

When to Leave Is Not Decided Inside the Patch

— the patch is identical in all five conditions, yet the optimal stay moves from **2.2875** to **7.4051**, a factor of **3.2372** — the only thing changed is the travel time — and at departure **50.35** per cent of the patch is still there — [Paper 387]

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— *From thinking to syncing.* —

Abstract

When to leave a patch seems like something the patch itself should decide. This paper shows that the optimal stay can be moved by a factor of **3.2372** without altering the patch at all—the only thing changed is the travel time to the next one. No new theorem or law is claimed. First, holding the gain curve $g(t) = 10(1 - e^{-0.30t})$ fixed, only the travel time T is moved from 1 to 20 (Section 2). Second, the optimal stay goes from 2.2875 to 7.4051, a factor of **3.2372** (Section 2). Third, and this is the core of the paper. The inside of the patch is identical in all five rows. What sets the stay is a quantity outside the patch (Section 3). Fourth, the marginal gain rate at departure agrees with the overall average rate to six digits (Section 2). Fifth, at $T = 1$ the optimum is to leave with **50.35** per cent of the patch's maximum still there (Section 3). Sixth, the separator is “did the patch change, or did the space between patches change” (Section 5).

Scope of this paper (scope note): No new theorem or law is claimed—the marginal value theorem (Charnov 1976) and its tangent construction are known. No ecology is built—empirical foraging behaviour and species differences are not entered. Only what sets the stay time is examined, in one model. It is not claimed that animals optimise—this paper computes an optimum; whether real animals behave that way is not the subject. Stochasticity is not treated—patch variance, incomplete information and predation risk are not the subject. Only a deterministic gain curve is used. Patch distribution is not treated—the environment is assumed homogeneous; learning in a heterogeneous one is not the subject. No application is claimed—transfers to exploration versus exploitation, allocation of attention, or when to exit an investment are stated only as sameness of shape. Relation to earlier papers: Paper 367 showed that pushing the sensitivity down pushes it up elsewhere—here a quantity outside decides a judgement inside. Paper 386 showed that the rate can move a thousandfold with the solubility fixed—here the stay moves 3.2372-fold with the patch fixed. Paper 357 showed that maximum power and maximum efficiency are different points—here maximum gain and optimal stay are different points. Paper 362 showed that lifting the stones is not the bottleneck—here the richness of the patch is not the bottleneck. What is added is putting numbers on stay and gain at five travel times, obtaining the 3.2372-fold span of the stay, confirming the tangency condition to six digits, giving the 50.35 per cent left behind at departure, and placing the separator at “the patch or the space between patches”.

1. Notation and the Question

Setting: staying t in one patch yields a cumulative gain $g(t)$; travel costs T .

$$g(t) = G(1 - e^{-\lambda t}), \quad G = 10, \lambda = 0.30$$

Find the stay t^* that maximises the long-run average rate.

$$R(t) = \frac{g(t)}{T + t} \rightarrow \max$$

Observation 1.1

g is concave and increasing. Staying longer always gains more, but at a declining rate—there is no point inside g that says “enough”.

Observation 1.2

T does not appear in g at all. It has nothing to do with the patch’s properties—and yet it sets t^* .

Observation 1.3

the optimum is a tangency condition. $g'(t^*) = R(t^*)$ —the marginal rate at the moment of leaving equals the overall average rate.

Question: where should one look to know when to leave?

2. The Stay Moves 3.2372-Fold with the Patch Untouched

Computation 2.1: optimal stay against travel time (computed in this paper).

Travel T	Stay t^*	Gain $g(t^*)$	Average R	Marginal $g'(t^*)$
1.0	2.2875	4.965379	1.510386	1.510386
2.0	3.0902	6.042823	1.187153	1.187153
5.0	4.4913	7.400840	0.779748	0.779748
10.0	5.8301	8.260577	0.521827	0.521827
20.0	7.4051	8.915583	0.325325	0.325325

Observation 2.2

the last two columns agree to six digits in all five rows. 1.510386 against 1.510386; 0.325325 against 0.325325—the tangency $g'(t^*) = R(t^*)$ is confirmed as two independently computed quantities.

Observation 2.3

the optimal stay goes from 2.2875 to 7.4051, a factor of 3.2372. The travel time went up twentyfold—the stay grows only logarithmically.

Observation 2.4

T never appears in the definition of g . $G = 10$ and $\lambda = 0.30$ hold in all five rows—the patch itself has not changed at all.

Observation 2.5

the average rate falls from 1.510386 to 0.325325, a factor of 4.6427. A larger drop than the rise in the stay——staying longer cannot recover the cost of travel.

3. The Optimum Is to Leave Half Behind

Computation 3.1: fraction remaining at departure (computed in this paper).

Travel T	Taken $g(t^*)/G$	Left behind
1.0	49.65 %	50.35 %
5.0	74.01 %	25.99 %
20.0	89.16 %	10.84 %

Observation 3.2

this is the core of the paper. At $T = 1$ the optimum is to leave with more than half the patch still there——the patch is not merely un-emptied; half of it remains. Leaving is still correct.

Observation 3.3

“there is still food” is not a reason to stay. Since g increases monotonically, food remains no matter when one leaves——looking at how much is left does not fix the time of departure.

Observation 3.4

the same patch is a “linger here” patch or a “move on” patch depending on the environment. 2.2875 or 7.4051—and the patch carries no information that distinguishes the two.

Observation 3.5

the shape is that of Paper 386. There the rate moved a thousandfold with the solubility fixed—here the stay moves 3.2372-fold with the patch fixed. In both, the name belongs to the thing that did not move.

Observation 3.6

maximum gain and optimal stay are different points. Maximising g alone would send $t \rightarrow \infty$ —the same structure as Paper 357, where maximum power and maximum efficiency were different points.

4. What the Tangency Condition Means

Observation 4.1

$g'(t^*) = R(t^*)$ is a bridge between two worlds. The left side is a quantity inside the patch, the right a quantity of the whole environment—the equality carries outside information inward.

Observation 4.2

that is why a purely local judgement can optimise. Know the environment’s average rate and the rest is comparing it with your own marginal rate—no survey of the whole is needed.

Observation 4.3

conversely, without the environment’s average rate nothing can be decided. Same patch, same marginal rate—and nothing to compare it against.

Observation 4.4

the shape appears elsewhere. Exploration versus exploitation, the allocation of attention, when to exit an investment——each compares “the rate I am getting” with “the rate if I switch”. This paper claims only sameness of shape.

Observation 4.5

the tangency depends on the patch being concave. If g were convex the optimum would jump to an endpoint——an interior solution exists only because of diminishing returns.

5. Separator: Did the Patch Change, or the Space Between Patches

Observation 5.1

separator——did the gain curve g move, or did the travel time T move.

What changed	g	T	Stay time
Patches got poorer	moves	fixed	moves
Patches got farther apart	fixed	moves	moves

Observation 5.2

the stay time alone cannot tell them apart. Both make it longer——the cause is not readable from the effect.

Observation 5.3

the distinction shows in the gain. If the patches got poorer, $g(t^*)$ falls——if they merely got farther apart, $g(t^*)$ rises from 4.965379 to 8.915583.

Observation 5.4

so “staying longer and taking more” is a symptom of a worsening environment. It looks like a good season—and the average rate has fallen by a factor of 4.6427.

Observation 5.5

to be honest— $G = 10$ and $\lambda = 0.30$ are constants of the model, not measurements. The figure 3.2372 depends on this λ —what does not move is that the tangency holds to six digits and that T does not enter g .

Observation 5.6

one last thing—“there is more to take” and “I should keep taking” are different statements. The first can be read off the patch; the second cannot—look in the wrong place and you never leave.

6. Roll Call and Closing

1. T does not enter g (Observation 2.4): $G = 10$, $\lambda = 0.30$ in all five rows.
2. The optimal stay goes 2.2875 to 7.4051 (Computation 2.1): a factor of 3.2372.
3. Tangency holds to six digits (Observation 2.2): two independently computed quantities.
4. The average rate falls by 4.6427 (Observation 2.5): more than the stay rises.
5. 50.35 per cent left behind at $T = 1$ (Computation 3.1): more than half remains.
6. “There is still food” is not a reason (Observation 3.3): food remains whenever one leaves.
7. The patch carries no distinguishing information (Observation 3.4): 2.2875 or 7.4051.
8. The tangency is a bridge between two worlds (Observation 4.1): it carries outside information inward.
9. Convex gain sends the optimum to an endpoint (Observation 4.5): diminishing returns are required.
10. Staying longer and taking more is a symptom (Observation 5.4): read the average rate.

Closing: when to leave a patch seems like something the patch itself should decide—but what decides it is a quantity outside the patch. Hold the gain curve $g(t) = 10(1 - e^{-0.30t})$ fixed and move only the travel time T from 1 to 20—the optimal stay goes from 2.2875 to 7.4051, a factor of 3.2372. T never appears in the definition of g ; $G = 10$ and $\lambda = 0.30$ hold in all five rows, and the patch itself has not changed at all. The optimum is the tangency $g'(t^*) = R(t^*)$ —in all five rows the independently computed marginal rate and average rate agreed to six digits (

1.510386 against 1.510386; 0.325325 against 0.325325). The left side is a quantity inside the patch, the right a quantity of the whole environment, and the equality carries outside information inward. That is why a purely local judgement can optimise—and why, without the environment’s average rate, nothing can be decided. And the optimum is to leave half behind—at $T = 1$ the correct move is to leave with **50.35** per cent of the patch’s maximum still there. Since g increases monotonically, food remains no matter when one leaves, so looking at how much is left does not fix the time of departure. “There is still food” is not a reason to stay. The same patch is a “linger here” patch or a “move on” patch depending on the environment, and the patch carries no information that distinguishes the two. One thing does the separating—did the gain curve move, or did the travel time move. The stay time alone cannot tell them apart, since both make it longer, and the cause is not readable from the effect. The distinction shows in the gain: if the patches got poorer, $g(t^*)$ falls; if they merely got farther apart, it rises from 4.965379 to 8.915583. So “staying longer and taking more” looks like a good season and is a symptom of a worsening environment—the average rate has fallen by a factor of 4.6427. To be honest— $G = 10$ and $\lambda = 0.30$ are constants of the model rather than measurements, and the figure 3.2372 depends on this λ . What does not move is that the tangency holds to six digits and that T does not enter g . The tangency also depends on the patch being concave; if g were convex the optimum would jump to an endpoint, so an interior solution exists only because of diminishing returns. One last thing—“there is more to take” and “I should keep taking” are different statements. The first can be read off the patch; the second cannot. Look in the wrong place and you never leave.

References

1. Charnov, E. L. (1976). Optimal foraging, the marginal value theorem. *Theoretical Population Biology*.
 2. Stephens, D. W., Krebs, J. R. (1986). *Foraging Theory*. Princeton: Princeton University Press.
 3. Nonacs, P. (2001). State dependent behavior and the marginal value theorem. *Behavioral Ecology*.
 4. Hayden, B. Y., Pearson, J. M., Platt, M. L. (2011). Neuronal basis of sequential foraging decisions in a patchy environment. *Nature Neuroscience*.
 5. Yamagishi, Y. (2026). *Maximum Power and Maximum Efficiency Are Not the Same Point* [Paper 357]. Zenodo. 10.5281/zenodo.22697180
 6. Yamagishi, Y. (2026). *Lifting the Stones of a Pyramid Is Not the Bottleneck* [Paper 362]. Zenodo. 10.5281/zenodo.22697498
 7. Yamagishi, Y. (2026). *Push the Sensitivity Down Somewhere and It Must Rise Somewhere Else* [Paper 367]. Zenodo. 10.5281/zenodo.22697709
 8. Yamagishi, Y. (2026). *The Solubility Did Not Move at All, and the Dissolution Became a Thousand Times Faster* [Paper 386]. Zenodo. 10.5281/zenodo.22699119
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