

# The Four Planck Quantities Are Four Instances of One Rule

— 3rd ed.: in the Yamagishi Complete Planck Unit Unpacking Theory, the mass dimension fixes the power of  $4\pi$ , and  $l_p/t_p=c$  holds in every unit system — [Paper 40]

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— *From thinking to syncing.* —

## ABSTRACT

This paper carried a table showing that under  $G = 4\pi$ ,  $\hbar = \frac{1}{4\pi}$  the four Planck quantities read  $l_p = c^{-3/2}$ ,  $t_p = c^{-5/2}$ ,  $m_p = \frac{\sqrt{c}}{4\pi}$ ,  $E_p = \frac{c^{5/2}}{4\pi}$ . The third edition supplies the reason for that table and removes from it two items that were never claims. First, two of the five listed results are identities.  $l_p/t_p = c$  and  $E_p t_p = \hbar$  use  $\hbar G = 1$  nowhere: in the ratio of  $l_p = \sqrt{\hbar G/c^3}$  and  $t_p = \sqrt{\hbar G/c^5}$  the factor  $\hbar G$  cancels. This edition checked ten assignments of  $\hbar$  and  $G$  (the patch, SI units, and random values with  $\hbar G$  ranging from 0.139 to  $6.4 \times 10^3$ ): the worst relative error was  $0.00 \times 10^0$  for  $l_p/t_p = c$  and  $1.81 \times 10^{-16}$  for  $E_p t_p = \hbar$ . They hold in every unit system and admit no way of failing. Since only what can fail is worth checking numerically, the second edition's line “all five items confirmed by numerical verification” placed two facts and two identities in the same column. The third edition counts them apart. Second, the  $4\pi$  is doing no work. Paper 116 showed the closure holds for  $G = a$ ,  $\hbar = 1/a$  with any  $a > 0$ . Writing  $4\pi$  rather than  $a$  is therefore a choice of name, not a result, and the phrase “expressible in the solid angle alone” does not require a solid angle. Moreover the  $4\pi$  in  $m_p$  is not independent: since  $\hbar = 1/G$  we have  $\hbar/G = 1/G^2$  exactly (verified here), so  $m_p = \sqrt{\hbar c/G} = \sqrt{c}/G$ . The  $4\pi$  in  $m_p$  is  $G$ 's  $4\pi$  squared and square-rooted back, not a second one (Paper 39). Third, what then explains the table? Mass dimension. Applying Paper 116's theorem  $\epsilon = \mu$ :  $l_p$  and  $t_p$  have  $\mu = 0$  and carry no  $4\pi$ ;  $m_p$  and  $E_p$  have  $\mu = 1$  and carry one. All four agree. This paper held a table saying “lengths and times do not carry it, masses and energies do” without saying why. Fourth, where do the half-integers come from? The determinant of the dimension matrix. For  $(\hbar, G, c)$  it is  $|\det| = 2$  (verified here in exact rational arithmetic). Solving the four exponent triples  $(p, q, N)$  by Cramer's rule gives denominator exactly 2 for all twelve exponents:  $(\frac{1}{2}, \frac{1}{2}, -\frac{3}{2})$  for  $l_p$ ,  $(\frac{1}{2}, \frac{1}{2}, -\frac{5}{2})$  for  $t_p$ ,  $(\frac{1}{2}, -\frac{1}{2}, \frac{1}{2})$  for  $m_p$ ,  $(\frac{1}{2}, -\frac{1}{2}, \frac{5}{2})$  for  $E_p$ ; checked against the SI values to a maximum relative error of  $1.52 \times 10^{-16}$ . The square roots are not a product of  $\hbar G = 1$  but of the determinant being 2 (Papers 38, 39). Fifth, what this paper therefore shows: the patch  $\hbar G = 1$  bites only on the forms of the four quantities, never on the two relations. Multiplying  $G$  by 1.3, or  $\hbar$  by 2, breaks all four forms while  $l_p/t_p = c$  survives (verified here). That asymmetry is itself the line between a fact and an identity. Scope: structural interpretation and self-audit of elementary established facts (Layer 2). No new theorem and no prediction. The exponent of  $c$  is not read as a spatial dimension and  $4\pi$  is not treated as a forced value.

**Standing of this paper (discipline):** the mathematics used here is **entirely known and elementary** — the definition of the Planck units (Planck 1899), the solution of a  $3 \times 3$  linear system, dimensional analysis (Buckingham, Bridgman). From within this series it uses **Paper 116's theorem  $\epsilon = \mu$ , the  $|\det| = 2$  of Papers 38 and 39, and Paper 44's form  $X = c^N/(4\pi)^\epsilon$ . Not one new theorem is proved.** The third edition's contribution is **a sorting and a reason:** (i) splitting the five items into **two facts and two identities**, (ii) showing by a one-parameter relaxation that the  $4\pi$  does no work, (iii) rereading the table as **four instances of  $\epsilon = \mu$** , (iv) tracing the half-integers to **the determinant**, and (v) restating the scope. **The most important restraints: an identity is not called a “check of self-consistency”** — an equation with no violable null hypothesis confirms nothing when verified numerically (Papers 21, 16).  **$4\pi$  is not called a forced value** (the closure holds for any  $a > 0$ ; Paper 116). **The exponent of  $c$  is not read as a spatial dimension. No new fact about the Planck system is asserted** — this is **a rewriting that follows from a choice of units** (Paper 95). Layer-3 lenses are quarantined under a non-scientific label.

## 1. Introduction: The Paper Held a Table and Did Not Hold Its Reason

The first and second editions of this paper set out four Planck quantities under  $G = 4\pi$ ,  $\hbar = \frac{1}{4\pi}$ .

| Quantity            | Form                    |
|---------------------|-------------------------|
| Planck length $l_p$ | $c^{-3/2}$              |
| Planck time $t_p$   | $c^{-5/2}$              |
| Planck mass $m_p$   | $\frac{\sqrt{c}}{4\pi}$ |
| Planck energy $E_p$ | $\frac{c^{5/2}}{4\pi}$  |

The table contains two regularities that were never written down.

**One:** the upper two carry no  $4\pi$  and the lower two do. **Why?** The second edition does not say.

**The other:** all four exponents are half-integers. **Why?** The second edition said only that they are “fixed uniquely by dimensional analysis”. **Being fixed uniquely and having denominator 2 are two different questions.**

**And outside the table, two relations stood as “results”** —  $l_p/t_p = c$  and  $E_pt_p = \hbar$ . The second edition included them in the line “all five items confirmed by numerical verification”.

The first thing the third edition does is recount those five items.

## 2. Two of the Five Are Identities

**Theorem 2.1** ( $l_p/t_p = c$  and  $E_p t_p = \hbar$  hold in every unit system)

With  $l_p = \sqrt{\hbar G/c^3}$ ,  $t_p = \sqrt{\hbar G/c^5}$ ,  $m_p = \sqrt{\hbar c/G}$  and  $E_p = m_p c^2$ , for every value of  $\hbar$  and  $G$ ,

$$\frac{l_p}{t_p} = \sqrt{\frac{\hbar G/c^3}{\hbar G/c^5}} = \sqrt{c^2} = c, \quad E_p t_p = \sqrt{\frac{\hbar c}{G}} c^2 \sqrt{\frac{\hbar G}{c^5}} = \sqrt{\frac{\hbar^2}{c^4}} c^2 = \hbar.$$

**Proof** In both computations  $\hbar G$  (and  $G$ ) cancels. Only powers of  $c$  remain.

**Numerical confirmation:** ten assignments of  $\hbar$  and  $G$  were tested — the Yamagishi patch ( $\hbar G = 1$ ), SI units ( $\hbar G \approx 7.0 \times 10^{-45}$ ),  $\hbar = G = c = 1$ , and six random assignments with  $\hbar G$  between  $0.139$  and  $6.4 \times 10^3$ .

| Relation          | Worst relative error over the ten |
|-------------------|-----------------------------------|
| $l_p/t_p = c$     | $0.00 \times 10^0$                |
| $E_p t_p = \hbar$ | $1.81 \times 10^{-16}$ (rounding) |

### Observation 2.1 (Only what can fail is worth checking)

The second edition wrote that “all five items follow from  $\hbar G = 1$  alone and have been confirmed by numerical verification”. **For two of them the first half is false** — they do not follow from  $\hbar G = 1$ ; **they follow from nothing**, being identities of the definitions. **And for those two the second half is empty. Verifying numerically an equation that cannot fail confirms nothing.** This series states the discipline elsewhere: **zero-tension requires a violable null hypothesis** (Paper 21), and **the equals sign has distinct roots, so an identity and an empirical law must not sit in one column** (Paper 16). **This paper was breaking that discipline inside itself. The third edition recounts as follows: four patch-dependent facts** (the forms of the four quantities) and **two unit-system-independent identities** (the two relations above).

Observation 2.2 (The asymmetry is the boundary line)

The patch was broken deliberately. Fixing  $c = 3.7$ , multiply  $G$  by 1.3, or  $\hbar$  by 2.

|                         | Patch | $G \rightarrow 1.3G$ | $\hbar \rightarrow 2\hbar$ |
|-------------------------|-------|----------------------|----------------------------|
| $l_p = c^{-3/2}$        | holds | fails                | fails                      |
| $t_p = c^{-5/2}$        | holds | fails                | fails                      |
| $m_p = \sqrt{c}/(4\pi)$ | holds | fails                | fails                      |
| $l_p/t_p = c$           | holds | holds                | holds                      |

Under the same perturbation every form breaks and only the ratio survives. That asymmetry is the line dividing a fact from an identity — and the line is seen by perturbing, not by arguing.

3. The  $4\pi$  Is Doing No Work

Established Fact 3.1 (Paper 116: the closure holds for any  $a > 0$ )

Setting  $G = a$ ,  $\hbar = 1/a$  for arbitrary  $a > 0$  makes  $\hbar G = 1$  hold **identically**, and the four quantities become

$$l_p = c^{-3/2}, \quad t_p = c^{-5/2}, \quad m_p = \frac{\sqrt{c}}{a}, \quad E_p = \frac{c^{5/2}}{a}.$$

Nothing anywhere requires  $a = 4\pi$ .

The second edition's conclusion — “all four Planck quantities are expressible in powers of  $c$  and **the solid angle  $4\pi$**  alone” — reads accurately as

expressible in powers of  $c$  and of the patch constant  $a$  alone.

Writing  $4\pi$  is a choice of name, not a result (in Paper 95's vocabulary, a convention). No meaning as a solid angle follows from these four quantities.

**Observation 3.1 (The  $4\pi$  in  $m_p$  is not a second  $4\pi$ )**

Since  $\hbar = 1/G$ , the relation  $\hbar/G = 1/G^2$  holds **exactly** (verified here for  $G = 4\pi, 2, 11.7$  to within  $10^{-14}$ ).  
Hence

$$m_p = \sqrt{\frac{\hbar c}{G}} = \sqrt{\frac{c}{G^2}} = \frac{\sqrt{c}}{G}.$$

The  $4\pi$  appearing in  $m_p$  is  $G$ 's  $4\pi$ , squared and then square-rooted back. It is not an independent  $4\pi$  coming from  $\hbar$  — Paper 39's remark that “ $\hbar = 1/(4\pi)$  is the reciprocal of  $G$ , not an independent  $4\pi$ ” applies here directly. In these four quantities the  $4\pi$  has exactly one address.

**4. The Reason for the Table:  $\epsilon = \mu$**

Why then do  $l_p, t_p$  carry no  $4\pi$  while  $m_p, E_p$  do? Paper 116 holds the answer.

**Established Fact 4.1 (Paper 116's theorem:  $\epsilon = \mu$ )**

Under the one-parameter patch  $G = a$ ,  $\hbar = 1/a$ ,  $X = \hbar^p G^q c^N = a^{q-p} c^N = a^{-\mu} c^N$ . That is, the power of the patch constant carried by  $X$  is fixed by the mass dimension  $\mu$  of  $X$ .

Applied to the four quantities:

| Quantity | Form             | $\epsilon$ (power of $4\pi$ ) | $\mu$ (mass dim.) | Agree |
|----------|------------------|-------------------------------|-------------------|-------|
| $l_p$    | $c^{-3/2}$       | 0                             | 0                 | yes   |
| $t_p$    | $c^{-5/2}$       | 0                             | 0                 | yes   |
| $m_p$    | $c^{1/2}/(4\pi)$ | 1                             | 1                 | yes   |
| $E_p$    | $c^{5/2}/(4\pi)$ | 1                             | 1                 | yes   |

All four agree.

**Observation 4.1 (The table was four instances of  $\epsilon = \mu$ )**

What decides whether a  $4\pi$  appears is **not the flavour of the words “geometric” and “material” but a number: the mass dimension**.  $l_p$  and  $t_p$  have  $\mu = 0$ , so nothing appears;  $m_p$  and  $E_p$  have  $\mu = 1$ , so one factor appears. **And because no quantity of mass dimension 2 or more occurs among the four, the power of  $4\pi$  never reaches 2. This paper held the table and did not hold its reason.** The reason lies in Paper 116, written later, and **these four quantities are the smallest roll-call of that theorem.**

**5. Where the Half-Integers Come From:  $|\det| = 2$**

All four exponents have denominator **2**. **This is not a product of  $\hbar G = 1$ .**

Arranging the dimensions of  $(\hbar, G, c)$  in  $(L, M, T)$ :

$$A = \begin{pmatrix} 2 & 3 & 1 \\ 1 & -1 & 0 \\ -1 & -2 & -1 \end{pmatrix}, \quad \det A = 2.$$

(Columns  $\hbar, G, c$ ; rows  $L, M, T$ . Verified here in exact rational arithmetic, and by numpy as 2.0.)

**Theorem 5.1 (The determinant fixes the denominators)**

If  $X = \hbar^p G^q c^N$  satisfies  $[X] = L^\lambda M^\mu T^\tau$ , then Cramer's rule determines  $(p, q, N)$  uniquely as **rational numbers with denominator  $\det A$** .

Solved exactly for the four quantities:

| Quantity | $[X]$          | $p(\hbar)$ | $q(G)$ | $N(c)$ |
|----------|----------------|------------|--------|--------|
| $l_p$    | $L$            | $1/2$      | $1/2$  | $-3/2$ |
| $t_p$    | $T$            | $1/2$      | $1/2$  | $-5/2$ |
| $m_p$    | $M$            | $1/2$      | $-1/2$ | $1/2$  |
| $E_p$    | $L^2 M T^{-2}$ | $1/2$      | $-1/2$ | $5/2$  |

**All twelve exponents came out with denominator exactly 2** (the set of reduced denominators is  $\{2\}$ ). Substituting the SI values of  $\hbar, G, c$  and comparing with the textbook Planck quantities gave a maximum relative error of  $1.52 \times 10^{-16}$ .

**Observation 5.1 (The square roots come from the determinant, not from cancellation)**

The second edition said the exponents are “fixed uniquely by dimensional analysis”. **True, but half the story:** uniqueness is given by  $\det A \neq 0$ , and the denominator **2** is given by  $|\det A| = 2$ . These are two different facts. Papers 38 and 39 treat the relation between  $|\det| = 2$  and half-integer exponents, and these four quantities are four instances of it.

6. What This Paper Therefore Shows

| Item                                  | What it is                                                       |
|---------------------------------------|------------------------------------------------------------------|
| $l_p = c^{-3/2}, t_p = c^{-5/2}$      | Patch-dependent fact (needs $\hbar G = 1$ )                      |
| $m_p = \sqrt{c}/a, E_p = c^{5/2}/a$   | Patch-dependent fact (needs $\hbar = 1/G$ )                      |
| The value $4\pi$                      | Convention (closure holds for any $a > 0$ ; Paper 116)           |
| Whether $\epsilon$ is 0 or 1          | Consequence of mass dimension (Paper 116, $\epsilon = \mu$ )     |
| Denominator <b>2</b> in the exponents | Consequence of the determinant ( $ \det A  = 2$ ; Papers 38, 39) |
| $l_p/t_p = c$                         | Identity (holds in every unit system)                            |
| $E_p t_p = \hbar$                     | Identity (holds in every unit system)                            |

Of the seven rows, only two carry a claim proper to this paper — the first two, namely the forms of the four quantities. The rest are conventions, consequences of other papers' theorems, or identities.

This is not a shrinking but an addressing.

Only by writing which row belongs to which root does the table become checkable (Paper 17).

7. Conclusion

This paper held a table of powers of  $c$ . The table itself is correct. What changed is how it is read.

The upper two carry no  $4\pi$  not because they are geometric but because their mass dimension is 0; the exponents have denominator 2 not because a square root sits in the definition but because the determinant is 2. And  $l_p/t_p = c$  is not a property of this unit system but of every unit system.

The table stayed, and all four of its explanations were replaced.  
And the five items counted as “verified” split into four facts and two identities.

**What is not claimed (restraint):** an identity is not called a “check of self-consistency” (no violable null hypothesis; Papers 21, 16) /  $4\pi$  is not called a forced value (closure holds for any  $a > 0$ ; Paper 116) /  $4\pi$  is not read as a solid angle (no such meaning follows from these four quantities) / the exponent of  $c$  is not read as a spatial dimension / the  $4\pi$  in  $m_p$  is not counted as an independent second one (Paper 39) / no new fact about the Planck system is asserted — it is a rewriting following from a choice of units (Paper 95) / the claim that  $c$  is the building material of dimension is neither supported nor denied (Layer 3, quarantined under a non-scientific label) / no new mathematics and no prediction (Layer 2).

## Revision Record

**Third edition (August 2026):** the table is unchanged. The forms  $l_p = c^{-3/2}$ ,  $t_p = c^{-5/2}$ ,  $m_p = \sqrt{c}/(4\pi)$ ,  $E_p = c^{5/2}/(4\pi)$  all still hold. What is added is a sorting and a reason. (i) What the second edition counted as “all five items confirmed by numerical verification” is split into four patch-dependent facts and two unit-system-independent identities:  $l_p/t_p = c$  and  $E_p t_p = \hbar$  hold for every  $\hbar$  and  $G$  because  $\hbar G$  cancels, confirmed over ten numerical assignments with worst relative errors  $0.00 \times 10^0$  and  $1.81 \times 10^{-16}$ . An equation with no violable null hypothesis confirms nothing when verified (Papers 21, 16). (ii) The  $4\pi$  was relaxed to a one-parameter  $a$ , and the closure shown to hold for any  $a > 0$  (Paper 116); “expressible in the solid angle alone” is accordingly restated as “expressible in powers of the patch constant alone”. The  $4\pi$  in  $m_p$  was shown to be the same single one inherited from  $G$  (Paper 39). (iii) The regularity in the table (the upper two carrying no  $4\pi$ ) was given its reason — Paper 116’s theorem  $\epsilon = \mu$ , of which the four quantities are four instances. (iv) The half-integer exponents were traced to  $|\det| = 2$  of the dimension matrix, with all twelve exponents verified in exact rational arithmetic to have denominator 2 (Papers 38, 39). (v) The scope was restated as a seven-row table, making explicit that the only claims proper to this paper are the forms of the four quantities. Items retracted in the second edition (the correspondence with Paper 34’s  $c^5$ , and the phrase “an algebraic proof of  $(3 + 1)$ -dimensional structure”) are not reinstated.

## References

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