

With a Million Candidates the Chance of Picking the Best Stays at 0.3679

— the fraction passed over and the probability of success converge separately to the same

$1/e = 0.3678794412$ — two different quantities going to one constant — **0.367880** at $n = 10^6$, 0.398690
at $n = 10$ — [Paper 388]

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September 2026

— *From thinking to syncing.* —

Abstract

More candidates ought to make it harder to pick the best one. This paper shows that the probability does not fall to zero, and that the fraction passed over and the probability of success—two separate quantities—converge to one and the same constant—about 0.3679 whether $n = 10$ or $n = 10^6$. No new theorem or law is claimed. First, the optimal number to pass over is **37** at $n = 100$ and **3679** at $n = 10000$ (Section 2). Second, the fraction passed over k/n runs $0.300000 \rightarrow 0.370000 \rightarrow 0.368000 \rightarrow 0.367900$ toward $1/e$ (Section 2). Third, and this is the core of the paper. The probability p runs $0.398690 \rightarrow 0.371043 \rightarrow 0.368196 \rightarrow 0.367911$ toward the same $1/e$. The two are different quantities (Section 2). Fourth, at $n = 10^6$ the probability is **0.367880** and has not fallen (Section 3). Fifth, a 200,000-trial simulation agrees with the theory to **0.000203** (Section 3). Sixth, the separator is “pick the best” versus “get a good one” (Section 5).

Scope of this paper (scope note): No new theorem or law is claimed—the secretary problem, the $1/e$ rule and the optimal stopping solution are all known. No theory of optimal stopping is built—general stopping problems and variational inequalities are not entered. Only the numbers of one classical problem are examined. Hiring is not discussed—applications to recruitment or mate choice are not the subject. Only sameness of shape is stated. The objective is not changed—only the probability of picking the very best is maximised. Expected rank or utility is a different problem with a different answer. The information assumptions are not changed—only relative ranks are visible, candidates arrive in uniformly random order, and there is no recall. The origin of $1/e$ is not proved—the limit is known; this paper lists finite- n values. Relation to earlier papers: Paper 364 showed that selection alone raises the mean and shrinks the spread—here the selection rule itself is optimised. Paper 387 showed that when to leave is decided outside the place—here too the question is when to stop. Paper 300 showed that same-root and different-root can be decided— k/n and p are different-root by that criterion and yet share a limit. Paper 379 showed that “twice” has three answers—here one constant plays two roles. What is added is putting numbers on the optimal k and the probability at four values of n , setting side by side the separate convergence of k/n and p to the same $1/e$, obtaining 0.367880 at $n = 10^6$, confirming agreement to 0.000203 in a 200,000-trial simulation, and placing the separator at “the best or a good one”.

1. Notation and the Question

Setting: n candidates arrive in uniformly random order. Only relative ranks are visible and there is no recall. Pass over the first k unconditionally, then take the first who beats all seen so far.

$$P(n, k) = \frac{k}{n} \sum_{i=k+1}^n \frac{1}{i-1} \quad (k \geq 1), \quad P(n, 0) = \frac{1}{n}$$

P is the probability that the one taken is the best overall.

Observation 1.1

too small a k and one takes someone far too early. At $k = 1$ the process nearly always stops at the second candidate—too large a k and one passes over the best.

Observation 1.2

so the optimum is interior. Both ends are bad—as in Paper 385, it is a product of two monotone effects.

Observation 1.3

intuition says the probability should fall to zero as n grows. One in a million——this paper shows with numbers that it does not.

Question: what happens to the probability as the number of candidates grows?

2. The Fraction Passed Over and the Probability Go to the Same Number

Computation 2.1: optimal cutoff and probability at each n (computed in this paper).

Candidates n	Optimal k	Fraction k/n	Probability p
10	3	0.300000	0.398690
100	37	0.370000	0.371043
1000	368	0.368000	0.368196
10000	3679	0.367900	0.367911
Limit	—	0.3678794412	0.3678794412

Observation 2.2

this is the core of the paper. The last two columns converge separately to the same number——one is a fraction of a procedure, “how many to pass over”; the other is the probability of an outcome, “getting it right”.

Observation 2.3

the two are different quantities. Different in unit and in meaning——and yet one constant, $1/e = 0.3678794412$, plays both roles.

Observation 2.4

at $n = 10$ they are far apart.0.300000 and 0.398690—a gap of 0.098690; at finite n one can see that they are different things.

Observation 2.5

at $n = 10000$ the gap is 0.000011.0.367900 and 0.367911—a thousandfold increase in n shrinks the gap by about a factor of 9000.

Observation 2.6

they are different-root by the criterion of Paper 300.One is a choice of k , the other a distribution over the position of the best—sharing a limit does not make them same-root.

3. It Does Not Fall Even at a Million

Computation 3.1: large n , and a simulation (computed in this paper).

Condition	Value
$n = 10^6, k = \lfloor n/e \rfloor$	0.367880
$n = 100, k = 37$ (theory)	0.371043
$n = 100, k = 37$ (200,000 trials)	0.370840
Difference	0.000203

Observation 3.2

a million candidates still give 0.367880.Only 0.03 below the 0.398690 at $n = 10$ —a hundred-thousandfold increase in candidates barely moves the probability.

Observation 3.3

intuition is wrong. “Picking one out of a million is impossible”——it works more than one time in three.

Observation 3.4

the simulation agrees with the theory to 0.000203. Two hundred thousand trials——within the binomial standard deviation $\sqrt{p(1-p)/T} = 0.00108$, confirming the closed form independently.

Observation 3.5

the direction of the discrepancy is not meaningful. The simulation is slightly lower——this is the size of the statistical fluctuation, not a systematic error.

Observation 3.6

the global maximum is 0.500000, reached at both $n = 2$ and $n = 3$. From there it is non-increasing in n down to $1/e$ ——the total fall is only 0.1321205588. From two candidates to a million, the probability stays between one half and one third.

4. Even Off the Optimum, It Is Not Bad

Computation 4.1: probability against k at $n = 100$ (computed in this paper).

Cutoff k	Probability p	Fraction of optimum
1	0.051774	0.1395
10	0.234841	0.6329
37	0.371043	1.0000
70	0.251179	0.6770
99	0.010000	0.0270

Observation 4.2

both $k = 10$ and $k = 70$ keep more than six tenths of the optimum. 0.6329 and 0.6770—the peak is flat and a rough guess still works. Only $k = 70$, however, clears two thirds (0.6667).

Observation 4.3

the sides are not symmetric. 0.1395 at $k = 1$, 0.0270 at $k = 99$ —passing over too many is worse than passing over too few.

Observation 4.4

the 0.010000 at $k = 99$ is $1/n$. One is left mechanically taking the last candidate—the same probability as chance; the rule is doing nothing at all.

Observation 4.5

the shape is that of Paper 385. There a product of two monotone factors had an interior maximum—the same here, with both endpoints bad.

5. Separator: Pick the Best, or Get a Good One

Observation 5.1

separator—is anything but first place worthless, or will second place do.

Objective	Optimal cutoff	Probability at large n
Pick the best	n/e	stays at $1/e$
Improve the expected rank	a different rule	outside this paper

Observation 5.2

change the objective and the answer changes. This paper maximises only the probability of first place—an extreme scoring in which second place earns zero.

Observation 5.3

the number 0.3679 is specific to that scoring. Change the scoring and $1/e$ does not appear—remember the constant first and you will misidentify the problem.

Observation 5.4

with recall there is no problem at all. Look at everyone, then choose—the difficulty comes from irreversibility, not from the number of candidates.

Observation 5.5

to be honest—both 0.367880 and 0.370840 hold under the assumption of uniformly random order. Bias the order and everything changes—what does not move is that k/n and p go separately to the same constant.

Observation 5.6

one last thing—“difficulty grows with scale” is not always true. Here a hundred-thousandfold increase in scale costs 0.03 of probability—what grows is the number passed over, not the rate of failure.

6. Roll Call and Closing

1. Optimal k is 3, 37, 368, 3679 (Computation 2.1): at four values of n .
2. k/n goes to $1/e$ (Computation 2.1): 0.300000 to 0.367900.
3. p goes to the same $1/e$ (Observation 2.2): 0.398690 to 0.367911.
4. The two are different quantities (Observation 2.3): different in unit and meaning.

5. A gap of **0.098690** at $n = 10$ (Observation 2.4): at finite n they are visibly different.
6. **0.367880** at $n = 10^6$ (Computation 3.1): it has not fallen.
7. Simulation and theory agree to **0.000203** (Computation 3.1): inside a standard deviation of 0.00108.
8. Over six tenths at both $k = 10$ and $k = 70$ (Observation 4.2): the peak is flat; 0.6329 and 0.6770.
9. $0.010000 = 1/n$ at $k = 99$ (Observation 4.4): the rule is doing nothing.
10. The difficulty comes from irreversibility (Observation 5.4): not from the count.

Closing: more candidates ought to make it harder to pick the best one——picking one out of a million is impossible, says intuition. Pass over the first k , then take the first who beats all seen so far——with

$$P(n, k) = (k/n) \sum_{i=k+1}^n 1/(i-1),$$

the optimal cutoff is **3** at $n = 10$, **37** at $n = 100$, **368** at $n = 1000$ and **3679** at $n = 10000$. The fraction passed over and the probability of success converge separately to the same number—— k/n runs $0.300000 \rightarrow 0.370000 \rightarrow 0.368000 \rightarrow 0.367900$ and p runs $0.398690 \rightarrow 0.371043 \rightarrow 0.368196 \rightarrow 0.367911$. One is a fraction of a procedure, the other the probability of an outcome; they differ in unit and in meaning. And yet one constant, $1/e = \mathbf{0.3678794412}$, plays both roles. At $n = 10$ the gap between them is 0.098690, so at finite n one can see they are different things; at $n = 10000$ the gap is 0.000011. By the criterion of Paper 300 they are different-root, and sharing a limit does not make them same-root. And the probability does not fall——at $n = 10^6$ it is **0.367880**, only 0.03 below the 0.398690 at $n = 10$. A hundred-thousandfold increase in candidates still works more than one time in three. A 200,000-trial simulation gave 0.370840 against the theoretical 0.371043, agreeing to **0.000203**——inside the binomial standard deviation of 0.00108, and so an independent confirmation of the closed form. Even off the optimum it is not bad——at $n = 100$, $k = 10$ gives 0.234841 and $k = 70$ gives 0.251179, both above six tenths of the optimum (the ratios are 0.6329 and 0.6770, and only the latter clears two thirds). The peak is flat and a rough guess still works. The sides are not symmetric, though: 0.1395 at $k = 1$ against 0.0270 at $k = 99$, so passing over too many is the worse error. The 0.010000 at $k = 99$ is exactly $1/n$ ——the same probability as chance, with the rule doing nothing at all. One thing does the separating——is anything but first place worthless, or will second place do. This paper maximises only the probability of first place, an extreme scoring in which second place earns zero. The number 0.3679 is specific to that scoring; change the scoring and $1/e$ does not appear. Remember the constant first and you will misidentify the problem. With recall there is no problem at all——look at everyone, then choose. The difficulty comes from irreversibility, not from the number of candidates. To be honest——both 0.367880 and 0.370840 hold under the assumption of uniformly random order, and biasing the order changes everything. What does not move is that k/n and p go separately to the same constant. One last thing——“difficulty grows with scale” is not always true. Here a hundred-thousandfold increase in scale costs 0.03 of probability. What grows is the number passed over, not the rate of failure.

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