

How Much You Remember Is Decided by How You Test

— One memory, one day later: **71.70** per cent by free recall and **91.07** per cent by savings in relearning — The gap widens with time, reaching **3.2698** after a month — And an exponential forgetting curve predicts 10^{-13} at a month, breaking down — [Paper 378]

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— *From thinking to syncing.* —

Abstract

A forgetting curve shows memory falling with time. This paper asks what quantity is falling—the answer is a different quantity for each way of testing, giving three answers one day later. No new mathematical theorem and no new law is claimed. First, there are at least three measures of retention: free recall, recognition, and savings in relearning (Section 2). Second, this is the core of the paper. One day later the answers split into **71.70**, **86.45** and **91.07** per cent (Section 2). Third, the gap widens with time, from 1.0142 after an hour to **3.2698** after a month (Section 3). Fourth, an exponential forgetting curve predicts 9.358×10^{-14} after a month (Section 4). Fifth, a power law gives 0.356937, which matches experience; the exponential's breakdown is evidence that forgetting is not exponential (Section 4). Sixth, the separator is which measure is used, not the amount of memory (Section 5).

Scope of this paper (scope note): No new mathematical theorem and no new law is claimed——Ebbinghaus’s forgetting curve, the savings method, and power-law retention are all standard. We do not build a theory of memory——whether loss occurs in encoding, storage, or retrieval is not entered. It is named and no more. We perform no experiment——no measurement on participants is made. The computation is on a model stated explicitly. We do not derive the exponents——the retention exponent $\beta = 0.30$ and the three measure exponents are assumptions. They are set to match the tendency in the literature. Individual differences are not treated——age, material, and amount of study are not the subject. Interventions are not discussed——study methods such as spaced repetition are not the subject. Relation to earlier papers: Paper 358 showed a depth-of-field number fixed by a convention——there the convention was the circle of confusion; here it is the measure. Paper 352 showed a transition temperature moving with the speed of measurement——there too the measurement made the number. Paper 300 showed whether two things share a root is decidable——by that test, no two of the three measures share a root. Paper 355 showed an empirical law naming its own breakdown——here the exponential names its own at a month. What is added is giving retention on three measures at six times, showing the three answers one day later, showing the gap widening with time, using the exponential’s breakdown at a month as a control, and putting the separator on which measure is used.

1. Notation and the Question

Setting: Posit power-law retention and express the three measures as powers of it.

$$R(t) = \left(1 + \frac{t}{S}\right)^{-\beta}, \quad S = 24 \text{ hours}, \quad \beta = 0.30$$

$$\text{free recall} = R^{1.6}, \quad \text{recognition} = R^{0.7}, \quad \text{savings} = R^{0.45}$$

Observation 1.1

There is more than one way to measure “remembering”. Producing it unprompted, identifying it when shown, or relearning it faster——all three are called retention.

Observation 1.2

They differ in severity. Free recall is strictest and savings in relearning most forgiving——different answers for one memory.

Observation 1.3

This paper gives the difference as a number. Not “it depends on the measure” but “by how much”—— knowing the size tells you whether results can be compared.

Question: Does “what percentage remains after a day” have one answer?

2. Three Answers One Day Later

Calculation 2.1: Retention on three measures (computed in this paper).

Elapsed [h]	Free recall	Recognition	Savings	Max / min
1	0.980596	0.991464	0.994504	1.0142
9	0.858252	0.935312	0.957920	1.1161
24	0.716978	0.864537	0.910670	1.2702
48	0.590176	0.793971	0.862162	1.4609
144	0.392964	0.664553	0.768974	1.9569
720	0.192374	0.486199	0.629022	3.2698

Observation 2.2

This is the core of the paper. One day later the answer is 71.70, 86.45, or 91.07 per cent——the same memory, the same moment, the same person; only what is measured differs.

Observation 2.3

The spread is 19.37 points. 91.07 against 71.70——“only seven-tenths remains” and “nine-tenths remains” describe the same state.

Observation 2.4

The ordering never changes. Free recall < recognition < savings—at every time. The order of severity is preserved.

Observation 2.5

At $t = 0$ all three give $1.R(0) = 1$ raised to any power—a check that the formulae close.

3. The Gap Widens with Time

Observation 3.1

After an hour the spread is only 1.0142.0.980596 against 0.994504—measured immediately, every measure gives nearly the same answer.

Observation 3.2

After a month it is 3.2698.0.192374 against 0.629022—more than threefold. The longer the interval, the more the choice of measure governs the conclusion.

Observation 3.3

This makes comparison hard. Short-interval and long-interval studies cannot be set side by side with different measures—a difference invisible at short intervals becomes threefold at long ones.

Observation 3.4

The verdict at a month can invert. 19.2 per cent by free recall and 62.9 by savings—“almost all forgotten” and “over sixty per cent remains” describe one memory.

4. The Exponential Breaks at a Month

Calculation 4.1: Power law against exponential (computed in this paper).

Elapsed	Power law $(1 + t/S)^{-\beta}$	Exponential $e^{-t/S}$
1 hour	0.987828	0.959189
720 hours (a month)	0.356937	9.358×10^{-14}

Observation 4.2

At an hour the two are close.0.987828 against 0.959189, a gap of 0.0286——over short intervals the two models cannot be told apart.

Observation 4.3

At a month they differ by orders of magnitude.0.356937 against 9.358×10^{-14} ——the exponential predicts that nothing whatever survives from a month ago.

Observation 4.4

That is plainly wrong.People recognise words learned a month earlier——the exponential’s breakdown is evidence that forgetting is not exponential.

Observation 4.5

The same shape as Paper 355.There an empirical law named its own breaking speed——here the exponential names its own limit by producing an impossible value on extrapolation.

Observation 4.6

To be stated honestly— $\beta = 0.30$ and the three measure exponents are assumptions made here. They match the tendency in the literature and are not measurements—what is shown is the structure that different measures give different answers, and its size.

5. The Separator: Which Measure Is Used

Observation 5.1

The separator—which measure is used; not the amount of memory.

Measure	What it asks	After a day	After a month
Free recall	can you produce it unprompted	71.70 per cent	19.24 per cent
Recognition	can you identify it when shown	86.45 per cent	48.62 per cent
Savings	do you relearn it faster	91.07 per cent	62.90 per cent

Observation 5.2

All three rows are about one memory. One state of the brain is being measured—there are three numbers because there are three questions.

Observation 5.3

No two share a root by the criterion of Paper 300. The presence of retrieval cues, a judgement task, and speed of relearning—one word “retention” naming three operations.

Observation 5.4

The separator can be moved but not removed. Fix one measure and results become comparable—but “which is the real retention” has no answer. All three are retention.

Observation 5.5

To be stated honestly—71.70 and 3.2698 are values on a stated model with assumed exponents. Other material or participants give other numbers—what does not move is the direction: stricter measures read lower, and the gap widens with time.

Observation 5.6

One last thing—“what percentage remains” acquires an answer only once the measure is stated. A report without its measure could be either 71.70 or 91.07—that one number is written does not mean there is one answer.

6. The Census and the Close

1. At least three measures (Observation 1.1): **all called retention.**
2. 71.70, 86.45, 91.07 per cent after a day (Calculation 2.1): **one memory.**
3. A spread of 19.37 points (Observation 2.3): **seven-tenths against nine.**
4. The ordering never changes (Observation 2.4): **severity is preserved.**
5. All three give 1 at $t = 0$ (Observation 2.5): **the formulae close.**
6. 1.0142 after an hour (Observation 3.1): **nearly the same answer.**
7. 3.2698 after a month (Observation 3.2): **more than threefold.**
8. The exponential gives 9.358×10^{-14} (Calculation 4.1): **nothing survives.**
9. The power law gives 0.356937 (Calculation 4.1): **this matches experience.**
10. The two models agree after an hour (Observation 4.2): **indistinguishable at short intervals.**

Closing: A forgetting curve shows memory falling with time—but the quantity that is falling differs with the way of measuring. There is more than one way to measure “remembering”—producing it unprompted (free recall), identifying it when shown (recognition), or relearning it faster (savings). All three are called retention, and they

differ in severity. So one day later there are three answers——**71.70, 86.45 and 91.07** per cent. The same memory, the same moment, the same person; only what is measured differs. The spread is **19.37** points, so “only seven-tenths remains” and “nine-tenths remains” describe the same state. The ordering never changes, and at $t = 0$ all three give 1. And the gap widens with time——after an hour it is only **1.0142**, so measured immediately every measure agrees; after a month it is **3.2698**. 19.2 per cent by free recall and 62.9 by savings — “almost all forgotten” and “over sixty per cent remains” describing one memory. Short-interval and long-interval studies cannot be set side by side with different measures. The same thing happens on the model’s side——power law and exponential give 0.987828 and 0.959189 after an hour, indistinguishable over short intervals. But after a month the power law gives 0.356937 and the exponential **9.358×10^{-14}** — it predicts that nothing whatever survives from a month ago. That is plainly wrong, and its breakdown is evidence that forgetting is not exponential. The same shape as the empirical law naming its own breaking speed in Paper 355. One thing separates them——which measure is used; not the amount of memory. All three rows are about one memory and one state of the brain: there are three numbers because there are three questions. Fix one measure and results become comparable, but “which is the real retention” has no answer — all three are retention. To be said honestly—— $\beta = 0.30$ and the three measure exponents are assumptions made here, set to match the tendency in the literature rather than measured. What is shown is the structure that different measures give different answers, and its size. One last thing——“what percentage remains” acquires an answer only once the measure is stated. A report without its measure could be either 71.70 or 91.07. That one number is written does not mean there is one answer.

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