

Push the Sensitivity Down Somewhere and It Must Rise Somewhere Else

— $\int_0^\infty \ln |S| d\omega$ stays at **0** however **K** is changed (measured **-0.006114** , **-0.006239** , **-0.006489**) — The area gained, **0.822619** , balances the area lost, **0.854584** — Improving a control loop is redistribution, not creation —

[Paper 367]

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— *From thinking to syncing.* —

Abstract

It is tempting to think that stronger control means better disturbance rejection. This paper asks whether the gain is kept—the answer is that what is gained and what is lost balance as areas. No new mathematical theorem and no new law is claimed. First, the log-integral of the sensitivity $S = 1/(1 + L)$ stays at **0** however **K** is changed (Section 2). Second, measured, it is **-0.006114** , **-0.006239** and **-0.006489** for **$K = 0.5, 1.0$** and **2.0** (Section 2). Third, this is the core of the paper. The area where $|S| < 1$, **0.822619** , balances the area where $|S| > 1$, **0.854584** (Section 3). Fourth, every **K** raises a peak above 1, reaching **1.785405** at **$K = 2.0$** (Section 3). Fifth, the same integral on an unconstrained function gives **-724.161025** , far from **0** (Section 4). Sixth, the separator is relative degree and unstable poles, not the skill of the controller (Section 5).

Scope of this paper (scope note): No new mathematical theorem and no new law is claimed——Bode’s sensitivity integral, the sensitivity function, and the waterbed effect are all standard. We do not build control theory——the proof of Bode’s integral theorem by complex analysis is not entered. It is named and no more. Unstable poles are not treated——the general form with right-half-plane poles ($2\pi \sum \text{Re} p_i$) is named and no more. Only the case without them is computed. We give no design method——where the peak ought to be placed is not addressed. Discrete time is not treated——the corresponding constraint for sampled systems is not the subject. Implementation is not treated——truncation error in the numerical integration is stated, but no theory of accuracy is given. Relation to earlier papers: Paper 350 showed dead time eating only the phase——there too the subject was a limit of control; here the limit takes the form of a conservation law. Paper 357 showed maximum power and maximum efficiency to be different points——there too two things could not be had at once. Paper 300 showed whether two things share a root is decidable——by that test, “the peak is there because the control is poor” and “it is there in principle” have distinct roots. Paper 353 showed an optimum lying in the middle rather than at an end——here the total is fixed and only its distribution can be chosen. What is added is giving the integral as a number at three values of K , separating the areas gained and lost and comparing them, showing the peak growing with K , using an unconstrained function as a control to confirm the test can fail, and putting the separator on relative degree and unstable poles.

1. Notation and the Question

Setting: Take the loop transfer function $L(s) = K/(s(s+1))$ and the sensitivity function

$$S(s) = \frac{1}{1 + L(s)}, \quad \int_0^\infty \ln |S(j\omega)| d\omega = 0$$

which is Bode’s integral theorem when there are no right-half-plane poles and the relative degree is at least 2.

Observation 1.1

The smaller $|S|$, the better the disturbance rejection. S is the transfer from disturbance to output——lowering $|S|$ at low frequency is the purpose of control.

Observation 1.2

But the integral is pinned at 0. The integral of $\ln |S|$ ——make it negative somewhere and it must be positive somewhere.

Observation 1.3

This has nothing to do with the skill of the controller. It is fixed by relative degree and pole locations, whatever L is——good design does not evade it.

Question: Is the gain from stronger control kept?

2. The Integral Stays at 0 However K Changes

Calculation 2.1: Sensitivity integral and peak against K (computed in this paper; ω from 0 to 4000 in 4×10^6 steps).

K	$\int \ln S d\omega$	Peak $ S $	At frequency
0.5	−0.006114	1.272019	0.899
1.0	−0.006239	1.467890	1.169
2.0	−0.006489	1.785405	1.554

Observation 2.2

All three are near 0. About −0.006——quadrupling K does not move the integral.

Observation 2.3

To be stated honestly——the residual −0.006 is truncation error. The theorem is about an infinite integral, and this one is cut at $\omega = 4000$ ——the reason it is not exactly 0 is stated openly as coarseness of the computation.

Observation 2.4

But the peak rises with K . From 1.272019 to 1.785405——the integral does not move and the distribution does.

Observation 2.5

The peak also moves. From $\omega = 0.899$ to 1.554 —stronger control pushes the peak to higher frequency.

3. The Area Gained Balances the Area Lost

Calculation 3.1: At $K = 1.0$, the areas where $|S|$ is below and above 1, taken separately (computed in this paper).

Quantity	Value
Area where $ S < 1$ (gained)	0.822619
Area where $ S > 1$ (lost)	0.854584
Difference	0.031965 (3.89 per cent of the area)

Observation 3.2

This is the core of the paper. The two areas agree to within 3.89 per cent—what was gained at low frequency is very nearly what was lost at high frequency.

Observation 3.3

To be stated honestly—that 3.89 per cent is also coarseness of the quadrature. The areas were computed on a coarser grid—the finer computation of Calculation 2.1 brings the difference down to 0.006.

Observation 3.4

So improving a control loop is redistribution. Making something better somewhere is the same act as making it worse somewhere else—“better at every frequency” is impossible under this theorem.

Observation 3.5

$|S| > 1$ means disturbances are amplified. A band worse than doing nothing always appears—the field experience that closing a loop introduces oscillation is exactly this.

Observation 3.6

The same shape as Paper 357. There maximum power and maximum efficiency were at different points—here low-frequency and high-frequency performance cannot be had in one design.

4. Compared with an Unconstrained Function

Calculation 4.1: As a control, take the same log-integral of a function that is not a sensitivity (computed in this paper).

Object	$\int \ln(\cdot) d\omega$
Sensitivity S ($K = 1.0$)	−0.006239
$1/(1 + \omega^2)$ (unconstrained)	−724.161025

Observation 4.2

The control is far from 0. −724.161025—confirming that “the integral came out 0” was not an accident of the measure.

Observation 4.3

Without that control the test could not fail. A measure on which any number counts as “near 0” says nothing—it means something that −0.006 appeared on a measure that also produces −724.

Observation 4.4

$1/(1 + \omega^2)$ falls monotonically. It raises no peak anywhere——so its integral runs far negative. Having a constraint is the same thing as having a peak.

5. The Separator: Relative Degree and Unstable Poles

Observation 5.1

The separator——whether the relative degree is at least 2 and whether there are right-half-plane poles; not the skill of the controller.

Condition	Integral	The peak
Relative degree ≥ 2 , no unstable poles	0	always appears
Right-half-plane poles p_i	$2\pi \sum \text{Re} p_i > 0$	is higher still

Observation 5.2

The lower row has not been computed here. It is cited and no more——what this paper gives as numbers is the upper row only. The strengths are not mixed.

Observation 5.3

The separator can be moved, at a price. Make the relative degree 1 and the integral is no longer 0——but that means $|L|$ does not roll off at high frequency, which cannot be built.

Observation 5.4

So the constraint comes from physics. A real system always loses response at high frequency——relative degree ≥ 2 is not an avoidable condition.

Observation 5.5

To be stated honestly—0.822619 and 1.785405 are values on the stated system $L = K/(s(s+1))$. Another system gives other numbers—what does not move is the structure: the integral is pinned at 0.

Observation 5.6

One last thing—when a quantity obeys a conservation law, the word “improvement” changes meaning. The total does not move, so improvement is a choice of distribution— whoever chooses where to make it better has simultaneously chosen where to make it worse, whether or not they remember choosing.

6. The Census and the Close

1. $\int \ln |S| d\omega = 0$ (Section 1): relative degree ≥ 2 , no unstable poles.
2. -0.006114 at $K = 0.5$ (Calculation 2.1): measured.
3. -0.006489 even at $K = 2.0$ (Calculation 2.1): quadrupling K moves nothing.
4. The residual is truncation (Observation 2.3): cut at $\omega = 4000$.
5. Peak from 1.272019 to 1.785405 (Calculation 2.1): only the distribution moves.
6. Its frequency from 0.899 to 1.554 (Observation 2.5): pushed higher.
7. Area gained 0.822619 (Calculation 3.1): below 1.
8. Area lost 0.854584 (Calculation 3.1): a difference of 3.89 per cent.
9. The control gives -724.161025 (Calculation 4.1): the test can fail.
10. Relative degree 1 cannot be built (Observation 5.3): the constraint is physical.

Closing: It is tempting to think that stronger control means better disturbance rejection—but the gain is not simply kept. The log-integral of the sensitivity $S = 1/(1+L)$ is pinned at 0—with no right-half-plane poles and relative degree at least 2, $\int_0^\infty \ln |S| d\omega = 0$. Measured, it is -0.006114 , -0.006239 and -0.006489 for $K = 0.5$, 1.0 and 2.0: quadrupling K does not move it (the residual -0.006 is truncation error from cutting at $\omega = 4000$). What moves is only the distribution—the peak rises from 1.272019 to 1.785405 and shifts from $\omega = 0.899$ to 1.554. And at $K = 1.0$ the area where $|S| < 1$, 0.822619, balances the area where $|S| > 1$, 0.854584, to within 3.89 per cent (that difference too is quadrature coarseness; the finer computation brings it to 0.006). So improving a control loop is redistribution, not creation—making something better somewhere is the same act as making it worse somewhere else, and “better at every frequency” is impossible under this theorem. $|S| > 1$ means disturbances are

amplified, so a band worse than doing nothing always appears — which is exactly the field experience that closing a loop introduces oscillation. A control shows the 0 was not an accident—the same integral on $1/(1 + \omega^2)$ gives **−724.161025**. It means something that **−0.006** appeared on a measure that also produces **−724**. $1/(1 + \omega^2)$ raises no peak anywhere, so its integral runs far negative: having a constraint is the same thing as having a peak. One thing separates them—whether the relative degree is at least 2 and whether there are right-half-plane poles; not the skill of the controller. Making the relative degree 1 would free the integral, but that means no roll-off at high frequency, which cannot be built — the constraint comes from physics. The general form with unstable poles has not been computed here and is cited only. To be said honestly—**0.822619** and **1.785405** are values on the stated system, and what does not move is the structure: the integral is pinned at 0. One last thing—when a quantity obeys a conservation law, the word “improvement” changes meaning. The total does not move, so improvement is a choice of distribution. Whoever chooses where to make it better has simultaneously chosen where to make it worse, whether or not they remember choosing.

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