

What Sets the Amplitude of a Predator--Prey Cycle Is the Initial Condition, Not the Equation

— With the coefficients unchanged, moving the start from **13** to **30** takes the amplitude ratio from **1.0518** to **6.8463** — There is no damping, so it never shrinks — The equation does not answer “how large is this ecosystem’s cycle” — [Paper 373]

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— *From thinking to syncing.* —

Abstract

Predator and prey are said to rise and fall in cycles. This paper asks what sets the size of that cycle—the answer is that the initial condition sets it, not the equation. No new mathematical theorem and no new law is claimed. First, the fixed point of the Lotka--Volterra system is $(c/d, a/b) = (13.3333, 10.0000)$ (Section 2). Second, this is the core of the paper. Starting at $(13, 10)$ gives an amplitude ratio of **1.0518**; at $(30, 10)$ it is **6.8463** (Section 2). Third, not one of the coefficients a, b, c, d was changed (Section 2). Fourth, the conserved quantity V is nearly constant along an orbit, drifting by 1.457×10^{-4} (integration error) (Section 3). Fifth, doubling the time does not shrink the amplitude, because there is no damping (Section 3). Sixth, the separator is whether the system has something to be drawn toward, not whether it oscillates (Section 5).

Scope of this paper (scope note): No new mathematical theorem and no new law is claimed—the Lotka–Volterra equations, neutral stability, and the conserved quantity are all standard. We do not build dynamical systems theory—the general theory as a Hamiltonian system and structural stability under perturbation are not entered. They are named and no more. We do not build ecology—what sets the oscillation of real populations is not the subject. Only the properties of the equations. We do not discuss numerics—Euler’s method is used and its error stated. Higher-order and symplectic integrators are not the subject. We do not extend the model—adding a logistic term or a functional response is not the subject. Adding either destroys the neutral stability. We use no data—records such as the Canadian lynx pelt series are not used. Relation to earlier papers: Paper 367 showed a conserved total with only the distribution free—there too a conserved quantity made the conclusion. Paper 357 showed “optimal” undetermined without an objective—here “amplitude” is undetermined without an initial condition. Paper 300 showed whether two things share a root is decidable—by that test, the period of an oscillation and its amplitude have distinct roots. Paper 371 showed a difference appearing with no cause—here the answer differs by 6.5 times with the equation unchanged. What is added is giving the amplitude ratio at four initial conditions, confirming that V is constant along an orbit, stating the numerical drift 1.457×10^{-4} openly, showing as a control that doubling the time does not shrink it, and putting the separator on whether there is an attractor.

1. Notation and the Question

Setting: Write prey x and predator y by the Lotka–Volterra equations.

$$\frac{dx}{dt} = ax - bxy, \quad \frac{dy}{dt} = -cy + dxy, \quad V = dx - c \ln x + by - a \ln y$$

Take $a = 1.000$, $b = 0.100$, $c = 1.000$, $d = 0.075$. V is a conserved quantity, independent of time.

Observation 1.1

The fixed point is $(c/d, a/b)$. With these coefficients, $(13.3333, 10.0000)$ —start there and nothing moves.

Observation 1.2

Away from it the system oscillates. But how far it swings is nowhere in the equations—the initial condition sets it.

Observation 1.3

There is no damping term.Nothing of the form $-\gamma x$ anywhere——so the amplitude does not shrink.

Question: What sets the size of the cycle?

2. The Initial Condition Alone Moves It by 6.5

Calculation 2.1: Prey excursion against initial condition (computed in this paper, Euler, $h = 10^{-4}$, $T = 40$).

Start (x_0, y_0)	Minimum	Maximum	Amplitude ratio	V
(13.0, 10.0)	12.9994	13.6729	1.0518	−2.892534
(16.0, 10.0)	10.9770	16.0053	1.4581	−2.875174
(20.0, 10.0)	8.3364	20.0144	2.4008	−2.798317
(30.0, 10.0)	4.3869	30.0344	6.8463	−2.453782

Observation 2.2

This is the core of the paper.The amplitude ratio runs from 1.0518 to 6.8463, a factor of 6.5——and not one of a, b, c, d was changed.

Observation 2.3

Nearer the fixed point, smaller.(13, 10) sits right beside the fixed point (13.3333, 10)——amplitude ratio 1.0518: it barely moves.

Observation 2.4

V differs by initial condition.From −2.892534 to −2.453782—— V is the orbit's name tag. The orbits nest and never cross.

Observation 2.5

So the equations give only a family of orbits. Which orbit one rides comes from outside the equations—the same shape as “optimal” being undetermined without an objective in Paper 357.

3. No Damping, So It Never Shrinks

Calculation 3.1: V along one orbit (computed in this paper, start $(20, 10)$).

Time	V
Start	-2.798317
Four samples along the way	$-2.798317 / -2.798271 / -2.798221 / -2.798172$
Total drift	1.457×10^{-4}

Observation 3.2

V is nearly constant. The drift is 1.457×10^{-4} —that is Euler-method error, not physical damping.

Observation 3.3

To be stated honestly—the drift grows in one direction because Euler’s method does not conserve V . A symplectic integrator would keep it bounded—this paper has not implemented one. It is stated openly as numerical coarseness.

Observation 3.4

Doubling the time does not shrink the amplitude. The ratio at $T = 40$ and $T = 80$ is nearly the same—as it must be with no damping term.

Observation 3.5

This is called neutral stability. A perturbation neither decays nor grows—it moves to another orbit and stays there.

Observation 3.6

So the whole history of disturbances remains. Shake it hard once and that amplitude continues indefinitely—a past that a damped system would forget is never erased here.

4. What Breaks It

Observation 4.1

Add a logistic term to the prey and the neutral stability breaks. A term of the form $ax(1 - x/K)$ —the fixed point becomes an attractor and the amplitude settles to one value.

Observation 4.2

Add a functional response and a limit cycle appears. One amplitude, independent of the start—in both cases the memory of the initial condition is erased.

Observation 4.3

This paper has computed neither. They are cited and no more—what is given numerically here is the bare Lotka–Volterra case.

Observation 4.4

Something can still be said. Asking “what is this ecosystem’s amplitude” of the bare system is not a well-formed question—the answer lives in the initial condition, not in the equation.

Observation 4.5

The period, by contrast, is set by the equation. Near the fixed point it is $2\pi/\sqrt{ac}$ —period and amplitude are fixed by different things: distinct roots by the criterion of Paper 300.

5. The Separator: Is There an Attractor?

Observation 5.1

The separator—whether the system has something to be drawn toward; not whether it oscillates.

System	Oscillates	Amplitude set by	Memory of the start
Lotka--Volterra (bare)	yes	the initial condition	kept
With damping	yes (decaying)	the equation (to 0)	erased
Limit cycle	yes	the equation	erased

Observation 5.2

All three oscillate. The observation “it is periodic” does not distinguish them—what distinguishes them is whether the amplitude depends on the start.

Observation 5.3

The rightmost column is the practical difference. If the memory is kept, past disturbances can be inferred—if erased, the present amplitude says nothing about the past.

Observation 5.4

The separator can be moved. Adding one logistic term moves the top row to the bottom—neutral stability is fragile against added terms.

Observation 5.5

To be stated honestly——**6.8463** and 1.457×10^{-4} are values under stated coefficients, initial conditions, and integrator. Another setting gives other numbers——what does not move is the structure: in the bare system the amplitude is set by the initial condition.

Observation 5.6

One last thing——do not put to an equation a question it does not answer. “What is this system’s amplitude” acquires meaning, in the bare Lotka–Volterra, only once an initial condition is named——when no answer comes back, sometimes the model is not deficient; the question is.

6. The Census and the Close

1. Fixed point (**13.3333, 10.0000**) (Section 1): $(c/d, a/b)$.
2. (**13, 10**) gives ratio **1.0518** (Calculation 2.1): **beside the fixed point**.
3. (**30, 10**) gives **6.8463** (Calculation 2.1): **a factor of 6.5**.
4. No coefficient was changed (Observation 2.2): **only the start**.
5. V differs by orbit (Observation 2.4): **the orbit’s name tag**.
6. V nearly constant along an orbit (Calculation 3.1): **drift 1.457×10^{-4}** .
7. The drift is integration error (Observation 3.3): **not physical damping**.
8. Doubling the time changes nothing (Observation 3.4): **no damping term**.
9. A logistic term breaks it (Observation 4.1): **a fragile property**.
10. The period is set by the equation (Observation 4.5): **a different root from amplitude**.

Closing: Predator and prey are said to rise and fall in cycles——but how far the cycle swings is nowhere in the equations. The fixed point of the Lotka–Volterra system is $(c/d, a/b) = (13.3333, 10.0000)$ ——start there and nothing moves; away from it the system oscillates, and the initial condition sets how far. With the coefficients unchanged, the start alone moves the amplitude by 6.5——(**13, 10**) gives an amplitude ratio of **1.0518** (right beside the fixed point, so it barely moves) and (**30, 10**) gives **6.8463**. Not one of a, b, c, d was changed. The conserved V runs from -2.892534 to -2.453782 , one value per orbit: it is the orbit’s name tag, and the orbits nest without crossing. And with no damping, it never shrinks——along one orbit V is nearly constant, drifting by 1.457×10^{-4} , which is Euler-method error and not physical damping. Doubling the time leaves the ratio nearly the same. This is

neutral stability: a perturbation neither decays nor grows, but moves to another orbit and stays. So the whole history of disturbances remains — shake it hard once and that amplitude continues indefinitely. The property is fragile, though——add a logistic term to the prey and the fixed point becomes an attractor; add a functional response and a limit cycle appears. In both cases the memory of the initial condition is erased. This paper has computed neither and cites them only. One thing separates them——whether the system has something to be drawn toward; not whether it oscillates. All three systems oscillate, so “it is periodic” does not distinguish them — what does is whether the amplitude depends on the start. If the memory is kept, past disturbances can be inferred; if erased, the present amplitude says nothing about the past. To be said honestly——the drift grows in one direction because Euler’s method does not conserve V ; a symplectic integrator would keep it bounded, and this paper has not implemented one. The 6.8463 holds for the stated coefficients and integrator. One last thing——do not put to an equation a question it does not answer. “What is this system’s amplitude” acquires meaning, in the bare Lotka–Volterra, only once an initial condition is named. When no answer comes back, sometimes the model is not deficient; the question is.

References

1. Lotka, A. J. (1925). *Elements of Physical Biology*. Baltimore: Williams & Wilkins.
2. Volterra, V. (1926). Variazioni e fluttuazioni del numero d’individui in specie animali conviventi. *Memorie della R. Accademia Nazionale dei Lincei*.
3. May, R. M. (1972). Limit Cycles in Predator-Prey Communities. *Science*.
4. Murray, J. D. (2002). *Mathematical Biology I: An Introduction* (3rd ed.). New York: Springer.
5. Hairer, E., Lubich, C., Wanner, G. (2006). *Geometric Numerical Integration* (2nd ed.). Berlin: Springer.
6. Yamagishi, Y. (2026). *The Three Hundredth Census: Whether Two Things Share a Root Is Decidable* [Paper 300]. Zenodo. 10.5281/zenodo.22171914
7. Yamagishi, Y. (2026). *Maximum Power and Maximum Efficiency Are Not the Same Point* [Paper 357]. Zenodo. 10.5281/zenodo.22697180
8. Yamagishi, Y. (2026). *Push the Sensitivity Down Somewhere and It Must Rise Somewhere Else* [Paper 367]. Zenodo. 10.5281/zenodo.22697709
9. Yamagishi, Y. (2026). *With No Cause at All, the Bottom Rises 1.76 Standard Deviations and the Top Falls 1.76* [Paper 371]. Zenodo. 10.5281/zenodo.22698226

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