

Is 6π Really $4\pi + 2\pi$?

— The Anatomy of Stokes Drag: the Same 4π Appears Twice, with Opposite Contents — [Paper 37]

Yuuki YAMAGISHI (山岸 勇輝)

Yamagishi Complexity Torus Lab, Japan

August 2026

— *From thinking to syncing.* —

ABSTRACT

The viscous drag on a sphere is $6\pi\mu Ua$ with a no-slip boundary and $4\pi\mu Ua$ with a perfectly slipping one. **It is tempting to read this as “ $6\pi = 4\pi + 2\pi$, so the 4π is the essential part and the 2π is what sticking adds”.** This paper refutes that reading by dissecting the drag. **NO NEW FLUID MECHANICS IS CLAIMED.** First, the no-slip 6 splits as $2 + 0 + 4$ (in units of $\pi\mu Ua$): 2 from pressure, 0 from the normal viscous stress, 4 from the tangential one (computed here) (Section 2). Second, the slip 4 splits as $4/3 + 8/3 + 0$: $4/3$ from pressure, $8/3$ from the normal viscous stress, 0 from the tangential one. The boundary conditions (zero normal velocity, zero tangential traction) and the residual of the Stokes equation were all verified to be 0 (Section 3). Third, and this is the core. The two 4 s have opposite contents. The 4 inside the no-slip split is 100% tangential, with the normal part exactly zero; the 4 that IS the slip drag is $2/3$ normal viscous and $1/3$ pressure, with the tangential part exactly zero. The arithmetic $6\pi = 4\pi + 2\pi$ is true, but that 4π is not the slip 4π (Section 4). Fourth, lifted to d dimensions the two coefficients share a factor. No-slip is $d \cdot \frac{d-2}{d-1}$ and slip is $2 \cdot \frac{d-2}{d-1}$ (both times $\text{Vol}(S^{d-1})$; checked at $d = 3, 4, 5, 6$). The $\frac{d-2}{d-1}$ is common, and the whole difference between the two boundary conditions collapses to the single factor d against 2 (Section 5). Fifth, the two $3/2$ s that appear at $d = 3$ are also distinct roots. One is the ratio $d/2$, the other the no-slip coefficient $d(d-2)/(d-1)$, and at $d = 4$ they part company as 2 against $8/3$ (Section 5). Sixth, the 6 of the Einstein–Stokes relation $D = k_B T / (6\pi\mu a)$ is not a property of the fluid but a modelling choice about the surface (Section 6).

Scope of this paper: NO NEW FLUID MECHANICS IS CLAIMED. Stokes flow, slip and no-slip boundary conditions and the Einstein–Stokes relation are all standard. What is added is splitting the drag into three parts and showing that the two 4 s have disjoint support, fencing the reading “ $6\pi = 4\pi + 2\pi$ ” by counterexample, showing that in d dimensions the two coefficients share a factor and differ only as d against 2 , and writing addresses for the two $3/2$ s at $d = 3$.

1. Notation and position

Let μ be the viscosity, U the uniform stream, a the radius. All drags are written in units of $\pi\mu Ua$ (in d dimensions, of $\text{Vol}(S^{d-1})\mu Ua^{d-2}$). The stress is $\sigma_{ij} = -p\delta_{ij} + \mu(\partial_i u_j + \partial_j u_i)$.

Position: Paper 43 argued that leaving a patch constant as a letter removes the ambiguity of a decomposition. This paper runs that check in the other direction: whether a decomposition that is numerically correct can carry more than one meaning. A companion piece (Paper 36) studies the same drag with the dimension moved, and shows that the π of 6π is a solid angle.

2. No-slip: $6 = 2 + 0 + 4$

Verification 2.1: Take the stream function $\psi = \frac{U \sin^2 \theta}{2} \left(r^2 - \frac{3}{2} ar + \frac{a^3}{2r} \right)$ and the pressure $p = -\frac{3}{2} \mu U a \cos \theta / r^2$. Then

$u_r _{r=a}$ and $u_\theta _{r=a}$	both 0
as $r \rightarrow \infty$	$U \cos \theta$ and $-U \sin \theta$
$\nabla p - \mu \nabla^2 u$	0

On the sphere $\sigma_{rr} = \frac{3\mu U \cos \theta}{2a}$ and $\sigma_{r\theta} = -\frac{3\mu U \sin \theta}{2a}$. Integrating,

contribution	in units of $\pi\mu Ua$
pressure	2
normal viscous $2\mu\partial_r u_r$	0
tangential viscous $\sigma_{r\theta}$	4
total	6

(QED)

Observation 2.2

Note that the normal viscous contribution is exactly zero. This matters later.

3. Slip: $4 = \frac{4}{3} + \frac{8}{3} + 0$

Verification 3.1: Perfect slip means zero normal velocity and zero tangential traction. With $\psi = \frac{U \sin^2 \theta}{2} (r^2 - ar)$ and $p = -\mu U a \cos \theta / r^2$,

$u_r _{r=a}$	0
$\sigma_{r\theta} _{r=a}$	0
$\nabla p - \mu \nabla^2 u$	0

and integrating,

contribution	in units of $\pi \mu U a$
pressure	4/3
normal viscous	8/3
tangential viscous	0 (by construction)
total	4

(QED)

4. The core: the two 4s have opposite contents

Observation 4.1

	pressure	normal viscous	tangential viscous	total
no-slip	2	0	4	6
slip	4/3	8/3	0	4

Observation 4.2

The 4 inside the no-slip split is made of tangential stress alone (the normal viscous part being exactly zero). The 4 that is the whole slip drag is made of 2/3 normal viscous stress and 1/3 pressure (the tangential part being exactly zero). The same number, and the supports of its components do not meet.

Observation 4.3

So the reading “ $6\pi = 4\pi + 2\pi$, the slip answer plus a 2π correction for sticking” is wrong. No component of the split is the slip flow — slip is a different solution, not a part of the no-slip one. The equation holds as arithmetic and fails as meaning.

Observation 4.4

This is a shape the corpus has met before — numbers agreeing does not make roots the same (Papers 43 and 45).

5. d dimensions: the difference is d against 2

Verification 5.1: Solving the same problem in d dimensions (in units of $\text{Vol}(S^{d-1})\mu Ua^{d-2}$),

d	no-slip	slip	ratio
3	$3/2$	1	$3/2$
4	$8/3$	$4/3$	2
5	$15/4$	$3/2$	$5/2$
6	$24/5$	$8/5$	3

All four agree with

$$\text{no-slip} = d \cdot \frac{d-2}{d-1}, \quad \text{slip} = 2 \cdot \frac{d-2}{d-1}, \quad \frac{\text{no-slip}}{\text{slip}} = \frac{d}{2}$$

(computed here). (QED)

Observation 5.2

The factor $\frac{d-2}{d-1}$ is common to both. The difference between clinging to a surface and sliding along it — physically two quite different conditions — collapses to the single factor d against 2.

Observation 5.3

And the two $3/2$ s that appear at $d = 3$ are distinct roots. One is the ratio $d/2$; the other is the no-slip coefficient $d(d-2)/(d-1)$. They coincide at $d = 3$ only because $2(d-2)/(d-1) = 1$ there, and at $d = 4$ they part as 2 against $8/3$.

Observation 5.4

Both carry $(d-2)$, so both die at $d = 2$. Stokes' paradox does not care which boundary condition was chosen.

6. The 6 of Einstein–Stokes

Observation 6.1

The **6** in the diffusion constant $D = k_B T / (6\pi\mu a)$ is the **no-slip coefficient**. With perfect slip the same relation reads $k_B T / (4\pi\mu a)$, so D is larger by $3/2$.

Observation 6.2

A factor of one and a half in a measured diffusion constant therefore rides on a modelling choice about the surface, not on a property of the fluid. The fluctuation side ($k_B T$) has not moved — only the mobility side has.

Observation 6.3

So 6π must not be treated as a universal constant. At molecular scales the slip length is known not to vanish, and the effective coefficient sits between 4π and 6π . The 6 is not a fact but the name of a boundary condition that was chosen.

7. The census

Observation 7.1

the number	where it comes from	what moves it
the no-slip 4	tangential stress alone	the boundary condition
the slip 4	$2/3$ normal viscous, $1/3$ pressure	the boundary condition
the π	inside $\text{Vol}(S^{d-1})$	the dimension
$(d-2)/(d-1)$	common to both	the dimension
d against 2	the boundary condition itself	the boundary condition

8. Conclusion

1. The no-slip 6 is $2 + 0 + 4$ (Ver. 2.1): the normal viscous part is exactly zero.
2. The slip 4 is $4/3 + 8/3 + 0$ (Ver. 3.1): the tangential part is exactly zero.
3. The two 4s have disjoint support (Obs. 4.2): “ 4π plus a correction” is wrong.
4. In d dimensions the common factor is $(d-2)/(d-1)$ and the difference is d against 2 (Ver. 5.1).
5. The two $3/2$ s at $d = 3$ are distinct roots (Obs. 5.3): 2 against $8/3$ at $d = 4$.
6. The 6 of $D = k_B T / (6\pi\mu a)$ is a modelling choice (Obs. 6.2).

Closing: On a calculator, $6\pi = 4\pi + 2\pi$ is correct. But one of those 4π s is the whole of a flow that slides past the sphere, and the other is a part of a flow that clings to it. Seeing the same number, the question to ask is not whether they are equal but what each is made of.

References

1. Stokes, G. G. (1851). *On the effect of the internal friction of fluids on the motion of pendulums*. Transactions of the Cambridge Philosophical Society, 9.
2. Einstein, A. (1905). *Über die von der molekularkinetischen Theorie der Wärme geforderte Bewegung von in ruhenden Flüssigkeiten suspendierten Teilchen*. Annalen der Physik, 322(8).
3. Hadamard, J. (1911). *Mouvement permanent lent d'une sphère liquide et visqueuse dans un liquide visqueux*. Comptes Rendus, 152.
4. Happel, J., & Brenner, H. (1983). *Low Reynolds Number Hydrodynamics*. Martinus Nijhoff.
5. Landau, L. D., & Lifshitz, E. M. (1987). *Fluid Mechanics*, 2nd ed. Pergamon.
6. Lauga, E., Brenner, M. P., & Stone, H. A. (2007). *Microfluidics: The No-Slip Boundary Condition*. In Springer Handbook of Experimental Fluid Mechanics.
7. Yamagishi, Y. (2026). *The π That Carries No Dimension* [Paper 31]. Zenodo. 10.5281/zenodo.22090430
8. Yamagishi, Y. (2026). *What Does the Exponent of $\mathfrak{c}$ Count?* [Paper 116]. Zenodo. 10.5281/zenodo.22003794
9. Yamagishi, Y. (2026). *Three Distinct Roots of Aggregation* [Paper 124]. Zenodo. 10.5281/zenodo.22057728
10. Yamagishi, Y. (2026). *A Vortex Line Cannot End Because of an Identity, Not a Law* [Paper 130]. Zenodo. 10.5281/zenodo.22079996
11. Yamagishi, Y. (2026). *The $\mathfrak{z}$ in Itô's Lemma Is Not a Dimension* [Paper 132]. Zenodo. 10.5281/zenodo.22080358

— *From thinking to syncing.* —

Yuuki YAMAGISHI (山岸 勇輝)

© 2026 Yamagishi Complexity Torus Lab, Japan · Licensed under Creative Commons Attribution 4.0 International (CC BY 4.0)