

An Hourglass Keeps Time Because the Height Is Not in the Formula

— Water flows **5.4772** times faster at **0.30** m than at **0.01** m — Sand stays at **0.024473** kg/s, with the height appearing nowhere — Both are “flow out of a hole”, and what separates them is whether the pressure at the bottom is set by the height — [Paper 366]

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— *From thinking to syncing.* —

Abstract

A water clock is hard and an hourglass is easy. This paper asks where that difference comes from—the answer is whether the remaining height appears in the flow formula. No new mathematical theorem and no new law is claimed. First, water follows Torricelli: **2.4257** m/s at **0.30** m of head and **0.4429** m/s at **0.01** m (Section 2). Second, the ratio of flows is **5.4772**, exactly the square root of the height ratio, $\sqrt{30}$ (Section 2). Third, this is the core of the paper. Beverloo’s law for sand contains no h at all, and the flow stays at **0.024473** kg/s (Section 3). Fourth, what acts instead is grain size: from 0 to 3 mm the flow falls to **0.256195** of its value (Section 3). Fifth, beyond a grain size of **7.1429** mm the formula predicts that flow stops (Section 3). Sixth, the separator is whether the pressure at the bottom is set by height, not the hole and not the material (Section 5).

Scope of this paper (scope note): No new mathematical theorem and no new law is claimed——Torricelli’s law, Beverloo’s law, and the Janssen effect are all standard. We do not build granular mechanics——the theory of force chains and arch formation is not entered. It is named and no more. We do not derive Beverloo’s law——it is posited as an empirical formula and the coefficients $C = 0.58$ and $k = 1.4$ are taken from the literature. No derivation is given. We perform no experiment——no measurement on a real hourglass is made. Jamming is not treated——the probability of an arch blocking the orifice is not the subject. We design no clock——the accuracy of hourglasses and their dependence on temperature and humidity are not the subject. Relation to earlier papers: Paper 343 showed the size of a hole not entering the stress concentration——there too what did not enter made the conclusion. Here it is the height. Paper 350 showed dead time not entering the amplitude——the same shape. Paper 300 showed whether two things share a root is decidable——by that test, water discharge and sand discharge have distinct roots. Paper 352 showed a transition temperature moving with measurement conditions——there a condition entered; here it does not. What is added is setting water and sand side by side under one condition, confirming the water ratio equals $\sqrt{30}$ exactly, showing numerically that the sand flow does not depend on height, giving the grain-size dependence at five points and solving backwards for the stopping boundary, and putting the separator on whether bottom pressure is set by height.

1. Notation and the Question

Setting: Discharge water and sand through a hole of radius 5 mm (area $7.853982 \times 10^{-5} \text{ m}^2$).

$$v_{\text{water}} = \sqrt{2gh}, \quad Q_{\text{sand}} = C\rho\sqrt{g}(D - kd)^{5/2}$$

with $C = 0.58$, $k = 1.4$, bulk density $\rho = 1500 \text{ kg/m}^3$, orifice $D = 10 \text{ mm}$, grain size $d = 0.3 \text{ mm}$.

Observation 1.1

Set the two formulae side by side and one has h and the other does not. Torricelli contains h ; Beverloo does not——this paper treats that single point.

Observation 1.2

The absence of h is not an omission. In a granular medium the pressure at the bottom does not grow with height——the walls carry part of the load (the Janssen effect).

Observation 1.3

As a result, sand falls at a constant rate. It can be a clock—a water clock is not one unless the vessel's shape compensates.

Question: Why does sand fall at a constant rate?

2. Water Varies by 5.4772 with the Head

Calculation 2.1: Water discharge against remaining head (computed in this paper).

Head h [m]	Efflux speed [m/s]	Flow [m^3/s]
0.30	2.4257	1.905135×10^{-4}
0.20	1.9806	1.555537×10^{-4}
0.10	1.4005	1.099930×10^{-4}
0.05	0.9903	7.777683×10^{-5}
0.01	0.4429	3.478285×10^{-5}

Observation 2.2

The flow at 0.30 m is **5.4772** times that at 0.01 m. The square root of the height ratio 30—exactly $\sqrt{30} = 5.477226$. The formula closes.

Observation 2.3

So a water clock has unequal time marks. Fast at first and slow at the end—to space the marks evenly, the vessel's cross-section must be designed as a function of height.

Observation 2.4

This is a property of pressure, not of water. Hydrostatic pressure is ρgh , proportional to height—more height, more push.

3. Sand Has No Height in It

Calculation 3.1: Sand flow (computed in this paper, Beverloo's law).

Head h [m]	Sand flow [kg/s]	Compare water
0.30	0.024473	water $1.905135 \times 10^{-4} \text{ m}^3/\text{s}$
0.10	0.024473	water $1.099930 \times 10^{-4} \text{ m}^3/\text{s}$
0.01	0.024473	water $3.478285 \times 10^{-5} \text{ m}^3/\text{s}$

Observation 3.2

This is the core of the paper. All three rows are the same 0.024473 kg/s—not approximately so: the formula contains no h at all.

Observation 3.3

What acts instead is grain size. It enters as $(D - kd)^{5/2}$ —what acts is an “effective orifice”, the diameter less 1.4 times the grain size.

Calculation 3.4: Sand flow against grain size (computed in this paper, orifice 10 mm).

Grain size d [mm]	Flow [kg/s]	Relative to $d = 0$
0.0	0.027245	1.000000
0.3	0.024473	0.898284
1.0	0.018686	0.685877
2.0	0.011984	0.439877
3.0	0.006980	0.256195

Observation 3.5

At 3 mm the flow is a quarter. Grains three tenths of the orifice——acting through a $5/2$ power, a small coarsening costs a great deal.

Observation 3.6

The formula names its own stopping point. Beyond $d = D/k = 7.1429$ mm, $(D - kd)$ goes negative——it predicts that flow stops. The same shape as the empirical law naming its own breakdown in Paper 355.

Observation 3.7

This orifice is large for an hourglass. It passes 1.4684 kg in a minute——a household hourglass has a much finer neck. A check that the numbers are realistic.

4. Why the Height Does Not Enter

Observation 4.1

In a granular medium the bottom pressure saturates with depth. This is the Janssen effect——grains interlock into force chains and the walls carry part of the load.

Observation 4.2

So piling metres more on top does not raise the pressure at the bottom. What does not rise cannot raise the flow—the disappearance of h from the formula is a consequence of that physics.

Observation 4.3

What acts instead is a local structure near the hole. A free-fall arch forms above the orifice—the flow is set by the size of that region, which is a function of orifice and grain size alone.

Observation 4.4

The same shape as Paper 343. There the size of the hole did not enter the stress concentration—here the height does not enter the flow. In both, what does not enter makes the conclusion.

Observation 4.5

To be stated honestly—this paper has not computed the Janssen effect. Beverloo's law is used empirically and only the absence of h from its form is observed—the explanation of the mechanism is left to the literature.

5. The Separator: Is Bottom Pressure Set by Height?

Observation 5.1

The separator—whether the pressure at the bottom grows with height; not the shape of the hole and not the kind of material.

	Bottom pressure	Does h enter the flow	Can it be a clock
Water	grows as ρgh	yes	only with a shaped vessel
Sand	saturates	no	as it is

Observation 5.2

The rightmost column is the whole of practice. Whether it can be a clock is settled by that one column——the hourglass worked from early on not because the material was special.

Observation 5.3

Distinct roots by the criterion of Paper 300. Both are “flow out of a hole”, but water is set by hydrostatic pressure and sand by a local arch——one name for a phenomenon covering two mechanisms.

Observation 5.4

The separator can be moved. Soak the sand and the interlocking weakens, so pressure starts depending on height——that is, it is not “because it is sand” but “because it is a dry granular medium”.

Observation 5.5

To be stated honestly——0.024473 kg/s and the factor 5.4772 are values on stated orifice, grain size, and bulk density. Another hourglass gives other numbers——what does not move is the structure: whether h enters the formula.

Observation 5.6

One last thing——a quantity absent from a formula moves the answer no matter how far it is changed. Pile the hourglass higher and it does not run faster——simply seeing which quantities enter tells you what cannot act, and that verdict comes before any computation.

6. The Census and the Close

1. Water gives 2.4257 m/s at $h = 0.30$ m (Calculation 2.1): Torricelli.
2. Flow ratio 5.4772 (Observation 2.2): exactly $\sqrt{30}$.
3. Sand gives 0.024473 kg/s at all three heads (Calculation 3.1): no h in the formula.

4. Grain size acts instead (Observation 3.3): $(D - kd)^{5/2}$.
5. **0.256195** at **3 mm** (Calculation 3.4): **down to a quarter**.
6. Stopping predicted at **7.1429 mm** (Observation 3.6): **the formula names its limit**.
7. **1.4684 kg per minute** (Observation 3.7): **a large orifice for an hourglass**.
8. Bottom pressure saturates (Observation 4.1): **the Janssen effect**.
9. The same shape as Paper 343 (Observation 4.4): **what does not enter decides**.
10. Wetting moves the separator (Observation 5.4): **not “because it is sand”**.

Closing: A water clock is hard and an hourglass is easy—and the difference is not the specialness of the material but whether the remaining height enters the flow formula. Water follows Torricelli—**2.4257 m/s** at **0.30 m** of head and **0.4429 m/s** at **0.01 m**. The flow ratio is **5.4772**, exactly the square root of the height ratio, $\sqrt{30} = 5.477226$. So a water clock runs fast at first and slow at the end, and spacing its marks evenly needs a vessel whose cross-section is designed as a function of height. The sand formula contains no h at all—on Beverloo’s law $Q = C\rho\sqrt{g}(D - kd)^{5/2}$, the flow at $h = 0.30$ m and at $h = 0.01$ m is the same **0.024473 kg/s**. Not approximately so: the height is simply not in the formula. What acts instead is grain size—an “effective orifice”, the diameter less 1.4 times the grain size, acting through a $5/2$ power, so that at **3 mm** the flow is **0.256195** of its value — a quarter. And beyond **7.1429 mm** the term goes negative and the formula predicts that flow stops, the same shape as the empirical law naming its own breakdown in Paper 355. The height disappears for a physical reason—grains interlock into force chains and the walls carry part of the load, so the bottom pressure saturates with depth (the Janssen effect). What does not rise cannot raise the flow. What acts instead is the free-fall arch above the orifice, whose size is a function of orifice and grain size alone. This paper has not computed the Janssen effect and only observes the absence of h from the formula. One thing separates them—whether the pressure at the bottom grows with height; not the shape of the hole and not the kind of material. Soak the sand and the interlocking weakens and pressure starts depending on height — so it is not “because it is sand” but “because it is a dry granular medium”. To be said honestly—**0.024473 kg/s** and the factor **5.4772** hold for the stated orifice, grain size and bulk density. What does not move is the structure: whether h enters the formula. One last thing—a quantity absent from a formula moves the answer no matter how far it is changed. Pile the hourglass higher and it does not run faster. Simply seeing which quantities enter tells you what cannot act, and that verdict comes before any computation.

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