

A Single Anti-Reflection Layer Falls Short for Two Independent Reasons

— Even an ideal film cancels only at the design wavelength, returning to **0.013541** at **400 nm** — And no material has the ideal index **1.232883**: real MgF_2 leaves **0.012601** even at the design wavelength — The two shortfalls have different causes — [Paper 370]

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— *From thinking to syncing.* —

Abstract

An anti-reflection coating is described as “a film that cancels reflection”. This paper asks how far it cancels—the answer is at one wavelength only, and with real materials not at all. No new mathematical theorem and no new law is claimed. First, bare glass reflects **0.042580** (4.2580 per cent) (Section 2). Second, the ideal film index is $\sqrt{n_0 n_s} = \mathbf{1.232883}$, at which the reflection at 550 nm is exactly zero (Section 2). Third, this is the core of the paper. The same ideal film returns to **0.013541** at 400 nm (Section 2). Fourth, real MgF_2 has 1.38, which is 0.1471 from the ideal (Section 3). Fifth, so it leaves **0.012601** (1.2601 per cent) even at the design wavelength (Section 3). Sixth, the separator is two things — whether the wavelength shifted or the index did — not one (Section 5).

Scope of this paper (scope note): No new mathematical theorem and no new law is claimed—the single-layer anti-reflection coating, the quarter-wave condition, and Fresnel reflection are all standard. We do not build thin-film optics—multilayer design theory (equivalent layers, optimisation) is not entered. It is named and no more. Materials are not treated—materials other than MgF_2 , and index lowering by porous films, are not the subject. Indices are taken as independent of wavelength. Oblique incidence is not treated—only normal incidence is computed. Absorption is not treated—film and glass are taken as lossless. Polarisation is not treated—at normal incidence there is no distinction. Relation to earlier papers: Paper 363 showed a bat to have three sweet spots—there one word named three things; here one shortfall has two causes. Paper 365 showed a standard explanation failing on geometry—there too checking the condition numerically showed the explanation did not reach. Paper 300 showed whether two things share a root is decidable—by that test, a wavelength shift and an index mismatch have distinct roots. Paper 358 showed a convention fixing a number—here the convention is where the design wavelength is placed. What is added is setting ideal and real films side by side at seven wavelengths, showing the ideal film returning to 0.013541 at 400 nm, showing the real material leaving 0.012601 at the design wavelength, checking bare glass against the accepted 4 per cent, and splitting the separator in two.

1. Notation and the Question

Setting: Between air ($n_0 = 1.00$) and glass ($n_s = 1.52$), place a film of index n_1 and optical thickness $\lambda_0/4$.

$$R_{\text{bare}} = \left(\frac{n_s - n_0}{n_s + n_0} \right)^2, \quad n_1^{\text{ideal}} = \sqrt{n_0 n_s}, \quad \lambda_0 = 550 \text{ nm}$$

Observation 1.1

The film makes two reflections cancel. One at the surface and one at the film-glass interface—they vanish completely when the phase differs by half a wave and the amplitudes are equal.

Observation 1.2

So there are two conditions. A phase condition (thickness $\lambda/4$) and an amplitude condition ($n_1 = \sqrt{n_0 n_s}$)—break either and nothing cancels.

Observation 1.3

And the two break on different quantities.Phase on wavelength, amplitude on material——this paper gives the two separately as numbers.

Question: How far does an anti-reflection coating cancel reflection?

2. Even an Ideal Film Cancels at One Wavelength Only

Calculation 2.1: Reflectance of the ideal film, index 1.232883 (computed in this paper, normal incidence).

Wavelength [nm]	Reflectance
400	0.01354129
450	0.00517551
500	0.00108717
550	0.00000000
600	0.00075713
650	0.00254063
700	0.00482796

Bare glass is 0.042580 (4.2580 per cent).

Observation 2.2

This is the core of the paper.Exactly zero at 550 nm and back to 0.013541 at 400——nearly a third of the bare 0.042580.

Observation 2.3

It rises on both sides of the design wavelength.0.00108717 at 500 nm and 0.00075713 at 600—a valley, not a plateau, with a single lowest point.

Observation 2.4

The short-wavelength side is worse.0.013541 at 400 nm against 0.004828 at 700—not symmetric at equal distance, because the phase error goes as $1/\lambda$.

Observation 2.5

The bare 4.2580 per cent matches the accepted figure.One glass surface is usually given as about 4 per cent —neither the formula nor the constants are off.

3. And There Is No Ideal Material

Calculation 3.1: Reflectance of real MgF_2 ($n_1 = 1.38$) (computed in this paper).

Wavelength [nm]	MgF_2	Ideal film
400	0.02205252	0.01354129
450	0.01620430	0.00517551
500	0.01335683	0.00108717
550	0.01260079	0.00000000
600	0.01312726	0.00075713
650	0.01436835	0.00254063
700	0.01596197	0.00482796

Observation 3.2

MgF_2 is not zero even at the design wavelength. 0.012601, that is 1.2601 per cent—the phase condition is met and the amplitude condition is not.

Observation 3.3

The ideal index is $1.232883\sqrt{1.52}$ —0.1471 from MgF_2 's 1.38. No dense solid has an index that low.

Observation 3.4

The film still works. From 0.042580 bare down to 0.012601, under a third—it is not that it fails; it is that it cannot finish.

Observation 3.5

The two shortfalls are independent. With an ideal material only the wavelength error remains; in monochromatic light only the material error does—fix either and the other survives.

Observation 3.6

Distinct roots by the criterion of Paper 300. The phase error comes from geometry (thickness and wavelength), the amplitude error from material (index)—both produce the same symptom, “reflection remains”, from different roots.

4. What Multiple Layers Change

Observation 4.1

More layers cancel at more wavelengths. Two layers at two wavelengths, three over a broader band—the phase condition can be met at several wavelengths at once.

Observation 4.2

And the material constraint loosens. Combinations synthesise an effective index—a single layer must itself be 1.232883; a stack need not.

Observation 4.3

So stacking loosens both shortfalls at once. One operation acting on two problems—which is why real lenses carry multilayer coatings.

Observation 4.4

To be stated honestly—this paper has not computed multilayer stacks. That more layers do better is stated as known—the numbers given here are for a single layer.

Observation 4.5

Knowing the single-layer limit is still worth something. 0.013541 at 400 nm and 0.012601 at the design wavelength—those two numbers are the answer to “why are stacks needed”.

5. The Separator: Phase, or Amplitude?

Observation 5.1

The separator—whether the broken condition is the phase one or the amplitude one.

Broken condition	Broken by	At 550 nm	At 400 nm
Neither (ideal film)	—	0.000000	0.013541
Phase only	wavelength	0.000000	0.013541
Amplitude only (MgF ₂)	material	0.012601	0.022053

Observation 5.2

The third row leaves something at 550 nm; the second does not. The value at the design wavelength distinguishes them—look there and you know which condition is broken.

Observation 5.3

So the symptom can be resolved into its causes. What remains at the design wavelength is the material shortfall; what grows away from it is the wavelength shortfall—one “reflection remains” decomposed into two.

Observation 5.4

The separator can be moved. Reset the design wavelength to 400 nm and it becomes zero there—which wavelength is made perfect can be chosen. “All of them” cannot be, with one layer.

Observation 5.5

To be stated honestly—0.013541 and 0.012601 are values under stated conditions: $n_s = 1.52$, $\lambda_0 = 550$ nm, and indices independent of wavelength. Another glass gives other numbers—what does not move is the structure: the two conditions break on different quantities.

Observation 5.6

One last thing—when one symptom has two causes, fixing one completely does not remove the symptom. Find the ideal material and the 0.013541 at 400 nm remains—miscount the causes and a fix that did not work gets mistaken for a shortfall in technique.

6. The Census and the Close

1. Bare glass 0.042580 (Calculation 2.1): matches the accepted 4 per cent.
2. Ideal index 1.232883 (Observation 3.3): $\sqrt{1.52}$.
3. Exactly zero at 550 nm (Calculation 2.1): both conditions met.

4. The same film gives **0.013541** at 400 nm (Calculation 2.1): **a third of bare.**
5. It rises on both sides (Observation 2.3): **a single lowest point.**
6. The short-wavelength side is worse (Observation 2.4): **phase goes as $1/\lambda$.**
7. MgF_2 is **1.38** (Observation 3.3): **0.1471 from the ideal.**
8. It leaves **0.012601** at the design wavelength (Observation 3.2): **amplitude broken.**
9. Still under a third of bare (Observation 3.4): **it cannot finish, that is all.**
10. The two shortfalls are independent (Observation 3.5): **fix one and the other survives.**

Closing: An anti-reflection coating is described as “a film that cancels reflection”——but it cancels only when two conditions hold at once. There is a phase condition and an amplitude condition——a thickness of $\lambda/4$ and an index of $\sqrt{n_0 n_s}$. And the two break on different quantities: phase on wavelength, amplitude on material. Even an ideal film cancels at one wavelength only——a film of index **1.232883** is exactly zero at 550 nm, and back to **0.013541** at 400 nm, nearly a third of bare glass’s 0.042580 (4.2580 per cent, matching the accepted 4). It is a valley with a single lowest point, and the short-wavelength side is worse because the phase error goes as $1/\lambda$. And there is no ideal material——no dense solid has an index as low as $\sqrt{1.52} = 1.232883$, and real MgF_2 is 1.38, 0.1471 away. So even at 550 nm, where the phase condition is met, it leaves **0.012601** (1.2601 per cent). It still brings bare glass to under a third, so it is not that it fails — it is that it cannot finish. The two shortfalls are independent——with an ideal material only the wavelength error remains; in monochromatic light only the material error does. Fix either and the other survives. By the criterion of Paper 300 they have distinct roots: one comes from geometry, the other from material. One thing separates them——whether the broken condition is the phase one or the amplitude one. What remains at the design wavelength is the material shortfall; what grows away from it is the wavelength shortfall, so one symptom decomposes into two. Reset the design wavelength to 400 nm and it becomes zero there — which wavelength is made perfect can be chosen, but “all of them” cannot be, with one layer. To be said honestly——this paper has not computed multilayer stacks. More layers cancel at more wavelengths and loosen the material constraint at the same time — one operation acting on two problems — which is why real lenses carry them. Those two numbers are the answer to “why are stacks needed”. One last thing——when one symptom has two causes, fixing one completely does not remove the symptom. Find the ideal material and the 0.013541 at 400 nm remains. Miscount the causes and a fix that did not work gets mistaken for a shortfall in technique.

References

1. Rayleigh, Lord (1886). On the Intensity of Light Reflected from Certain Surfaces at Nearly Perpendicular Incidence. *Proceedings of the Royal Society of London*.
2. Macleod, H. A. (2010). *Thin-Film Optical Filters* (4th ed.). Boca Raton: CRC Press.
3. Born, M., Wolf, E. (1999). *Principles of Optics* (7th ed.). Cambridge: Cambridge University Press.

4. Raut, H. K., Ganesh, V. A., Nair, A. S., Ramakrishna, S. (2011). Anti-reflective coatings: A critical, in-depth review. *Energy & Environmental Science*.
5. Hecht, E. (2016). *Optics* (5th ed.). Boston: Pearson.
6. Yamagishi, Y. (2026). *The Three Hundredth Census: Whether Two Things Share a Root Is Decidable* [Paper 300]. Zenodo. 10.5281/zenodo.22171914
7. Yamagishi, Y. (2026). *The “Depth” of Depth of Field Is a Convention, Not a Measurement* [Paper 358]. Zenodo. 10.5281/zenodo.22697216
8. Yamagishi, Y. (2026). *A Bat Has Three Sweet Spots, and They Lie 23 cm Apart* [Paper 363]. Zenodo. 10.5281/zenodo.22697505
9. Yamagishi, Y. (2026). *The Usual Explanation of the Brazil-Nut Effect Fails on the Size of the Gap* [Paper 365]. Zenodo. 10.5281/zenodo.22697622

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