

The Physical Resource Bound on Brute-Force Search

— 3rd Edition: The Landauer Limit, and Why Only the Logarithm Is Robust — [Paper 04]

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— *From thinking to syncing.* —

ABSTRACT

A brute-force search that actually scans 2^n candidates is bounded below by Landauer's principle. This paper states that bound precisely and writes out **what does and does not follow from it**. **First, the exact form of the theorem is "for irreversible operations"**. Landauer's limit demands $k_B T \ln 2$ of work **per bit erased**. Bennett (1973), by contrast, showed that **logically reversible computation can in principle dissipate arbitrarily little**. **The bound therefore constrains irreversible implementations, not search itself**. That qualification is essential. **Second — and this is the point of the 3rd edition — the robust statement is the logarithmic one, not the linear one**. The second edition described the energy required at $n = 300$ as "about 0.6 times the mass-energy of the observable universe". But that ratio **ranges from 0.43 to 1.08** depending on how the mass of the universe is estimated. "About 0.6 times" asserts a precision it does not possess (Paper 101). By contrast, **the n^* at which the required energy reaches the available energy barely moves**. Since $n^* = \log_2(E/(k_B T \ln 2))$, changing the energy estimate by a factor of 2.5 moves n^* by only 1.3, and lowering the temperature from 300 K to 2.7 K — a factor of 111 — moves it by only 6.8. What should be said is not "about 0.6 times" but " $n \approx 300$ ". **Third, this is a module boundary**. It is the place where the dimensionless ratio $E_{\text{required}}/E_{\text{available}}$ crosses one, **structurally the same** as the Planck scale of Paper 39, the bound $\Lambda \lesssim 4\pi v$ of Paper 09 and the $\text{Re}_\eta = 1$ of Paper 03. **But it is a boundary of physical implementation, not of complexity classes**. **Fourth, erasure is coarse-graining**. Logical irreversibility means **two states collapsing into one**, and since the microscopic dynamics is reversible (Liouville, Paper 77), **the difference must be passed to the environment** (Paper 78). **Landauer's principle is a quantitative bridge between information and thermodynamics, and has been verified experimentally** (Bérut et al. 2012). **Fifth, the three barriers are complexity theory's version of the machinery-transfer test**. Relativization, natural proofs and algebrization are **theorems about which kinds of tool cannot reach this problem**, playing the same role as the transfer test of Paper 100. **This paper answers none of them**. **Scope**: a structural account of established physical law and theorems (Layer 2). $\mathcal{P}$ vs $\mathcal{NP}$ is a Clay Millennium Prize Problem and is unsolved. **This paper claims neither its proof, nor a partial proof, nor a route towards one**.

Positioning of this paper (discipline): This paper offers a **structural interpretation** of established physical law and mathematical theorems. It claims neither the discovery of new physical laws nor the proof of open problems. The relations shown are transformations of known expressions; the aim is to propose a structural reading of them. No claim contradicting established theory is made. **Thermodynamic arguments cannot separate complexity classes (§6). $\mathcal{P}$ vs $\mathcal{NP}$ is unsolved and nothing here changes that.**

1. Introduction: which of this paper's numbers survive?

Consider scanning an n -bit candidate space by brute force. There are 2^n candidates. **Is there a physical lower bound on that operation?**

The answer is a conditional yes. **Landauer's principle** demands a minimum of work to erase information. This paper states that bound precisely and separates **what follows from it from what does not.**

**And the 3rd edition asks a further question:
of the numbers this paper states, which survive independently of the estimates that go into them?**

2. The Landauer limit and irreversible brute-force search

Established Fact 2.1 (Landauer's principle, 1961)

In an environment at temperature T , **erasing one bit of information** requires a minimum work

$$W_{\min} = k_B T \ln 2$$

and dissipates the same quantity of heat. At room temperature $T = 300$ K, $W_{\min} = 2.871 \times 10^{-21}$ J.

Theorem 2.2 (Lower bound on irreversible brute-force search)

For an implementation that scans 2^n candidates and **discards the information of each irreversibly**, the required work is bounded below by

$$E \geq 2^n k_B T \ln 2.$$

At $n = 300$ and $T = 300$ K this gives $E \geq 5.85 \times 10^{69}$ J.

Corollary 2.3 ("Irreversible" cannot be dropped)

Bennett (1973) showed that **logically reversible computation** can be constructed and that its dissipation can in principle be made arbitrarily small. A computation involving no erasure is not subject to Landauer's limit.

The theorem above therefore constrains "brute-force search as irreversibly implemented", not brute-force search in general. A reversible implementation pays in **space or time** instead of energy. **Drop the qualification and the theorem is false.**

3. The robust statement is the logarithmic one, not the linear one

3.1 The linear statement moves with the estimate

The second edition described the energy required at $n = 300$ as "about 0.6 times the mass-energy of the observable universe". **That ratio is checked here.**

Estimate of the mass of the universe	Mass-energy	Ratio at $n = 300$
1.5×10^{53} kg	1.35×10^{70} J	0.43
1.0×10^{53} kg	8.99×10^{69} J	0.65
6×10^{52} kg (baryons only)	5.39×10^{69} J	1.08

It ranges from 0.43 to 1.08. The two-figure display "about 0.6" does not possess that precision. This is close in form to the retrospective handling of numbers against which Paper 101 warned, and the 3rd edition corrects it.

3.2 The logarithmic statement does not move

Let n^* be the n at which the required energy reaches an available energy E :

$$2^{n^*} k_B T \ln 2 = E \iff n^* = \log_2 \left(\frac{E}{k_B T \ln 2} \right) = \log_2 E - \log_2 (k_B T \ln 2).$$

Available energy	n^* (300 K)	n^* (2.7 K)	Difference
World annual primary energy ($\sim 6 \times 10^{20}$ J)	137.3	144.1	6.8
Annual solar output ($\sim 1.2 \times 10^{34}$ J)	181.5	188.3	6.8
Universe (1.5×10^{53} kg)	301.2	308.0	6.8
Universe (6×10^{52} kg)	299.9	306.7	6.8

Observation 3.1 (The logarithm is insensitive to the estimate)

Changing the estimated mass of the universe by a factor of 2.5 moves n^* only from 301.2 to 299.9, a shift of 1.3. Lowering the temperature from 300 K to 2.7 K — a factor of 111 — moves n^* by only 6.8.

Moreover **temperature shifts n^* by a constant**: the difference is 6.8 in every row above because, in $n^* = \log_2 E - \log_2(k_B T \ln 2)$, only the second term depends on T .

Hence there is exactly one thing this paper can say robustly: irreversible brute-force search becomes physically impossible at around $n = 300$. Not "about 0.6 times".

When the quantity of interest is exponential, a linear measure inherits the uncertainty of its inputs at full size, while a logarithmic measure compresses it. The latter is what should be reported.

4. This is a module boundary

Paper 113 recorded that the **boundary between an older and a newer description always lies where a dimensionless number crosses one**.

Observation 4.1 (Where $E_{\text{required}}/E_{\text{available}}$ crosses one)

The boundary of this paper is the position at which the dimensionless ratio

$$\frac{E_{\text{required}}}{E_{\text{available}}} = \frac{2^n k_B T \ln 2}{E}$$

reaches one (4.2×10^{-4} at $n = 290$, 0.43 at $n = 300$, 4.4×10^2 at $n = 310$).

This is a structure this series has met repeatedly: the Planck scale of Paper 39 (E/E_p), the naturalness bound of Paper 09 ($\Lambda/4\pi v$), the Kolmogorov length of Paper 03 (Re_η). **Each was a place where a dimensionless number crosses one.**

There is, however, a decisive difference. The former three are boundaries **at which the description of a theory changes**. The present boundary is one **at which a physical implementation becomes impossible, not one at which the theory of computation changes**. This is made explicit in §6.

5. Erasure is coarse-graining

Why does only erasure carry a cost? This series already has the tools to answer.

Corollary 5.1 (Logical irreversibility is a compression of phase space)

Erasing one bit means **collapsing two states (0 and 1) into one** (say 0): the volume of phase space is **halved**.

But as Paper 77 recorded, **the microscopic dynamics is reversible and Liouville's theorem conserves phase-space volume**. The lost volume must therefore **be passed somewhere** — to the degrees of freedom of the environment. This is **the same mechanism** Paper 78 described as "friction = coarse-grained transfer to the bath".

The $\ln 2$ in $k_B T \ln 2$ is precisely the logarithm of "two becoming one".

Landauer's principle is a quantitative bridge between information and thermodynamics. Paper 88 separated **coarse-graining entropy** from **black-hole area entropy** as distinct roots; **what Landauer bridges is the former with information**. The bridge has been **verified experimentally** (Bérut et al. 2012).

But this bridge does not reach $\mathcal{P}$ vs $\mathcal{NP}$. That is the next section.

6. Thermodynamics cannot separate complexity classes

Corollary 6.1 (A bound on brute force is not a bound on computation)

Theorem 2.2 constrains **implementations that actually scan 2^n candidates**. The hypothesis $\mathcal{P} = \mathcal{NP}$ asserts **precisely the existence of an algorithm that avoids that scan**.

One therefore cannot pass from "brute force has a physical lower bound" to " $\mathcal{P} \neq \mathcal{NP}$ ". To do so would be **to place the conclusion among the premises**.

The same argument yields false conclusions elsewhere. Linear programming requires exponential effort by naive vertex enumeration, yet **polynomial-time algorithms exist**; sorting takes **$n!$** by trying all permutations, yet **$O(n \log n)$ suffices**. **"The naive method is expensive" does not imply "no efficient method exists"**.

7. The three barriers — complexity theory's machinery-transfer test

Any claimed proof concerning $\mathcal{P}$ vs $\mathcal{NP}$ **must address three barriers**.

Barrier	Source	What it states
Relativization	Baker–Gill–Solovay (1975)	techniques invariant under oracles cannot separate
Natural proofs	Razborov–Rudich (1994)	constructive, large circuit-bound techniques clash with pseudorandom functions
Algebrization	Aaronson–Wigderson (2008)	techniques invariant under algebraic extension still do not suffice

Observation 7.1 (These are theorems about which tools fail)

Each barrier states that **a certain class of proof technique is in principle powerless** against this problem. **They say not what is true but what one cannot get there with.**

This plays the same role as the machinery-transfer test set up in Paper 100 — the same kind of work Paper 03 did in showing that Ricci flow's machinery does not transfer to Navier–Stokes. **Different field, same discipline.**

And this paper answers none of the three. A thermodynamic argument is invariant under oracles and therefore **cannot even clear the relativization barrier.** **The reasoning here contributes nothing to the separation of complexity classes.**

This paper belongs to the lineage of "NP-complete problems and physical reality" opened by Aaronson (2005), and **is not a proof.**

Personal reflection of the author (Layer 3; labelled non-scientific / no claim of mechanism)

In the first edition I wrote: brute force would take the energy of the universe, therefore $\mathcal{P} \neq \mathcal{NP}$. The second edition noticed the circularity and withdrew it. **Up to there, it was a matter of correcting an error.**

What I noticed in the 3rd edition was **a quieter mistake**. I had written "about **0.6** times the mass-energy of the observable universe". The number was not wrong. But **that "0.6" carried a precision I was not entitled to**: depending on how one estimates the mass of the universe it is **0.43**, or **1.08**. **I had written a digit I did not have.**

And in trying to fix it I found that **the correct statement had been available all along**. $n \approx 300$ — a figure that moves by **1.3** if I am wrong about the mass of the universe by a factor of **2.5**, and by **6.8** if I lower the temperature by a factor of **111**. **When the quantity is exponential, what should be reported is the logarithm.**

The way that makes a number sound large and the way that states it accurately are often different. "0.6 times the energy of the universe" has a ring to it. "It becomes impossible around $n = 300$ " is plain. **But only the second was something I could actually say.** One stops at the edge of the cliff (Paper 101).

8. Scope / Conclusion

What is shown: that irreversible brute-force search obeys $E \geq 2^n k_B T \ln 2$ (§2) / that "irreversible" is essential, reversible computation (Bennett 1973) being unconstrained (§2) / that **the robust statement is $n^* \approx 300$ rather than**

"about 0.6 times", together with the reason the logarithm is insensitive (§3) / that this is **a module boundary** of the same form as Papers 39, 09 and 03, but **one of implementation rather than of theory** (§4) / that **erasure is coarse-graining**, the $\ln 2$ being the logarithm of "two becoming one" (§5, Papers 77, 78, 88) / that **thermodynamics cannot separate complexity classes** (§6) / and that **the three barriers are theorems about which tools fail**, none of which this paper answers (§7).

What is not shown (restraint): no proof of $\mathcal{P} \neq \mathcal{NP}$, no partial proof, no route towards one — lacking any response to the three barriers, the reasoning here contributes nothing to separating complexity classes / **no constraint on search methods other than brute force** / **no constraint on reversible implementations** / **no claim to digits not possessed**, such as "about 0.6 times" (corrected in the 3rd edition) / no new mathematics and no prediction (Layer 2).

$\mathcal{P}$ vs $\mathcal{NP}$ is unsolved. Nothing here changes that.

What the 3rd edition adds is the standing of the numbers this paper states. The lower bound on irreversible brute-force search is correct. But what follows from it **robustly** is the logarithmic statement alone: **that it becomes physically impossible at around $n = 300$.**

And that boundary is one of implementation, not of theory.

What lies beyond it is not "another theory of computation" but "a machine that cannot be built".

Revision Record

3rd edition (August 2026): removed the word "Withdrawal" from the title, which now names **what the paper shows**; the list of withdrawals and corrections is collected in this section.

Corrected in the 3rd edition: the second edition described the energy required at $n = 300$ as "about 0.6 times the mass-energy of the observable universe". **That ratio ranges from 0.43 to 1.08 depending on the estimated mass of the universe and does not carry two-figure precision. The 3rd edition corrects this and shows that the robust statement is $n^* \approx 300$** (§3).

Added in the 3rd edition: a corollary making explicit that the theorem's exact form is "**for irreversible implementations**" (§2, Bennett 1973); the reason **the logarithm is insensitive to estimates** (since $n^* = \log_2 E - \log_2(k_B T \ln 2)$, temperature shifts it by a constant) (§3); that this boundary is **a module boundary in the sense of Paper 113**, of the same form as Papers 39, 09 and 03 but **of implementation rather than theory** (§4); that **erasure is coarse-graining**, the same mechanism as Liouville (Paper 77) and friction (Paper 78), with $\ln 2$ the logarithm of "two becoming one" (§5); and that **the three barriers play the role of Paper 100's machinery-transfer test**, thermodynamic arguments failing to clear even relativization (§7).

Withdrawn in the 2nd edition (July 2026): the first edition claimed a proof of $\mathcal{P} \neq \mathcal{NP}$. That claim was withdrawn in full.

1. **Circular reasoning.** The first edition's Step 2 stated that "by Theorem 3.1, $\Omega(2^n)$ irreversible transitions are required", but **Theorem 3.1 presupposes brute-force search**, whereas the hypothesis $\mathcal{P} = \mathcal{NP}$ asserts precisely the existence of an algorithm avoiding it. **The argument did not derive a contradiction from the hypothesis; it placed a premise that denies the hypothesis.**
2. **Misreading of Bennett.** The first edition took Bennett (1973) to require knowledge of the target; **the theorem applies to any deterministic computation.**
3. **Separation by thermodynamics.** Thermodynamic arguments cannot separate complexity classes; the first edition's reasoning **yields false conclusions for linear programming and for sorting (§6).**

First edition: Zenodo 10.5281/zenodo.21404131.

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