

The Significant Wave Height Is Neither the Mean Nor the Maximum

— in a sea of “significant wave height **3 m**” the mean is **1.8776 m**, the highest tenth averages **3.8134 m**, and the three-hour maximum is **5.8305 m** — the representative value does not represent, because a human eye is inside the definition — [Paper 391]

Yuuki YAMAGISHI (山岸 勇輝)

Yamagishi Complexity Torus Lab, Japan

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— *From thinking to syncing.* —

Abstract

Hear “waves of three metres” and you expect a three-metre wave. This paper shows that the number is neither the mean nor the maximum—the mean is only **1.8776 m**, and what actually arrives is **5.8305 m**. No new theorem or law is claimed. First, drawing 4×10^6 samples from a Rayleigh distribution gives a mean of **0.886139** and a top-third mean of **1.415837** (Section 2). Second, the mean is only **0.6259** of the significant wave height (Section 2). Third, and this is the core of the paper. The maximum over three hours (1080 waves) is **1.9435** times the significant height—very nearly double (Section 3). Fourth, an independent simulation gave **1.9359**, agreeing with the closed form to **0.4 per cent** (Section 3). Fifth, there is nothing statistically necessary about “one third”; it was chosen because it matches what trained observers report by eye (Section 4). Sixth, the separator is whether the representative is a mean, a mean of the top, or an extreme (Section 5).

Scope of this paper (scope note): No new theorem or law is claimed—the Rayleigh distribution of wave heights, the definition of significant wave height, and expected extremes are all known in ocean engineering. The ocean is not forecast—wave hindcasting and meteorology are not the subject. Only the numbers inside one distribution are examined. Narrow-bandedness is assumed—the Rayleigh distribution is an approximation for narrow-band linear waves; breaking and shoaling are not the subject. No structure is designed—design loads for breakwaters or hulls are not given. Only the meaning of the representative value is examined. Freak waves are not treated—whether rogue waves depart from the Rayleigh distribution is not the subject. Period is not treated—only wave height; the distribution of periods and the wave form are not the subject. Relation to earlier papers: Paper 379 showed that “twice as hot” has three answers—here one “wave height” has three. Paper 364 showed that selection alone raises the mean and shrinks the spread—here the act of taking the top third is itself the subject. Paper 386 showed that “poorly soluble” names two things—here “wave height” names three. Paper 300 showed that same-root and different-root can be decided—mean and extreme are different-root by that criterion. What is added is putting numbers on three representative values from 4×10^6 samples, showing that the mean is 0.6259 of the significant height, obtaining 1.9435 for the three-hour maximum, confirming agreement between simulation and closed form to 0.4 per cent, and placing the separator at “which representative”.

1. Notation and the Question

Setting: for narrow-band waves the height H follows a Rayleigh distribution.

$$p(H) = \frac{2H}{H_{\text{rms}}^2} \exp\left(-\frac{H^2}{H_{\text{rms}}^2}\right)$$

The significant wave height H_s is defined as the mean of the highest third of the waves. Below, $H_{\text{rms}} = 1$.

Observation 1.1

the number “one third” sits inside the definition. A convention of selection, not a property of the distribution—why one third does not follow from the distribution.

Observation 1.2

the mean and the maximum come from the same distribution. All three describe the same sea—and the numbers are entirely different.

Observation 1.3

the maximum depends on how many waves were watched. The mean and the significant height do not—— only the maximum carries the observation time inside it.

Question: when we say “wave height”, what are we saying?

2. The Mean Is Only 0.6259 of the Significant Height

Computation 2.1: 4×10^6 samples from a Rayleigh distribution (computed in this paper).

Quantity	Value ($H_{\text{rms}} = 1$)	Ratio to H_s
Mean H_{mean}	0.886139	0.625876
Mean of the top third H_s	1.415837	1.000000
Mean of the top tenth	1.799701	1.271121

Observation 2.2

all three agree with the known values. The standard 0.886, 1.416 and 1.800——a sampling computation reproduced the closed form independently.

Observation 2.3

the mean is 0.6259 of the significant height. In a three-metre sea the mean is 1.8776 m——not “three-metre waves are coming” but “most waves are around 1.9 m”.

Observation 2.4

even the top tenth reaches only 1.2711. 3.8134 m in a three-metre sea——looking at the upper end still does not reach double.

Observation 2.5

the significant height sits between the mean and the top tenth. Between 0.6259 and 1.2711——deliberately placed where it is neither.

3. The Three-Hour Maximum Is Very Nearly Double

Computation 3.1: expected maximum against number of waves (computed in this paper).

Waves	Maximum / H_s	If $H_s = 3$ m
100 (about 17 minutes)	1.6105	4.8315 m
1080 (3 hours at a 10-second period)	1.9435	5.8305 m
3000 (about 8 hours)	2.0703	6.2109 m
Simulation (1080 waves, 2000 runs)	1.9359	5.8077 m

Observation 3.2

this is the core of the paper. In a sea reported as “three metres”, a 5.8305 m wave arrives once in three hours——1.9435 times the number you were told, very nearly double.

Observation 3.3

the simulation agrees with the closed form to 0.4 per cent. 1.9359 against 1.9435——two independent routes to the same number.

Observation 3.4

the maximum grows only with the logarithm of the count. From 1.6105 at 100 waves to 2.0703 at 3000——thirty times the waves buys 1.29 times the height.

Observation 3.5

so “wait longer for a bigger wave” does not work well. The growth is $\sqrt{\ln N}$ —double the waiting and the maximum barely moves.

Observation 3.6

only the maximum carries the observation time. The mean and the significant height do not—the same sea returns a different number depending on how long you watch.

4. “One Third” Comes from the Human Eye

Observation 4.1

the distribution gives no reason to choose one third. A quarter or a fifth would write down just as well—it was not the distribution that chose.

Observation 4.2

the reason is that it matched what trained observers reported by eye. The number a mariner gives when asked how high the waves are today—the statistic closest to that answer was the mean of the top third.

Observation 4.3

so a human being is inside the definition. The eye fails to count the small waves—and that bias was absorbed on the side of the statistic.

Observation 4.4

the shape is that of Paper 379. There the scale decided the ratio—here a habit of the eye decides the definition of the representative value. In both, a convention on the measuring side makes the number.

Observation 4.5

they are different-root by the criterion of Paper 300. The mean is a first moment of the distribution; the significant height a distribution plus a convention; the maximum a distribution plus an observation time——all three are called “wave height”, with three roots.

5. Separator: A Mean, a Mean of the Top, or an Extreme

Observation 5.1

separator——count everything, count only the upper part, or look only at the single highest.

Representative	Set by	Depends on watching time	If $H_s = 3$ m
Mean	distribution	no	1.8776 m
Significant height	distribution + convention	no	3.0000 m
Maximum	distribution + time	yes	5.8305 m

Observation 5.2

the third column is the test. Double the observation time and see whether the number moves——if it moves it is an extreme; if not it is a property of the distribution.

Observation 5.3

the last column spans a factor of 3.1. From 1.8776 m to 5.8305 m——it is the same sea.

Observation 5.4

which one you use changes the design. The mean for comfort, the maximum for overtopping——fixing the representative value is fixing the question.

Observation 5.5

to be honest——both 5.8305 m and 1.8776 m assume a Rayleigh distribution. Breaking cuts off the top, and in shallow water the distribution itself changes——what does not move is that three representative values come from one sea and span a factor of three.

Observation 5.6

one last thing——“significant” is not a word of statistics but a word about people. It means conspicuous——the height of the waves one notices, not the height of a typical wave.

6. Roll Call and Closing

1. Mean 0.886139, H_s 1.415837, top tenth 1.799701 (Computation 2.1): matching known values.
2. The mean is 0.6259 of H_s (Computation 2.1): 1.8776 m in a three-metre sea.
3. The top tenth is only 1.2711 (Observation 2.4): still short of double.
4. The three-hour maximum is 1.9435 (Computation 3.1): 5.8305 m.
5. The simulation gave 1.9359 (Computation 3.1): agreeing to 0.4 per cent.
6. The maximum grows as $\sqrt{\ln N}$ (Observation 3.5): thirty times buys 1.29.
7. Only the maximum carries the time (Observation 3.6): that is the test.
8. “One third” came from the eye (Observation 4.2): the distribution gave no reason.
9. All three are different-root (Observation 4.5): distribution, convention, time.
10. A span of 3.1 (Observation 5.3): it is the same sea.

Closing: hear “waves of three metres” and you expect a three-metre wave——but the number is neither the mean nor the maximum. Narrow-band wave heights follow a Rayleigh distribution, and the significant height is defined as the mean of the highest third——drawing 4×10^6 samples gives a mean of 0.886139, a top-third mean of 1.415837 and a top-tenth mean of 1.799701 at $H_{rms} = 1$, matching the standard 0.886, 1.416 and 1.800. The mean is only 0.6259 of the significant height——in a three-metre sea the mean is 1.8776 m, so it is not “three-metre waves are coming” but “most waves are around 1.9 m”. Even the top tenth reaches only 1.2711 (3.8134 m), and the significant height sits between them, deliberately placed where it is neither. And yet the three-hour maximum is very nearly double——over 1080 waves at a 10-second period the maximum is 1.9435 times the significant height, so in a sea reported as “three metres” a 5.8305 m wave arrives once in three hours. An independent simulation (1080 waves,

2000 runs) gave 1.9359, agreeing with the closed form to 0.4 per cent. The maximum grows only as $\sqrt{\ln N}$, so thirty times the waves buys 1.29 times the height, and “wait longer for a bigger wave” does not work well. Only the maximum carries the observation time, so the same sea returns a different number depending on how long you watch. “One third” comes from the human eye—the distribution gives no reason to choose it; a quarter or a fifth would write down just as well. It was chosen because the statistic closest to what trained observers report by eye is the mean of the top third. The eye fails to count the small waves, and that bias was absorbed on the side of the statistic—a human being is inside the definition. As in Paper 379, where the scale decided the ratio, a convention on the measuring side makes the number. One thing does the separating—count everything, count only the upper part, or look only at the single highest. The test is to double the observation time and see whether the number moves: if it moves it is an extreme; if not it is a property of the distribution. In a three-metre sea the three values are 1.8776 m, 3.0000 m and 5.8305 m—a span of 3.1, and it is the same sea. By the criterion of Paper 300 all three are different-root: distribution, distribution plus convention, distribution plus time. To be honest—both 5.8305 m and 1.8776 m assume a Rayleigh distribution; breaking cuts off the top and in shallow water the distribution itself changes. What does not move is that three representative values come from one sea and span a factor of three. One last thing—“significant” is not a word of statistics but a word about people. It means conspicuous: the height of the waves one notices, not the height of a typical wave.

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