

Not Everyone Needs Immunity; and Yet Everyone May Not Be Enough

— at $R_0 = 3$ it stops at **66.6667** per cent — but at $R_0 = 12$ with a vaccine of **90** per cent efficacy the required coverage is **101.85** per cent — the boundary is where the efficacy equals the threshold itself — [Paper 401]

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— *From thinking to syncing.* —

Abstract

It is sometimes said that herd immunity requires everyone to be immune. This paper shows that it does not, and that everyone can still be insufficient—the two kinds of “not enough” are different things. No new theorem or law is claimed. First, the threshold is $1 - 1/R_0$; at $R_0 = 3$ it stops at **66.6667** per cent (Section 2). Second, at exactly the threshold the effective reproduction number is exactly **1** (Section 2). Third, however large R_0 becomes, the threshold never reaches **1** (Section 2). Fourth, and this is the core of the paper. At $R_0 = 12$ with **90** per cent efficacy the required coverage is **101.85** per cent—unreachable even by vaccinating everyone (Section 3). Fifth, the boundary is where the efficacy equals the threshold itself (**91.6667** per cent at $R_0 = 12$) (Section 3). Sixth, the separator is whether what is short is the coverage or the efficacy (Section 5).

Scope of this paper (scope note): No new theorem or law is claimed—the basic reproduction number, the herd-immunity threshold $1 - 1/R_0$ and the division by efficacy are all known in epidemiology. Policy is not discussed—what should be done about any disease is not the subject. Only the numbers inside one formula. Homogeneous mixing is assumed—contacts are taken as uniform; age structure, spatial structure and contact heterogeneity are not the subject, and they can move the threshold in either direction. A single R_0 is assumed— R_0 is not a property of the pathogen alone but also of the contact rate of a society, and it varies with place and time. Waning is not treated—immunity that declines over time is not the subject. Infection-blocking and disease-blocking are not distinguished—“efficacy” here means the fraction of transmission blocked, a different quantity from efficacy against disease. Relation to earlier papers: Paper 400 is the four hundredth census—this is the paper that follows it. Paper 388 showed that the probability does not fall as the number of candidates grows—here the threshold does not reach 1 as R_0 grows. Paper 386 showed that “poorly soluble” names two things—here “not enough” names two. Paper 300 showed that same-root and different-root can be decided—coverage and efficacy are different-root by that criterion. What is added is putting numbers on threshold and required coverage at five values of R_0 , confirming by inversion that the effective reproduction number is exactly 1, obtaining the impossible figure of 101.85 per cent at 90 per cent efficacy, showing that the boundary is “efficacy equals threshold”, and placing the separator at “coverage or efficacy”.

1. Notation and the Question

Setting: let one infected person produce R_0 secondary infections. With an immune fraction p ,

$$R_{\text{eff}} = R_0(1 - p), \quad R_{\text{eff}} \leq 1 \iff p \geq 1 - \frac{1}{R_0}$$

To reach an immune fraction p with a vaccine of efficacy e requires a coverage $c = p/e$.

Observation 1.1

the threshold is not 1. It is $1 - 1/R_0$ —for any finite R_0 it is strictly less than one.

Observation 1.2

coverage and immune fraction are different quantities. $c = p/e$ —unless the efficacy is one, they do not coincide.

Observation 1.3

coverage has a ceiling $c \leq 1$ —once p/e exceeds one, that instrument cannot reach.

Question: how many people need immunity to stop an epidemic?

2. At $R_0 = 3$ It Stops at Two Thirds

Computation 2.1: threshold and required coverage against R_0 (computed in this paper).

R_0	Threshold $1 - 1/R_0$ [%]	Coverage at 95 % efficacy	Coverage at 90 %
2	50.0000	52.6316	55.5556
3	66.6667	70.1754	74.0741
5	80.0000	84.2105	88.8889
12	91.6667	96.4912	101.8519
18	94.4444	99.4152	104.9383

Observation 2.2

the threshold at $R_0 = 3$ is 66.6667 per cent. Two thirds—one person in three may remain without immunity.

Observation 2.3

at exactly the threshold the effective reproduction number is exactly $1.3 \times (1 - 2/3) = 1.000000000$ —inverting the definition returns to one.

Observation 2.4

however large R_0 becomes, the threshold never reaches 1.0.999999 even at $R_0 = 10^6$ ——“everyone must be immune” holds for no R_0 .

Observation 2.5

but at $R_0 = 18$ it is 94.4444 per cent, effectively almost everyone. One person in eighteen——the room left by “not everyone” is narrow.

Observation 2.6

the threshold increases monotonically in R_0 . 50.0000 $\rightarrow$ 94.4444——rising, but bounded above.

3. The Impossible Figure of 101.85 Per Cent

Computation 3.1: required coverage at $R_0 = 12$, a representative value for measles (computed in this paper).

Efficacy e [%]	Required coverage $c = p/e$ [%]	Enough if everyone is vaccinated
95	96.4912	yes
91.6667	100.0000	exactly (the boundary)
90	101.8519	no
80	114.5833	no

Observation 3.2

this is the core of the paper. At 90 per cent efficacy the required coverage is 101.85 per cent——a figure above 100 cannot be achieved. Vaccinating everyone does not reach.

Observation 3.3

the boundary is where the efficacy equals the threshold itself. 91.6667 per cent at $R_0 = 12$ —the threshold formula and the lower bound on efficacy give the same number.

Observation 3.4

the inversion closes. Set the efficacy to $1 - 1/R_0$ and the required coverage is exactly 1.000000—not a coincidence but the structure of $c = (1 - 1/R_0)/e$.

Observation 3.5

between 95 and 90 per cent the answer changes from possible to impossible. 96.4912 and 101.8519—five points straddle the boundary.

Observation 3.6

raising the coverage cannot solve it. The ceiling is 100 per cent—here only the efficacy can be raised.

4. The Two Kinds of “Not Enough” Are Repaired in Different Places

Observation 4.1

short coverage is an operational problem. Some people were not reached—the remedy is to reach further.

Observation 4.2

short efficacy is a product problem. The vaccine was given and immunity did not follow—the remedy is a different vaccine or a booster.

Observation 4.3

both look like “coverage is short”.The same figure appears in the field—— $c < \text{the required value}$.

Observation 4.4

the distinction shows in whether the required value exceeds 100.If it does, the problem is the efficacy——and reaching further cannot solve it.

Observation 4.5

they are different-root by the criterion of Paper 300.Coverage is set by operations, efficacy by the immune response——one phrase, “not enough”, with two roots.

Observation 4.6

the shape is that of Paper 386.There “poorly soluble” named solubility and dissolution rate——here “not enough” names coverage and efficacy.

5. Separator: Is the Coverage Short, or the Efficacy

Observation 5.1

separator——does the required coverage $(1 - 1/R_0)/e$ exceed 100 per cent or not.

What is short	Required coverage	Reaching further	Remedy
Coverage	below 100 %	solves it	reach further
Efficacy	100 % or above	cannot solve it	different vaccine, booster

Observation 5.2

the third column is the test. Does reaching further solve it—if not, the problem is on the product side.

Observation 5.3

the boundary is a single inequality. $e \geq 1 - 1/R_0$ —unless the efficacy is at least the threshold, no coverage suffices.

Observation 5.4

the separator can be moved. Reduce contacts to lower the effective R_0 and the threshold falls, so the same efficacy now reaches—a third remedy that is neither of the two.

Observation 5.5

to be honest—both 91.6667 and 101.85 per cent come from a model with homogeneous mixing and a single R_0 . Contact heterogeneity can move the threshold either way—what does not move is that the threshold never reaches 1, and that an efficacy below the threshold leaves everyone insufficient.

Observation 5.6

one last thing—“everyone must be immune” and “everyone is not enough” are both wrong and can both arise. The first holds for no R_0 —the second arises whenever the efficacy is low. Fail to separate them and the remedy points the wrong way.

6. Roll Call and Closing

1. The threshold is $1 - 1/R_0$ (Section 1): **not 1.**
2. 66.6667 per cent at $R_0 = 3$ (Computation 2.1): **two thirds.**
3. $R_{\text{eff}} = 1.000000000$ at the threshold (Observation 2.3): **the inversion closes.**
4. 0.999999 even at $R_0 = 10^6$ (Observation 2.4): **it never reaches 1.**

5. **94.4444** per cent at $R_0 = 18$ (Computation 2.1): **little room left**.
6. **101.85** per cent at **90 per cent efficacy** (Computation 3.1): **unachievable**.
7. **The boundary is efficacy = threshold** (Observation 3.3): **91.6667 %** at $R_0 = 12$.
8. **Five points cross from possible to impossible** (Observation 3.5): **96.4912** and **101.8519**.
9. **Different-root** (Observation 4.5): **operations and immune response**.
10. **A third remedy: reduce contacts** (Observation 5.4): **lower the threshold itself**.

Closing: it is sometimes said that herd immunity requires everyone to be immune—it does not; and yet everyone can still be insufficient. The threshold is $1 - 1/R_0$ and not 1—at $R_0 = 3$ it is **66.6667** per cent, so one person in three may remain without immunity. At exactly the threshold the effective reproduction number is exactly 1.000000000, and inverting the definition returns to one. Even at $R_0 = 10^6$ the threshold is 0.999999, so “everyone must be immune” holds for no R_0 . At $R_0 = 18$, however, it is **94.4444** per cent, and the room left by “not everyone” is narrow. But the required coverage can exceed 100 per cent—reaching an immune fraction p with efficacy e needs coverage $c = p/e$, so at $R_0 = 12$ with 90 per cent efficacy the required coverage is **101.8519** per cent. A figure above 100 cannot be achieved: vaccinating everyone does not reach. At 95 per cent efficacy it is **96.4912** and does reach, so five points of efficacy straddle the boundary between possible and impossible. The boundary is where the efficacy equals the threshold itself—**91.6667** per cent at $R_0 = 12$. Setting the efficacy to $1 - 1/R_0$ makes the required coverage exactly 1.000000, which is not a coincidence but the structure of $c = (1 - 1/R_0)/e$. And there, raising the coverage cannot solve it, because the ceiling is 100 per cent; only the efficacy can be raised. The two kinds of “not enough” are repaired in different places—short coverage is operational and the remedy is to reach further; short efficacy is a product problem and the remedy is a different vaccine or a booster. Both appear in the field as the same figure, “coverage is short”. The distinction shows in whether the required value exceeds 100—if it does, reaching further cannot solve it. By the criterion of Paper 300 they are different-root: operations on one side, the immune response on the other, the shape of Paper 386. One thing does the separating—does the required coverage $(1 - 1/R_0)/e$ exceed 100 per cent or not. The test is whether reaching further solves it; if not, the problem is on the product side. The boundary is the single inequality $e \geq 1 - 1/R_0$. And the separator can be moved: reduce contacts to lower the effective R_0 , the threshold falls, and the same efficacy now reaches—a third remedy that is neither of the two. To be honest—both **91.6667** and **101.85** per cent come from a model with homogeneous mixing and a single R_0 , and contact heterogeneity can move the threshold either way. What does not move is that the threshold never reaches 1, and that an efficacy below the threshold leaves everyone insufficient. One last thing—“everyone must be immune” and “everyone is not enough” are both wrong and can both arise. The first holds for no R_0 ; the second arises whenever the efficacy is low. Fail to separate them and the remedy points the wrong way.

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