

An Achromat Meets at Two Wavelengths Only

— A single lens shifts focus by **1.558361** mm with colour (**1.5584** per cent of its focal length) — Two glasses cut that to **0.044604** mm, a factor of **34.9374**, but not to zero — The cause of the residue can be named by one number: the difference in partial dispersion — [Paper 377]

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— *From thinking to syncing.* —

Abstract

An achromatic lens is called a lens with the chromatic aberration removed. This paper asks how far it is removed — the answer is at two wavelengths only, with a third colour remaining. No new mathematical theorem and no new law is claimed. First, a single lens shifts focus with colour by the focal length divided by the Abbe number, **1.558361** mm (Section 2). Second, the achromatic condition gives $f_1 = 43.322425$ mm and $f_2 = -76.436624$ mm (Section 3). Third, the second element must be negative, that is, diverging (Section 3). Fourth, this is the core of the paper. Even with two glasses, **0.044604** mm remains (Section 4). Fifth, the improvement is **34.9374**-fold, and the residue's cause is named by one number: the difference in partial dispersion (Section 4). Sixth, the separator is first-order against second-order dispersion, not the number of elements (Section 5).

Scope of this paper (scope note): No new mathematical theorem and no new law is claimed—the Abbe number, the achromatic condition, and the secondary spectrum are all standard. We do not build lens design—apochromat design and the selection of anomalous-dispersion glasses are not entered. They are named and no more. Other aberrations are not treated—spherical aberration, coma, and astigmatism are not the subject. The computation is thin-lens and paraxial. We measure no glass—Abbe numbers and partial dispersions are taken as representative figures from the literature. They are not the values of particular products. Manufacture is not treated—cementing and tolerances are not the subject. Diffraction is not treated—comparison with the diffraction limit is not the subject. Relation to earlier papers: Paper 370 showed a single anti-reflection layer cancelling at one wavelength only—there one layer for one wavelength; here two glasses for two. The same counting. Paper 363 showed a bat to have three sweet spots—there too the number of conditions matched the number of points. Paper 300 showed whether two things share a root is decidable—by that test, first-order and second-order dispersion have distinct roots. Paper 368 showed algebraically equal forms returning different digits—there too the residue’s cause could be named by one number. What is added is giving the single-lens shift as a number, solving the achromatic condition for the two focal lengths, confirming the combined focal length returns, showing the secondary spectrum falling by 34.9374 without reaching zero, and putting the separator on first against second order.

1. Notation and the Question

Setting: Build an achromat of combined focal length $f = 100$ mm from two glasses of Abbe number V .

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}, \quad \frac{1}{f_1 V_1} + \frac{1}{f_2 V_2} = 0$$

Take crown $V_1 = 64.17$ and flint $V_2 = 36.37$.

Observation 1.1

The Abbe number acts like an inverse dispersion. The larger it is, the less the colours separate—a single lens’s chromatic focal shift is f/V .

Observation 1.2

Two equations fix two unknowns. f_1 and f_2 —the number of equations is the number of wavelengths that can be brought together.

Observation 1.3

There is no equation for a third colour. Only two conditions have been imposed——this paper gives the remainder as a number.

Question: How far does an achromat remove chromatic aberration?

2. A Single Lens Shifts by 1.558361 mm

Calculation 2.1: Chromatic focal shift of a single crown lens (computed in this paper, $f = 100$ mm).

Quantity	Value
Abbe number V	64.17
Focal shift f/V	1.558361 mm
As a fraction of the focal length	1.5584 per cent

Observation 2.2

1.5584 per cent is not small. 1.5 mm on a 100 mm lens——blue and red focus 1.5 mm apart, which in a photograph is a clear coloured fringe.

Observation 2.3

The shift scales with focal length. Because it is f/V ——the longer the lens, the larger in absolute terms. That is why telescopes met the problem first, historically.

3. The Second Element Must Diverge

Calculation 3.1: Focal lengths from the achromatic condition (computed in this paper).

Quantity	Value
Crown f_1	43.322425 mm
Flint f_2	−76.436624 mm
Check $1/f_1 + 1/f_2$	0.0100000000 ($1/f = 0.0100000000$)
Residual of the achromatic condition	0.000×10^0

Observation 3.2

The second element is negative.—76.436624 mm——without adding a diverging lens the colour does not cancel.

Observation 3.3

So the first is stronger than the target.43.322425 mm is shorter than 100 mm——it must bend extra to survive the diverging element.

Observation 3.4

The formulae close.The combination gives 0.0100000000 and the achromatic residual is exactly 0——not an arithmetic error.

Observation 3.5

The difference of Abbe numbers sits in the denominator. $64.17 - 36.37 = 27.80$ ——a small difference makes both f_1 and f_2 extreme. The two must be glasses of different dispersion.

4. The Secondary Spectrum Leaves 0.044604 mm

Calculation 4.1: Residual from the difference in partial dispersion (computed in this paper).

Quantity	Value
Crown partial dispersion P_1	0.3076
Flint partial dispersion P_2	0.2952
Secondary spectrum $f(P_1 - P_2)/(V_1 - V_2)$	0.044604 mm
Relative to the single lens	1/34.9374
With $P_1 = P_2$	0.000×10^0 mm

Observation 4.2

This is the core of the paper. It falls by 34.9374 and does not reach zero——0.044604 mm remains.

Observation 4.3

The cause is named by one number. The difference in partial dispersion, $P_1 - P_2 = 0.0124$ ——set it to zero and the secondary spectrum is exactly zero. The residue's origin is identified.

Observation 4.4

So it is not that “two elements are not enough”. Two elements satisfy the two conditions——what is missing is a pair of glasses whose partial dispersions agree.

Observation 4.5

The same shape as Paper 370. There a single layer demanded a material of the ideal index——here achromatism demands a pair of matched partial dispersions. Both reduce to whether a material meeting the condition exists.

Observation 4.6

To be stated honestly——this paper does not treat anomalous-dispersion glasses. With fluorite and the like the secondary spectrum falls further——cited and no more; what is computed here is the ordinary two-glass case.

5. The Separator: First Order, or Second?

Observation 5.1

The separator——whether the difference in Abbe number (first order) or in partial dispersion (second order) has been cancelled; not the number of elements.

	Cancelled term	Residue	Relative to a single lens
Single lens	none	1.558361 mm	1
Achromat (two)	first order (Abbe)	0.044604 mm	1/34.9374
Matched partial dispersions	first and second	0.000000 mm	0

Observation 5.2

The third row is a condition on materials, not on design. Add elements and the residue survives if the partial dispersions differ——how far it can be removed is settled on the side of the available glass.

Observation 5.3

Distinct roots by the criterion of Paper 300. First-order dispersion comes from the Abbe number, second-order from partial dispersion——both produce the same symptom, “the colours miss”, from different roots.

Observation 5.4

The separator can be moved. Anomalous-dispersion glasses shrink $P_1 - P_2$ —but they are expensive and hard to work. It can be moved, at a price.

Observation 5.5

To be stated honestly—0.044604 mm and 34.9374 are values under stated Abbe numbers and partial dispersions, thin-lens and paraxial. Another glass gives other numbers—what does not move is the structure: as many wavelengths meet as conditions imposed, and beyond that the material decides.

Observation 5.6

One last thing—one may say “removed” only for as many wavelengths as conditions were imposed. Impose two equations and two wavelengths meet, with no guarantee whatever for a third colour—to count what has been removed, count how many conditions were imposed.

6. The Census and the Close

1. Single lens shifts 1.558361 mm (Calculation 2.1): 1.5584 per cent of f .
2. The shift scales with f (Observation 2.3): longer lenses are worse.
3. $f_1 = 43.322425$ mm (Calculation 3.1): shorter than 100 mm.
4. $f_2 = -76.436624$ mm (Calculation 3.1): a diverging element is required.
5. The combination gives 0.0100000000 (Observation 3.4): the formulae close.
6. The achromatic residual is exactly 0 (Calculation 3.1): both conditions met.
7. Secondary spectrum 0.044604 mm (Calculation 4.1): a factor of 34.9374.
8. With $P_1 = P_2$ it is exactly 0 (Observation 4.3): the cause is named.
9. What is missing is a pair of glasses (Observation 4.4): not more elements.
10. The Abbe difference is in the denominator (Observation 3.5): different dispersions are needed.

Closing: An achromatic lens is called a lens with the chromatic aberration removed—but what is removed is as many wavelengths as conditions were imposed. A single lens shifts focus with colour by the focal length divided by the Abbe number—1.558361 mm at $f = 100$ mm and $V = 64.17$, 1.5584 per cent of the focal length. Blue and

red focus 1.5 mm apart on a 100 mm lens, a clear coloured fringe in a photograph. Since the shift is f/V it grows with focal length, which is why telescopes met the problem first. Two equations fix two focal lengths——
 $f_1 = 43.322425$ mm and $f_2 = -76.436624$ mm. The second must be negative — without a diverging element the colour does not cancel. The first is therefore shorter than the target 100 mm, bending extra to survive it. The combination gives 0.010000000 and the achromatic residual is exactly 0. And still a third colour remains——the secondary spectrum from the difference in partial dispersion is **0.044604** mm, a fall by **34.9374** that does not reach zero. And its cause is named by one number: set the partial-dispersion difference $P_1 - P_2 = 0.0124$ to zero and the secondary spectrum is exactly zero. So it is not that two elements are not enough — what is missing is a pair of glasses whose partial dispersions agree. The same shape as Paper 370——there a single layer demanded a material of the ideal index, and here achromatism demands a matched pair. Both reduce to whether a material meeting the condition exists. One thing separates them——whether the first-order (Abbe) or the second-order (partial dispersion) difference has been cancelled; not the number of elements. Add elements and the residue survives if the partial dispersions differ, so how far it can be removed is settled on the side of the available glass. Anomalous-dispersion glasses move it, at the price of cost and workability. To be said honestly——0.044604 mm and 34.9374 hold for the stated Abbe numbers and partial dispersions, thin-lens and paraxial. This paper does not treat anomalous-dispersion glasses and computes the ordinary two-glass case. One last thing——one may say “removed” only for as many wavelengths as conditions were imposed. Impose two equations and two wavelengths meet, with no guarantee whatever for a third colour. To count what has been removed, count how many conditions were imposed.

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