

In a Gravity Assist the Probe Speeds Up, and in the Planet's Frame Its Speed Never Changed

— Against Jupiter's orbital speed of **13.07** km/s, the gain seen from the Sun is at most **26.14** km/s — Jupiter thereby loses **2.4310×10^{-20}** m/s of orbital speed — The probe's change in speed is **1.0753×10^{24}** times Jupiter's — [Paper 381]

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— *From thinking to syncing.* —

Abstract

A gravity assist is described as accelerating on a planet's gravity. This paper asks in which frame the acceleration happened—the answer is that from the Sun it accelerated and from the planet its speed never changed. No new mathematical theorem and no new law is claimed. First, in the planet's frame the speed is unchanged and only the direction turns (as in elastic scattering) (Section 2). Second, this is the core of the paper. Seen from the Sun the gain is at most $2U$, twice the planet's orbital speed (Section 2). Third, that is **26.14** km/s at Jupiter and **70.04** km/s at Venus (Section 2). Fourth, a 1000 kg probe gains **6.0305×10^{11}** J at Jupiter (Section 3). Fifth, Jupiter thereby loses **2.4310×10^{-20}** m/s, a ratio of **1.0753×10^{24}** (Section 3). Sixth, the separator is which frame the speed is measured in, not the strength of the gravity (Section 5).

Scope of this paper (scope note): No new mathematical theorem and no new law is claimed—the gravity assist, elastic scattering, and conservation of momentum are all standard. We do not build orbital mechanics—the validity of the patched-conic approximation and real trajectory design are not entered. They are named and no more. We do not derive the maximum—the geometry achieving $2U$ (the head-on limit) is used as a result and not derived. No real mission is treated—the trajectory and actual gain of any particular spacecraft are not used. The numbers are on stated assumptions. Relativity is not treated—speeds are taken as small compared with light. Tides are not treated—tidal forces during the pass and effects on rotation are not the subject. Relation to earlier papers: Paper 367 showed a conserved total with only the distribution free—there too something looked created and had only moved. Paper 356 showed what flows and what works to be different quantities—here one speed takes two values in two frames. Paper 300 showed whether two things share a root is decidable—by that test, the speed in the planet's frame and in the Sun's have distinct roots. Paper 373 showed the initial condition setting the amplitude—here the frame settles the conclusion. What is added is giving the maximum gain at five planets, computing the kinetic energy gained at Jupiter, solving backwards for Jupiter's loss of 2.4310×10^{-20} m/s, confirming the energy inversion closes, and putting the separator on the frame.

1. Notation and the Question

Setting: Let U be the planet's orbital speed and v the probe's speed in the planet's frame.

$$\text{planet frame : } |v_{\text{in}}| = |v_{\text{out}}|, \quad \text{Sun frame : } \Delta V_{\text{max}} = 2U$$

Gravity is conservative, so the speed in the planet's frame is the same before and after.

Observation 1.1

In the planet's frame this is elastic scattering. The speed is unchanged and only the direction turns—the same shape as a ball bouncing off a wall.

Observation 1.2

If the wall is moving, the speed seen from outside changes. Bounce off an approaching wall and you go faster—a planet is in orbit, so it is a moving wall.

Observation 1.3

So the ceiling on the gain is set by the planet’s speed. Not by the strength of its gravity——this paper gives the value planet by planet.

Question: In which frame did the probe accelerate?

2. The Ceiling Is $2U$

Calculation 2.1: Orbital speed and maximum gain by planet (computed in this paper).

Planet	Orbital speed U [km/s]	Maximum gain $2U$ [km/s]
Venus	35.02	70.04
Earth	29.78	59.56
Mars	24.07	48.14
Jupiter	13.07	26.14
Saturn	9.69	19.38

Observation 2.2

This is the core of the paper. The ceiling is set by orbital speed, not by mass——Venus is far lighter than Jupiter and gives more than twice as much, 70.04 against 26.14.

Observation 2.3

Mass acts elsewhere. In how far the path can be bent——at a light planet one cannot approach $2U$, so the real gain falls below the ceiling. This paper has not computed that geometry.

Observation 2.4

A stationary body gives nothing. $2 \times 0 = 0$ —however massive, a body at rest cannot speed anything up. A check that the formula is doing the work.

Observation 2.5

It also works in reverse. Pass behind the planet and $2U$ is lost—probes bound for the inner planets use this.

3. Jupiter Pays 2.4310×10^{-20} m/s

Calculation 3.1: A 1000 kg probe accelerating at Jupiter (computed in this paper).

Quantity	Value
Probe mass	1000 kg
Speed in the Sun's frame	10.00 $\rightarrow$ 36.14 km/s
Kinetic energy gained	6.0305×10^{11} J
Jupiter's mass	1.898×10^{27} kg
Orbital speed Jupiter loses	2.4310×10^{-20} m/s
Ratio (probe / Jupiter)	1.0753×10^{24}

Observation 3.2

2.4310×10^{-20} m/s is unmeasurable. Less than 10^{16} times smaller than 1 mm/s—and it is not zero.

Observation 3.3

That it is not zero is the point. No energy has been created—it is taken from Jupiter's orbital motion. The counterpart simply weighs 1.898×10^{27} kg, so the quotient becomes unmeasurably small.

Observation 3.4

The ratio is 1.0753×10^{24} . Between the probe's and Jupiter's change in speed—the mass ratio, appearing directly.

Observation 3.5

The inversion closes. Substituting into $\Delta E = MU\Delta U$ returns 6.0305×10^{11} J—not an arithmetic error.

Observation 3.6

So it is not acceleration for free. The price is paid—the payer is simply so large that the invoice is invisible.

4. One Motion, Two Descriptions

Observation 4.1

In the planet's frame the speed is the same before and after. $|v_{\text{in}}| = |v_{\text{out}}|$ —no acceleration whatever has occurred.

Observation 4.2

In the Sun's frame 26.14 km/s has been added. The same pass, the same moment—only the vantage point differs.

Observation 4.3

Both are correct. Because velocity is a frame-dependent quantity—“which is really true” has no answer.

Observation 4.4

Distinct roots by the criterion of Paper 300. The planet-frame speed is fixed by gravity being conservative; the Sun-frame speed by the planet's motion—one word “speed” naming two quantities.

Observation 4.5

The same shape as Paper 367. There the total sensitivity was conserved and only its distribution moved—here the total momentum is conserved and only its distribution moves.

5. The Separator: Which Frame Measures the Speed

Observation 5.1

The separator—which frame the speed is measured in; not the strength of the gravity.

Frame	The probe's speed	What changes	The energy
Planet	unchanged	direction only	conserved
Sun	up to $2U$ higher	speed as well	moves from the planet

Observation 5.2

The rightmost column is where the confusion lives. Read the top row alone and “nothing happened”—read the bottom alone and “energy appeared”. Only both together show a transfer.

Observation 5.3

The separator can be moved. Write everything in the planet-fixed frame and the acceleration disappears—what disappears is the description, not the fact that the probe reaches further.

Observation 5.4

Which frame to use is settled by purpose. The planet's frame for the geometry of the pass, the Sun's for whether the next planet is reached—chosen for convenience, not correctness.

Observation 5.5

To be stated honestly—26.14 km/s is a ceiling, not an achieved gain. It assumes ideal geometry—real missions get less, and this paper has not computed the shortfall.

Observation 5.6

One last thing—saying “it increased” requires saying relative to what. One pass changes nothing in one frame and gives 26.14 km/s in another—an increase without a stated reference is not even settled as an increase.

6. The Census and the Close

1. The planet-frame speed is unchanged (Section 1): as in elastic scattering.
2. Up to $2U$ in the Sun's frame (Calculation 2.1): set by orbital speed.
3. 26.14 km/s at Jupiter (Calculation 2.1): 70.04 at Venus.
4. Speed acts, not mass (Observation 2.2): Venus gives more.
5. A stationary body gives zero (Observation 2.4): the formula works.
6. Energy gained 6.0305×10^{11} J (Calculation 3.1): for 1000 kg.
7. Jupiter loses 2.4310×10^{-20} m/s (Calculation 3.1): not zero.
8. Ratio 1.0753×10^{24} (Calculation 3.1): the mass ratio itself.
9. The inversion closes (Observation 3.5): $\Delta E = MU\Delta U$.
10. Only both frames show the transfer (Observation 5.2): either alone misleads.

Closing: A gravity assist is described as accelerating on a planet's gravity—but in the planet's frame the speed never changed. In the planet's frame this is elastic scattering—gravity is conservative, so the speed is unchanged and only the direction turns. It is the shape of a ball bouncing off a wall, and because that wall is in orbit, the speed seen from outside changes. So the ceiling on the gain is set by orbital speed, not mass—at most $2U$: 26.14 km/s at

Jupiter and **70.04** km/s at Venus. Venus is far lighter and gives more than twice as much. A body at rest, however massive, gives nothing. (Mass acts in how far the path can be bent, so at a light planet one cannot approach $2U$; this paper has not computed that geometry.) And the price is paid—a 1000 kg probe going from 10.00 to 36.14 km/s at Jupiter gains 6.0305×10^{11} J, and Jupiter loses 2.4310×10^{-20} m/s of orbital speed. That is unmeasurable — less than 10^{16} times smaller than 1 mm/s — and it is not zero. No energy was created; it was taken from Jupiter’s orbital motion, and the counterpart simply weighs 1.898×10^{27} kg. The ratio 1.0753×10^{24} is the mass ratio itself, and $\Delta E = MU\Delta U$ returns the original energy. One thing separates them—which frame the speed is measured in; not the strength of the gravity. Read the planet’s frame alone and “nothing happened”; read the Sun’s alone and “energy appeared” — only both together show a transfer. Both are correct, because velocity is frame-dependent, and “which is really true” has no answer. To be said honestly—26.14 km/s is a ceiling assuming ideal geometry, not an achieved gain; real missions get less, and this paper has not computed the shortfall. One last thing—saying “it increased” requires saying relative to what. One pass changes nothing in one frame and gives 26.14 km/s in another. An increase without a stated reference is not even settled as an increase.

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