

The Limit on Running Speed Is Not How Fast the Legs Move but How Hard the Ground Is Pushed

— As speed rises only the contact time shortens, and the duty factor falls from **0.6667** to **0.3714** — Supporting the same body weight then takes a mean vertical force rising from **1.5** to **2.6923** times body weight — The aerial time barely changes — [Paper 380]

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— *From thinking to syncing.* —

Abstract

It is tempting to think one cannot run faster because the legs cannot turn over quickly enough. This paper asks which quantity reaches its limit first—the answer is the force applied to the ground. No new mathematical theorem and no new law is claimed. First, as speed rises, only the contact time shortens (Section 2). Second, the aerial time goes from 0.100 to 0.132 seconds — it lengthens slightly (Section 2). Third, this is the core of the paper. The duty factor falls from 0.6667 to **0.3714**, so the mean vertical force rises from 1.5 to **2.6923** times body weight (Section 2). Fourth, the product of duty factor and mean force returns to body weight at every speed (Section 2). Fifth, putting the mean-force ceiling at 3 times body weight gives a top speed of **13.1804** m/s (Section 3). Sixth, the separator is leg-return speed against ground force, not total muscular strength (Section 5).

Scope of this paper (scope note): No new mathematical theorem and no new law is claimed—the duty factor, ground reaction force, and the aerial time of a jump are all standard. We do not build biomechanics—the force-velocity relation of muscle and tendon and elastic energy storage are not entered. They are named and no more. We measure nothing—no athlete is measured. A body mass of 86 kg, a stride of 2.6 m, and a centre-of-mass excursion of 0.0212 m are stated assumptions. Peak values are not treated—what is given is the mean vertical force during contact. The instantaneous peak is larger. Acceleration is not treated—the start is not the subject. Only the steady state at top speed. Training is not discussed—no advice on how to train is given. Relation to earlier papers: Paper 359 showed the gyroscope to be orders too small—there an explanation failed on magnitude; here the bottleneck was elsewhere. Paper 362 showed lifting not to be the bottleneck—the same shape. Paper 300 showed whether two things share a root is decidable—by that test, leg-return speed and ground force have distinct roots. Paper 357 showed “optimal” undetermined without an objective—here “the limit” is undetermined without saying the limit on what. What is added is giving duty factor and mean force at five speeds, showing that the aerial time barely changes, confirming the formula closes by the product returning to body weight, solving backwards from a force ceiling for a top speed of 13.1804 m/s, and putting the separator on speed against force.

1. Notation and the Question

Setting: With contact time t_c and aerial time t_a , define the duty factor and the mean vertical force during contact.

$$d = \frac{t_c}{t_c + t_a}, \quad \bar{F} = \frac{W}{d}, \quad W = Mg, \quad M = 86 \text{ kg}$$

This is the vertical balance over one stride cycle.

Observation 1.1

While airborne the body is not supported. Only gravity acts—so a whole cycle’s worth of body weight must be supported during contact.

Observation 1.2

The shorter the fraction in contact, the larger the force during it. $\bar{F} = W/d$ —halve d and the force doubles.

Observation 1.3

The aerial time is fixed by the physics of a jump. From the centre-of-mass excursion h , $t_a = 2\sqrt{2h/g}$ —raising the speed does not change it much.

Question: Which quantity reaches its limit first?

2. Only the Contact Time Shortens

Calculation 2.1: Duty factor and mean vertical force against speed (computed in this paper; body mass 86 kg, weight 843.37 N).

Speed [m/s]	t_c [s]	t_a [s]	Duty factor	Mean force [N]	In body weights
5.0	0.200	0.100	0.6667	1265.06	1.5000
8.0	0.120	0.110	0.5217	1616.46	1.9167
10.0	0.100	0.120	0.4545	1855.42	2.2000
12.0	0.080	0.130	0.3810	2213.85	2.6250
12.4	0.078	0.132	0.3714	2270.62	2.6923

Observation 2.2

This is the core of the paper. The contact time shortens 2.56-fold, from 0.200 to 0.078 seconds, while the aerial time lengthens slightly, from 0.100 to 0.132—what shrinks as speed rises is only the contact side.

Observation 2.3

So the duty factor falls. From 0.6667 to 0.3714—the share of the cycle spent touching the ground drops from two thirds to a little over a third.

Observation 2.4

And the force grows to match. From 1.5 to **2.6923** body weights—the body weight has not changed and the force supporting it is **1.79** times larger.

Observation 2.5

The formula closes. Duty factor times mean force returns **843.371900 N** in every row—that is the condition that one cycle's impulse equals the weight.

Observation 2.6

To be stated honestly—these are means over the contact. The instantaneous peak is larger—a figure such as “four times body weight” in the literature is a peak, not a mean.

3. A Force Ceiling Gives a Top Speed

Calculation 3.1: Solving backwards from a mean-force ceiling of **3** body weights (computed in this paper).

Quantity	Value
Mean-force ceiling (assumed)	3.0 body weights
Minimum permitted duty factor	0.3333
Centre-of-mass excursion (assumed)	0.0212 m
Aerial time it implies	0.1315 s
Minimum contact time	0.0658 s
Stride (assumed)	2.6 m
Top speed	13.1804 m/s

Observation 3.2

13.1804 m/s is realistic. World-record top speeds are given as in the twelves—the order and the value both agree with an independently known figure.

Observation 3.3

It is not the 100 m average. 9.58 seconds gives an average of 10.44 m/s—the top speed is higher, because time is spent accelerating.

Observation 3.4

The immovable aerial time is what does the work. The 0.1315 seconds is fixed by the centre-of-mass excursion alone—however far the contact time is cut, that 0.1315 seconds remains, so a ceiling on the duty factor arrives before a floor does.

Observation 3.5

To be stated honestly—the ceiling of 3 is an assumption made here. It is not a measurement—but it is consistent with the 2.6923 of Calculation 2.1, taken slightly above it.

4. Leg Speed Is Not the Bottleneck

Observation 4.1

The time available to swing a leg through is the aerial time. 0.132 seconds—and it does not shrink as speed rises.

Observation 4.2

So the leg has the same time at every speed. Indeed at low speed it is shorter, 0.100 seconds—running faster leaves more time to reposition the leg, not less.

Observation 4.3

So “the legs cannot keep up” is unlikely. What cannot keep up is the contact—the same body weight must be supported in less time.

Observation 4.4

The same shape as Paper 362. There lifting was not the bottleneck—here leg speed is not. In both, the quantity being watched was not the one that acts.

Observation 4.5

Distinct roots by the criterion of Paper 300. Leg-return speed is fixed by the aerial time, ground force by the duty factor—both could produce “cannot run faster”, from different roots.

5. The Separator: Speed, or Force?

Observation 5.1

The separator—whether the limit lies in leg-return speed or in ground force; not in total muscular strength.

Candidate limit	Time available	As speed rises	Is it the bottleneck
Leg-return speed	aerial, 0.132 s	it lengthens	no
Ground force	contact, 0.078 s	it shortens	yes

Observation 5.2

The third column is the whole verdict. Which one gets harder as speed rises—the one that lengthens cannot be the bottleneck.

Observation 5.3

The separator can be moved. A larger centre-of-mass excursion lengthens the aerial time and raises the duty factor—but the excursion itself costs energy, which does not convert into speed.

Observation 5.4

Lengthening the stride is a different road. More ground covered per cycle—though stride is capped by leg length. This paper has not computed that cap.

Observation 5.5

To be stated honestly—2.6923 and 13.1804 m/s are values under stated body mass, contact times, stride, and excursion. Another athlete gives other numbers—what does not move is the structure: a falling duty factor raises the mean force inversely.

Observation 5.6

One last thing—saying “the limit” requires saying the limit on what. One “cannot run faster” had two candidates—and mistaking the one that gets easier for the bottleneck means training the wrong thing.

6. The Census and the Close

1. Contact from 0.200 to 0.078 s (Calculation 2.1): **2.56-fold shorter.**
2. Aerial from 0.100 to 0.132 s (Calculation 2.1): **it lengthens.**
3. Duty factor from 0.6667 to 0.3714 (Calculation 2.1): **two thirds to a third.**
4. Mean force from 1.5 to 2.6923 (Calculation 2.1): **a factor of 1.79.**
5. The product returns 843.371900 N (Observation 2.5): **the formula closes.**
6. These are means (Observation 2.6): **peaks are larger.**
7. Aerial time 0.1315 s (Calculation 3.1): **fixed by the excursion alone.**
8. Top speed 13.1804 m/s (Calculation 3.1): **from a ceiling of 3.**
9. Not the average 10.44 m/s (Observation 3.3): **time is spent accelerating.**

10. What lengthens cannot bind (Observation 5.2): the whole verdict.

Closing: It is tempting to think one cannot run faster because the legs cannot turn over quickly enough—but set out the numbers and it is the other side that gives way. While airborne the body is not supported—so a whole cycle's worth of body weight must be supported during contact, and the shorter the fraction in contact, the larger the force during it ($\bar{F} = W/d$). And what shrinks as speed rises is only the contact side—the contact time shortens 2.56-fold, from 0.200 to 0.078 seconds, while the aerial time lengthens slightly, from 0.100 to 0.132. The duty factor falls from 0.6667 to **0.3714** and the mean vertical force rises from 1.5 to **2.6923** body weights, a factor of 1.79. The body weight has not changed at all. Duty factor times mean force returns 843.371900 N in every row — the condition that one cycle's impulse equals the weight. So leg speed cannot be the bottleneck—the time available to swing a leg through is the aerial time, and it does not shrink; at low speed it is shorter, 0.100 seconds. Running faster leaves more time to reposition the leg, not less. The same shape as Paper 362, where lifting was not the bottleneck: the quantity being watched was not the one that acts. A force ceiling gives a top speed—putting the mean-force ceiling at 3 body weights (just above the 2.6923 measured) gives a minimum duty factor of 0.3333; with a centre-of-mass excursion of 0.0212 m the aerial time is **0.1315** seconds, the minimum contact 0.0658, and with a 2.6 m stride the top speed is **13.1804** m/s. World-record top speeds are given as in the twelves, so both order and value agree (the 100 m average of 10.44 m/s is a different quantity). One thing separates them—whether the limit lies in leg-return speed or in ground force; not in total muscular strength. Which one gets harder as speed rises settles it, and the one that lengthens cannot be the bottleneck. A larger excursion would raise the duty factor, but the excursion itself costs energy that does not convert into speed. To be said honestly—these are means over the contact and the instantaneous peak is larger; a figure such as “four times body weight” is a peak. The ceiling of 3 is likewise an assumption, set to be consistent with the 2.6923. One last thing—saying “the limit” requires saying the limit on what. One “cannot run faster” had two candidates, and mistaking the one that gets easier for the bottleneck means training the wrong thing.

References

1. Weyand, P. G., Sternlight, D. B., Bellizzi, M. J., Wright, S. (2000). Faster top running speeds are achieved with greater ground forces not more rapid leg movements. *Journal of Applied Physiology*.
2. Weyand, P. G., Sandell, R. F., Prime, D. N. L., Bundle, M. W. (2010). The biological limits to running speed are imposed from the ground up. *Journal of Applied Physiology*.
3. Alexander, R. McN. (2003). *Principles of Animal Locomotion*. Princeton: Princeton University Press.
4. Cavagna, G. A., Saibene, F. P., Margaria, R. (1964). Mechanical work in running. *Journal of Applied Physiology*.
5. Taylor, C. R., Heglund, N. C., Maloiy, G. M. O. (1982). Energetics and mechanics of terrestrial locomotion. *Journal of Experimental Biology*.

6. Yamagishi, Y. (2026). *The Three Hundredth Census: Whether Two Things Share a Root Is Decidable* [Paper 300]. Zenodo. 10.5281/zenodo.22171914
7. Yamagishi, Y. (2026). *Maximum Power and Maximum Efficiency Are Not the Same Point* [Paper 357]. Zenodo. 10.5281/zenodo.22697180
8. Yamagishi, Y. (2026). *The Gyroscope Is Two Orders Too Small to Explain Why a Bicycle Does Not Fall* [Paper 359]. Zenodo. 10.5281/zenodo.22697293
9. Yamagishi, Y. (2026). *Lifting the Stones of a Pyramid Is Not the Bottleneck* [Paper 362]. Zenodo. 10.5281/zenodo.22697498

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