

Halving the Area Does Not Halve the Species — And Halving the Species Takes Cutting the Area to **6.25 Per Cent**

— With exponent $z = 0.25$, halving the area leaves **0.840896** of the species — Cutting to a tenth still leaves **0.562341** — One formula reads either for or against conservation — [Paper 372]

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— *From thinking to syncing.* —

Abstract

One hears that losing half the forest loses half the species. This paper asks whether that proportionality holds—the answer is that it does not, and the exponent settles the whole departure. No new mathematical theorem and no new law is claimed. First, the species-area relation is $S = cA^z$, with z about **0.25** on islands (Section 2). Second, this is the core of the paper. Halving the area leaves **0.840896** of the species (Section 2). Third, cutting to a tenth still leaves **0.562341** (Section 2). Fourth, conversely, halving the species takes cutting the area to **0.0625** (6.25 per cent) (Section 3). Fifth, taking z to **0.15** and **0.35** splits the survival at half area into **0.9013** and **0.7846** (Section 4). Sixth, the separator is the exponent z , not the area and not the taxon (Section 5).

Scope of this paper (scope note): No new mathematical theorem and no new law is claimed—the species-area relation, Arrhenius’s power law, and island biogeography are all standard. We do not build ecology—the equilibrium theory of island biogeography and extinction debt are not entered. They are named and no more. We use no real data—species counts for particular regions are not used. $z = 0.25$ is a stated assumption. We do not discuss conservation policy—no normative judgement about how much to protect is made. Only the consequences of the formula. Time is not treated—the time from area loss to the new equilibrium is not the subject. Only equilibrium values are compared. Taxa are not separated—how z varies with taxon and geography is not the subject. Relation to earlier papers: Paper 354 showed length ceasing to be the quantity that measures a curve—there too an exponent governed the conclusion. Paper 345 showed one logarithm counting a ratio in one place and items in another—there too the question was what is being counted. Paper 300 showed whether two things share a root is decidable—by that test, loss of area and loss of species have distinct roots. Paper 358 showed a convention fixing a number—here the issue is whether z comes from observation or from assumption. What is added is giving the surviving fraction at four area ratios, solving backwards for the 0.0625 that halves the species, confirming the value returns exactly 0.5, showing how far the conclusion moves with z , and putting the separator on z .

1. Notation and the Question

Setting: Relate species number S to area A by a power law.

$$S = cA^z, \quad \frac{S_{\text{after}}}{S_{\text{before}}} = \left(\frac{A_{\text{after}}}{A_{\text{before}}} \right)^z, \quad z = 0.25$$

Observation 1.1

At $z = 1$ it is proportional. Half the area, half the species—intuition assumes this case.

Observation 1.2

But the measured z is far below 1. About 0.25 on islands—and that is the size of the departure.

Observation 1.3

The direction of the departure is single. With $z < 1$, species fall more slowly than area—this paper gives the size.

Question: Halve the area, and how many species remain?

2. Halving Still Leaves 0.840896

Calculation 2.1: Surviving species against the fraction of area kept (computed in this paper, $z = 0.25$).

Area kept	Species kept	Species lost	Area ratio / species ratio
0.90	0.974004	0.025996	0.9240
0.50	0.840896	0.159104	0.5946
0.10	0.562341	0.437659	0.1778
0.01	0.316228	0.683772	0.0316

Observation 2.2

This is the core of the paper.Halving the area still leaves **0.840896** of the species—the loss is 15.9104 per cent, not 50.

Observation 2.3

A tenth of the area still leaves more than half.**0.562341**—lose 90 per cent of the area and only 43.7659 per cent of the species.

Observation 2.4

The rightmost column measures the departure.At an area ratio of 0.10 the species ratio is **0.562341**, so the column reads **0.1778**—the fraction of species kept is **5.6234** times the fraction of area kept.

Observation 2.5

Cut to one per cent and **0.316228** remain.A hundredth of the area holding a third of the species—the consequence of a small power-law exponent.

3. Halving the Species Takes 6.25 Per Cent

Calculation 3.1: Solving for the area at which species halve (computed in this paper).

Quantity	Value
Area fraction halving the species	0.062500 (6.25 per cent)
Species fraction on substituting back	0.500000000000
Closed form	$0.5^{1/z} = 0.5^4$

Observation 3.2

0.0625 is 0.5^4 . Because $z = 0.25$ makes $1/z = 4$ —the round number is no accident; the reciprocal of the exponent is an integer.

Observation 3.3

The formula closes. $0.0625^{0.25} = 0.500000000000$ —not an arithmetic error.

Observation 3.4

So it is asymmetric. Halving the area costs 16 per cent of the species, while halving the species takes cutting 94 per cent of the area—the same word “half” tells entirely different stories depending on which quantity it refers to.

Observation 3.5

That asymmetry gives two readings. “Lose half and 84 per cent of species remain” and “you must cut 94 per cent to halve them”—one and the same formula.

4. The Exponent Governs the Conclusion

Calculation 4.1: Survival at half area against z (computed in this paper).

z	Species kept at half area	Area that halves the species
0.15	0.901250	0.009843
0.25	0.840896	0.062500
0.35	0.784584	0.138011
1.00	0.500000	0.500000

Observation 4.2

Taking z to 0.15 and 0.35 splits survival into 0.9013 and 0.7846. A gap of 0.1167—the answer to “what fraction survives half the area” moves by over ten points on the estimate of z alone.

Observation 4.3

The inverse question moves far more. 0.009843 against 0.138011, a factor of 14.02—against a survival gap of 0.1167, the inverse problem is far more sensitive to z .

Observation 4.4

At $z = 1$ proportionality returns. Half the area, half the species—a check that the formula is doing the work.

Observation 4.5

To be stated honestly— z is measured, not chosen. But it varies with taxon, region, and the range of areas—this paper does not treat its estimation and simply posits 0.25.

5. The Separator: The Exponent z

Observation 5.1

The separator—the exponent z ; not the area and not the taxon.

z	At half area	To halve the species	Proportional
0.15	90.1 per cent kept	cut to 0.98 per cent	no
0.25	84.1 per cent kept	cut to 6.25 per cent	no
1.00	50.0 per cent kept	cut to 50 per cent	yes

Observation 5.2

One number separates the three rows. The area and the taxon are identical— z alone moves the answer from 90.1 to 50.0.

Observation 5.3

The same shape as Paper 354. There the exponent of length growth fixed the dimension—here the exponent on area fixes the species count. In both, one exponent governs everything.

Observation 5.4

The separator cannot be moved. z is a property of the ecosystem, not a design variable—it can only be measured, which is exactly why any argument assuming a z must state where it came from.

Observation 5.5

To be stated honestly—0.840896 and 0.0625 are values under the stated assumption $z = 0.25$. Another z gives other numbers—what does not move is the direction: with $z < 1$, species fall more slowly than area.

Observation 5.6

One last thing—two sentences giving opposite impressions can be built from one formula. “Lose half and 84 per cent remain” and “you must cut 94 per cent to halve them”—both are correct. Whoever chooses which to say has chosen an emphasis, not a formula.

6. The Census and the Close

1. $S = cA^z$ with $z = 0.25$ (Section 1): the island figure.
2. 0.840896 kept at half area (Calculation 2.1): a loss of 15.9104 per cent.
3. 0.562341 kept at a tenth (Calculation 2.1): still over half.
4. 0.316228 at one per cent (Calculation 2.1): a third remains.
5. Halving species takes 0.0625 (Calculation 3.1): 0.5^4 .
6. Substituting back gives 0.500000000000 (Observation 3.3): the formula closes.
7. 0.901250 at $z = 0.15$ (Calculation 4.1): 0.784584 at $z = 0.35$.
8. The inverse moves by 14.02 (Observation 4.3): sensitive to z .
9. $z = 1$ restores proportionality (Observation 4.4): the formula works.
10. z is measured (Observation 5.4): it cannot be designed.

Closing: One hears that losing half the forest loses half the species—that proportionality holds only when the exponent is 1. But the measured exponent is far below 1—in the species-area relation $S = cA^z$, z is about 0.25 on islands, and with $z < 1$ species fall more slowly than area. Halving the area still leaves 0.840896 of the species—the loss is 15.9104 per cent, not 50. Cutting to a tenth still leaves 0.562341: lose 90 per cent of the area and only 43.7659 per cent of the species. Cut to one per cent and 0.316228 remain. Ask it the other way and the story reverses—solving for the area at which species halve gives 0.0625, a cut to 6.25 per cent (with $z = 0.25$ that is 0.5^4 , and substituting back returns 0.500000000000). Halving the area costs 16 per cent of the species, while halving the species takes cutting 94 per cent of the area. The same word “half” tells entirely different stories depending on which quantity it refers to. And the exponent governs the conclusion—taking z to 0.15 and 0.35 splits survival at half area into 0.901250 and 0.784584, while the area that halves the species moves from 0.009843 to 0.138011, a factor of 14.02. The inverse problem is far more sensitive to z . Setting $z = 1$ restores proportionality, confirming the formula is doing the work. One thing separates them—the exponent z ; not the area and not the taxon. With area and taxon identical, z alone moves the answer from 90.1 to 50.0 per cent. The same shape as the exponent of length growth fixing the dimension in Paper 354. And z is measured, not designed, which is why any argument assuming a z must state where it came from. To be said honestly—0.840896 and 0.0625 hold under the stated $z = 0.25$, and

this paper does not treat how z is estimated. What does not move is the direction: with $z < 1$, species fall more slowly than area. One last thing——two sentences giving opposite impressions can be built from one formula. “Lose half and 84 per cent remain” and “you must cut 94 per cent to halve them” — both are correct. Whoever chooses which to say has chosen an emphasis, not a formula.

References

1. Arrhenius, O. (1921). Species and Area. *Journal of Ecology*.
2. Preston, F. W. (1962). The Canonical Distribution of Commonness and Rarity. *Ecology*.
3. MacArthur, R. H., Wilson, E. O. (1967). *The Theory of Island Biogeography*. Princeton: Princeton University Press.
4. Rosenzweig, M. L. (1995). *Species Diversity in Space and Time*. Cambridge: Cambridge University Press.
5. He, F., Hubbell, S. P. (2011). Species-area relationships always overestimate extinction rates from habitat loss. *Nature*.
6. Yamagishi, Y. (2026). *The Three Hundredth Census: Whether Two Things Share a Root Is Decidable* [Paper 300]. Zenodo. 10.5281/zenodo.22171914
7. Yamagishi, Y. (2026). *One and the Same $\log_2$ Counts a Ratio in One Law and Items in the Other* [Paper 345]. Zenodo. 10.5281/zenodo.22682287
8. Yamagishi, Y. (2026). *Length Can Stop Being the Quantity That Measures a Curve* [Paper 354]. Zenodo. 10.5281/zenodo.22696977
9. Yamagishi, Y. (2026). *The “Depth” of Depth of Field Is a Convention, Not a Measurement* [Paper 358]. Zenodo. 10.5281/zenodo.22697216

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