

# The Same Flow Is Two Different States: Measuring the Flow Does Not Tell You Which One You Are In

— densities of **50** and **100** vehicles per km both deliver **3333.3333** vehicles per hour — the speeds are **66.6667** and **33.3333** km/h, exactly a factor of two apart — at capacity the speed is exactly half the free-flow speed —

[Paper 394]

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— *From thinking to syncing.* —

## Abstract

Whether a road is congested seems like something a flow measurement would settle. This paper shows that the same flow is realised in two states—one comfortable, the other a jam. No new theorem or law is claimed. First, with  $v = v_f(1 - k/k_j)$  and  $q = kv$ , capacity is **3750** vehicles per hour at  $k = k_j/2$  (Section 2). Second, the speed at capacity is exactly half the free-flow speed (**50.0000** km/h) (Section 2). Third, and this is the core of the paper. Densities of 50 and 100 both deliver **3333.3333** vehicles per hour, at speeds a factor of two apart (Section 2). Fourth, one vehicle per km above capacity and one below both reduce the flow (Section 3). Fifth, at a density of 100 the flow is **416.6667** vehicles per hour below capacity (Section 3). Sixth, the separator is whether the density is above or below critical, and the flow cannot decide it (Section 5).

Scope of this paper (scope note): No new theorem or law is claimed—the Greenshields linear relation, the fundamental diagram and the definition of capacity are all known in traffic engineering. No road is designed—lane counts and signal control are not given. Only the numbers inside one relation are examined. A linear speed–density relation is assumed—measured fundamental diagrams are not linear and show capacity drop and two-phase structure; the qualitative two-valuedness is unchanged. The onset of congestion is not treated—shock waves, car-following models and the dynamics of the transition are not the subject. Only steady states. Drivers are not treated—reaction times and headway distributions are not the subject. Policy is not discussed—the merits of tolls or regulations are not the subject. Relation to earlier papers: Paper 379 showed that “twice as hot” has three answers—here one flow has two states. Paper 391 showed that the significant wave height is neither the mean nor the maximum—here the flow does not represent the state. Paper 357 showed that maximum power and maximum efficiency are different points—here maximum flow and maximum speed are different points. Paper 300 showed that same-root and different-root can be decided—density and flow are different-root by that criterion. What is added is putting numbers on speed and flow at five densities, showing the same flow arising at two densities, showing that the speed at capacity is exactly half, bracketing the capacity point from both sides, and placing the separator at “above or below critical density”.

## 1. Notation and the Question

Setting: let the speed  $v$  fall linearly with density  $k$  (Greenshields).

$$v = v_f \left(1 - \frac{k}{k_j}\right), \quad q = kv = v_f k \left(1 - \frac{k}{k_j}\right)$$

Take  $v_f = 100$  km/h (free-flow speed) and  $k_j = 150$  veh/km (jam density).

### Observation 1.1

$q$  is a downward parabola in  $k$ . Zero at  $k = 0$  and at  $k = k_j$ —with a maximum in between.

### Observation 1.2

a parabola takes each value at two values of  $k$ . One on each side of the vertex—measure  $q$  and  $k$  is not determined.

### Observation 1.3

$v$ , by contrast, is monotone in  $k$ . One to one——measure the speed and the density is determined.

Question: to know the state of a road, what should be measured?

## 2. The Same 3333.3333 Appears Twice

Computation 2.1: speed and flow against density (computed in this paper).

| Density $k$ [veh/km] | Speed [km/h] | Flow $q$ [veh/h] |
|----------------------|--------------|------------------|
| 25                   | 83.3333      | 2083.3333        |
| 50                   | 66.6667      | 3333.3333        |
| 75                   | 50.0000      | 3750.0000        |
| 100                  | 33.3333      | 3333.3333        |
| 125                  | 16.6667      | 2083.3333        |

### Observation 2.2

this is the core of the paper.  $k = 50$  and  $k = 100$  deliver the same 3333.3333 veh/h to the last decimal——yet the speeds are 66.6667 and 33.3333 km/h, exactly a factor of two apart.

### Observation 2.3

the same happens at  $k = 25$  and  $k = 125$ . Both 2083.3333 veh/h——not a coincidence but the symmetry of the parabola.

#### Observation 2.4

the speed at capacity is exactly half the free-flow speed. 50.0000 km/h——at the moment of greatest throughput the traffic is already down to half speed.

#### Observation 2.5

maximum flow and maximum speed are different points. Maximum speed at  $k = 0$ , maximum flow at  $k = 75$ ——the structure of Paper 357: two “bests” that do not coincide.

#### Observation 2.6

only the speed column is monotone.  $83.3333 \rightarrow 16.6667$ ——that is the one to measure.

### 3. The Flow Falls on Both Sides of Capacity

Computation 3.1: near capacity (computed in this paper).

| Density $k$ | Flow $q$ [veh/h] | Difference from capacity |
|-------------|------------------|--------------------------|
| 74          | 3749.3333        | −0.6667                  |
| 75          | 3750.0000        | 0                        |
| 76          | 3749.3333        | −0.6667                  |
| 100         | 3333.3333        | −416.6667                |

#### Observation 3.2

the drop is identical on both sides. −0.6667 at 74 and at 76——at a vertex the first-order term vanishes.

### Observation 3.3

so near capacity the flow barely distinguishes the states. A difference of 0.6667 veh/h——buried in measurement noise.

### Observation 3.4

further away the difference opens. —416.6667 veh/h at  $k = 100$ ——the loss per extra vehicle grows with the density.

### Observation 3.5

the sign of  $dq/dk$  changes at  $k = 75$ .  $v_f(1 - 2k/k_j)$  is positive at 74 and negative at 76——the boundary is bracketed from both sides.

### Observation 3.6

past critical, one more entering vehicle reduces the whole. Your passing prevents others from passing——the act of joining a busy road lowers that road's throughput.

## 4. The Flow Is Not a Representative Value

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### Observation 4.1

the flow is “how many passed in an hour”. A quantity of outcome——not a quantity of state.

### Observation 4.2

the same outcome comes from two states. Few fast vehicles, or many slow ones——counting vehicles cannot tell them apart.

#### Observation 4.3

the shape is that of Paper 391. There the significant wave height was neither the mean nor the maximum——here the flow does not represent the state. In both, one number claims to represent and does not.

#### Observation 4.4

they are different-root by the criterion of Paper 300. Density is set by headway, speed by driver judgement, flow by their product——the flow is a derived quantity, not a primary one.

#### Observation 4.5

so “the same traffic volume as last year” does not mean the same state. The vertex may have been crossed——the same figure can be evidence of improvement or of deterioration.

### 5. Separator: Is the Density Above or Below Critical

#### Observation 5.1

separator——is the density less than  $k_j/2$  or greater.

| State          | Density  | One more vehicle | Distinguishable by flow |
|----------------|----------|------------------|-------------------------|
| Free flow      | $k < 75$ | raises the flow  | no                      |
| Congested flow | $k > 75$ | lowers the flow  | no                      |

#### Observation 5.2

the last column reads “no” in both rows. The same flow arises from either——without changing what is measured, no decision is possible.

### Observation 5.3

measure speed or density and it is settled at once. Both are one to one with  $k$ —what is needed is not more observation but a different observation.

### Observation 5.4

the separator can be moved. Meter the inflow to hold the density below 75 and the two-valuedness disappears—which is why ramp metering works.

### Observation 5.5

to be honest—both 3750 and 3333.3333 veh/h come from a model with a linear speed--density relation. Measured diagrams are not linear and show a capacity drop—what does not move is that the same flow arises at two densities and that the speed is monotone.

### Observation 5.6

one last thing—the point of greatest throughput is already the point of half speed. “Move many” and “move fast” are not maximised at the same point—until one of them is chosen as the goal, the optimisation itself is undefined.

## 6. Roll Call and Closing

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1.  $q$  is a downward parabola (Section 1): each value at two densities.
2. Capacity 3750 veh/h at  $k = 75$  (Computation 2.1):  $k_j/2$ .
3.  $k = 50$  and  $k = 100$  both give 3333.3333 (Observation 2.2): to the last decimal.
4. Speeds 66.6667 and 33.3333 (Observation 2.2): exactly a factor of two.
5. Half the free-flow speed at capacity (Observation 2.4): 50.0000 km/h.
6.  $-0.6667$  at both 74 and 76 (Observation 3.2): the first-order term vanishes.
7.  $-416.6667$  at  $k = 100$  (Computation 3.1): the difference opens.
8.  $dq/dk$  changes sign at  $k = 75$  (Observation 3.5): bracketed both sides.

9. Only the speed is monotone (Observation 2.6): that is the one to measure.

10. Two goals, two maxima (Observation 5.6): undefined until one is chosen.

Closing: whether a road is congested seems like something a flow measurement would settle—but the same flow is realised in two states. If the speed falls linearly with density,  $q = kv$  is a downward parabola—with  $v_f = 100$  km/h and  $k_j = 150$  veh/km, capacity is **3750** veh/h at  $k = 75$ . A parabola takes each value twice, once on each side of the vertex, so measuring  $q$  does not determine  $k$ . Densities of 50 and 100 deliver the same **3333.3333** veh/h to the last decimal—yet the speeds are 66.6667 and 33.3333 km/h, exactly a factor of two apart. The same happens at  $k = 25$  and  $k = 125$  (both 2083.3333 veh/h), which is the symmetry of the parabola and not a coincidence. And the speed at capacity is exactly half the free-flow speed (**50.0000** km/h)—at the moment of greatest throughput the traffic is already down to half speed. Maximum speed is at  $k = 0$  and maximum flow at  $k = 75$ : as in Paper 357, two “bests” that do not coincide. The flow falls by the same amount on both sides of capacity— $-0.6667$  veh/h at  $k = 74$  and at  $k = 76$ , because at a vertex the first-order term vanishes. Near capacity, then, the flow barely distinguishes the states, and  $0.6667$  veh/h is buried in measurement noise. Further away the difference opens: at  $k = 100$  the flow is **416.6667** veh/h below capacity. The sign of  $dq/dk = v_f(1 - 2k/k_j)$  is positive at 74 and negative at 76, bracketing the boundary. Past critical, one more entering vehicle reduces the whole—the act of joining a busy road lowers that road’s throughput. The flow is not a representative value—it is “how many passed in an hour”, a quantity of outcome rather than of state. Few fast vehicles or many slow ones give the same count. As in Paper 391, where the significant wave height was neither the mean nor the maximum, one number claims to represent and does not. By the criterion of Paper 300 they are different-root: density is set by headway, speed by driver judgement, flow by their product. So “the same traffic volume as last year” does not mean the same state, and the same figure can be evidence of improvement or of deterioration. One thing does the separating—is the density less than  $k_j/2$  or greater. Neither free flow nor congested flow is distinguishable by the flow itself. Measure speed or density and it is settled at once, since both are one to one with  $k$ —what is needed is not more observation but a different observation. Meter the inflow to hold the density below 75 and the two-valuedness disappears, which is why ramp metering works. To be honest—both 3750 and 3333.3333 veh/h come from a model with a linear speed–density relation, and measured diagrams are not linear and show a capacity drop. What does not move is that the same flow arises at two densities and that the speed is monotone. One last thing—the point of greatest throughput is already the point of half speed. “Move many” and “move fast” are not maximised at the same point. Until one of them is chosen as the goal, the optimisation itself is undefined.

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