usual Explanation of the Brazil-Nut Effect Fails on the Size of the Gap

— The gap between three **20** mm spheres admits only **3.0940** mm — For “small grains fall through the gaps” to work, the diameter ratio must be at least **6.4641**, while the effect is seen at about **4** — It is the explanation that fails, on geometry — [Paper 365]

Yuuki YAMAGISHI (山岸 勇輝)

Yamagishi Complexity Torus Lab, Japan

September 2026

— *From thinking to syncing.* —

Abstract

Shake a mixture and the large grains come to the top. This paper asks whether the usual explanation survives the geometry—the answer is that it does not: the gap is too small. No new mathematical theorem and no new law is claimed. First, the sphere that fits in the gap between three equal spheres has **0.154701** of their diameter (Section 2). Second, for **20** mm grains that gap admits only **3.0940** mm (Section 2). Third, this is the core of the paper. Percolation requires a diameter ratio of at least **6.4641** (Section 2). Fourth, at a ratio of **4** (a **5** mm grain) it does not fit; at **10** (a **2** mm grain) it does (Section 2). Fifth, the buoyancy analogy reverses its prediction if the large grain is made dense (Section 3). Sixth, the separator is whether the geometry permits it, not density and not how it is shaken (Section 5).

Scope of this paper (scope note): No new mathematical theorem and no new law is claimed—the Brazil-nut effect, percolation, and granular convection are all standard. We do not build granular mechanics—the theory of granular convection and the conditions for the reverse Brazil-nut effect are not entered. They are named and no more. We perform no experiment—no measurement on real grains is made. The computation is on hard-sphere geometry. We do not settle the mechanism—this paper says only that percolation cannot explain it; it does not settle what does. Packing is not discussed—close packing and random packing fractions themselves are not the subject. The loosest case, three mutually touching spheres, is taken. Vibration is not treated—dependence on frequency, amplitude, and acceleration is not the subject. Relation to earlier papers: Paper 359 showed the gyroscope to be orders too small—there an explanation failed on magnitude; here one fails on geometry. Paper 363 showed a bat to have three sweet spots—there too several mechanisms stood behind one phenomenon. Paper 300 showed whether two things share a root is decidable—by that test, percolation and convection have distinct roots. Paper 361 showed a classifier failing when tried on a known instance—here the explanation fails at the ratio where the effect is actually seen. What is added is giving the gap ratio 0.154701 as a number, solving backwards for the required diameter ratio 6.4641, checking both sides of the boundary (ratios 4 and 10), using the buoyancy analogy's density reversal as a control, and putting the separator on whether the geometry permits it.

1. Notation and the Question

Setting: For three spheres of diameter D mutually touching, find the diameter d_{gap} of the sphere that fits in the central gap.

$$\frac{d_{\text{gap}}}{D} = \frac{2}{\sqrt{3}} - 1 = 0.154701$$

Observation 1.1

The usual explanation is that small grains fall into the gaps. Each shake lets small grains work downward and pushes large ones up—this is called percolation.

Observation 1.2

That explanation presumes the small grains fit through the gaps. If they do not fit, they cannot work downward—so there is a geometric condition.

Observation 1.3

This paper takes the loosest arrangement. The central gap of three mutually touching spheres in a plane——real packings are tighter, so the condition only gets harder.

Question: Does the usual explanation survive the geometry?

2. The Gap Is Only 0.1547 of the Diameter

Calculation 2.1: Admissible diameter and required ratio (computed in this paper).

Quantity	Value
Gap ratio $2/\sqrt{3} - 1$	0.15470054
Gap for 20 mm grains	3.0940 mm
Diameter ratio required for percolation	6.4641 or more

Observation 2.2

This is the core of the paper. Unless the diameter ratio is at least 6.4641, the small grain cannot pass——“the smaller one falls through” is usable only when it is small enough.

Calculation 2.3: Both sides of the boundary (computed in this paper, large grains 20 mm).

Small grain diameter	Ratio	Fits the gap
5 mm	4.0	no
3.0940 mm	6.4641	exactly the boundary
2 mm	10.0	yes

Observation 2.4

The ratio at which the effect is actually seen is about 4. Brazil nuts and peanuts; soybeans and sesame——these mostly do not reach 6.46.

Observation 2.5

So it is the explanation that fails. The phenomenon happens while the mechanism is forbidden by geometry——the same shape as the gyroscope failing on order of magnitude in Paper 359.

Observation 2.6

The estimate was taken in percolation's favour. Three touching spheres in a plane is the case with the largest gap——a real three-dimensional packing is tighter, and this is the result even so.

3. The Buoyancy Analogy Fails Too

Observation 3.1

The other explanation is buoyancy. A large grain is lighter than the mass of grains around it, so it floats——an analogy with a fluid.

Observation 3.2

That explanation is decided by density. With the large grain at 900 kg/m^3 and the small at 1400, it predicts “floats”——the direction is right.

Observation 3.3

But make the large grain dense and it reverses. At 2000 kg/m^3 it predicts “sinks”——and heavy large grains are known to rise anyway.

Observation 3.4

So both explanations are falsifiable in shape. Percolation is conditioned on geometry, buoyancy on density——neither says “it always rises”.

Observation 3.5

And in both cases the phenomenon happens outside the condition. A ratio of 4, and a dense large grain——there, both explanations fall silent.

4. What Is Left

Observation 4.1

The literature names granular convection. Vibration sets up a downward flow along the container walls and an upward flow in the middle——a large grain rides the upward flow and cannot re-enter the narrow wall channel to come back down.

Observation 4.2

That mechanism carries no diameter-ratio condition. All it needs is that the large grain cannot enter the narrow wall channel——it works at a ratio of 4.

Observation 4.3

This paper has not tested that mechanism. It is cited and no more——what this paper says independently is only that percolation cannot explain it.

Observation 4.4

A volume count is worth adding. A diameter ratio of 4 gives a volume ratio of 64——the space vacated by one large grain takes 64 small ones.

Observation 4.5

The number of shakes also comes out. Rising one small-grain layer (5 mm) per shake, a depth of 100 mm takes 20——matching the experience of a few tens of shakes bringing it to the surface.

5. The Separator: Does the Geometry Permit It?

Observation 5.1

The separator——whether the geometry permits the mechanism; not density and not how it is shaken.

Mechanism	Condition	At ratio 4	With a dense large grain
Percolation	ratio ≥ 6.4641	no	yes
Buoyancy analogy	large grain lighter	yes	no
Convection (literature)	cannot enter the wall channel	yes	yes

Observation 5.2

The two right-hand columns do the deciding. With a ratio of 4 and a dense large grain, only the third row survives——conditions narrow the mechanism.

Observation 5.3

All three have distinct roots by the criterion of Paper 300. Passing through geometry, a density difference, and riding a flow——all produce “the large grain rises” from different roots.

Observation 5.4

The separator can be moved. Make the small grains finer and percolation switches on——the boundary is 3.0940 mm: with the same large grains, the size of the small ones exchanges one mechanism for another.

Observation 5.5

To be stated honestly——**3.0940 mm and 6.4641** are values on a hard-sphere model. Real grains have corners and deform——what does not move is the structure: percolation has a lower bound on the diameter ratio.

Observation 5.6

One last thing——that a phenomenon occurs is not evidence that its explanation is right. Brazil nuts do come to the top——but at the ratio where the effect is actually seen, the usual explanation is forbidden by geometry. Unless the explanation is checked, its correctness cannot be borrowed from the phenomenon.

6. The Census and the Close

1. Gap ratio **0.15470054** (Calculation 2.1): **$2/\sqrt{3} - 1$** .
2. **3.0940 mm for 20 mm grains** (Calculation 2.1): **the admissible limit**.
3. Ratio **6.4641** required (Calculation 2.1): **the reciprocal of the gap ratio**.
4. Ratio **4 (5 mm)** does not fit (Calculation 2.3): **percolation is off**.
5. Ratio **10 (2 mm)** fits (Calculation 2.3): **the other side of the boundary**.
6. The observed ratio is about **4** (Observation 2.4): **short of 6.46**.
7. The estimate favours percolation (Observation 2.6): **the loosest arrangement**.
8. Buoyancy reverses for a dense grain (Observation 3.3): **it falls silent there too**.
9. Volume ratio **64** (Observation 4.4): **the cube of the diameter ratio**.
10. **20 shakes for 100 mm** (Observation 4.5): **one layer at a time**.

Closing: Shake a mixture and the large grains come to the top——the usual explanation is that small grains fall into the gaps and push the large ones up. But that explanation carries a geometric condition——the sphere fitting the gap between three equal spheres has **0.154701** of their diameter. For 20 mm grains that is **3.0940 mm**, so percolation needs a diameter ratio of at least **6.4641**. And three touching spheres in a plane is the case with the largest gap; a real three-dimensional packing is tighter. Meanwhile the ratio at which the effect is actually seen is about **4**——Brazil nuts and peanuts, soybeans and sesame. At a ratio of **4** (a 5 mm grain) the small grain does not fit; at **10** (a 2 mm grain) it does. So the phenomenon happens while the mechanism is forbidden by geometry — the same shape as the gyroscope failing on order of magnitude in Paper 359. The other explanation fails too——the buoyancy analogy

predicts “floats” with the large grain at 900 kg/m^3 against 1400, but predicts “sinks” at 2000 kg/m^3 , and heavy large grains are known to rise anyway. Both explanations are falsifiable in shape, and in both cases the phenomenon happens outside the condition. The literature names granular convection——vibration sets up a downward flow along the walls and an upward flow in the middle, and a large grain rides the upward flow but cannot re-enter the narrow wall channel to come back down. That mechanism carries no diameter-ratio condition and works at a ratio of 4. This paper has not tested it and cites it only. One thing separates them——whether the geometry permits the mechanism; not density and not how it is shaken. With a ratio of 4 and a dense large grain, only convection survives, so conditions narrow the mechanism. And the separator can be moved: make the small grains finer than 3.0940 mm and, with the same large grains, percolation switches on. To be said honestly——3.0940 mm and 6.4641 are values on a hard-sphere model, and real grains have corners and deform. What does not move is the structure: percolation has a lower bound on the diameter ratio. One last thing——that a phenomenon occurs is not evidence that its explanation is right. Brazil nuts do come to the top, but at the ratio where the effect is actually seen the usual explanation is forbidden by geometry. Unless the explanation is checked, its correctness cannot be borrowed from the phenomenon.

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