

The Fourth Satellite Is for the Clock, Not for the Position

— space has three dimensions and four satellites are needed because there are four unknowns — a quartz clock good to 10^{-6} is off by **299.79** m after a second and **1079.25** km after an hour — the receiver obtains atomic-clock time without carrying an atomic clock — [Paper 402]

Yuuki YAMAGISHI (山岸 勇輝)

Yamagishi Complexity Torus Lab, Japan

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— *From thinking to syncing.* —

Abstract

Position is three-dimensional, so three satellites ought to be enough. This paper shows that the fourth is not there for the position—it is there because the receiver's clock is unknown. No new theorem or law is claimed. First, light travels **0.299792** m in a nanosecond, so a time error is directly a distance error (Section 1). Second, a quartz clock good to 10^{-6} is off by **299.79** m after a second, **17.99** km after a minute and **1079.25** km after an hour (Section 2). Third, and this is the core of the paper. The unknowns are three of position and one of clock: four, which is why four satellites are needed (Section 2). Fourth, with a known clock three suffice. The fourth is not there for the geometry (Section 2). Fifth, the relativistic correction is **+45.7185** general and **−7.2136** special, **+38.5048** $\mu\text{s/day}$ in total; neglect it and the error is **11.54** km a day (Section 3). Sixth, the separator is whether there are three unknowns or four (Section 5).

Scope of this paper (scope note): No new theorem or law is claimed——positioning from time of arrival, simultaneous estimation of the receiver clock bias, and the relativistic corrections in GPS are all known.No receiver is designed——signal acquisition, tracking and code correlation are not entered. Only the count of unknowns and the size of the time error.No accuracy is claimed——real accuracy is set by the ionosphere, troposphere, multipath and geometry, all outside this paper.Relativity is not derived——numbers are put into known formulae. The Earth is taken as a sphere and the orbit as a circle.Other navigation systems are not treated——inertial navigation and terrestrial systems are not the subject.Time distribution is not discussed——that GPS is used as a time source is stated, not examined.Relation to earlier papers: Paper 384 showed that the frequency term of free-space loss comes from the receiving side——here the fourth satellite exists for an unknown on the receiving side.Paper 394 showed that the flow alone does not fix the state——here three ranges alone do not fix the position.Paper 392 showed that the boiling point is a property of a pair——here a range comes from a pair of position and time.Paper 300 showed that same-root and different-root can be decided——the position unknowns and the clock unknown are different-root by that criterion.What is added is converting the clock error into distance at three elapsed times, counting the unknowns against the number of satellites, showing that three satellites narrow to two points when the clock is known, giving the breakdown of the relativistic correction and the daily error, and placing the separator at “how many unknowns”.

1. Notation and the Question

Setting: let satellite i be at A_i and transmit at t_i . The receiver position X and its clock bias b are unknown.

$$|X - A_i| = c(t_{\text{recv}} + b - t_i)$$

The unknowns are the three components of X together with b : four in all.

Observation 1.1

light travels 0.299792 m in a nanosecond.A time error is directly a distance error——one nanosecond corresponds to 30 cm.

Observation 1.2

the satellite clocks are atomic. t_i may be taken as known——the receiver’s clock may not.

Observation 1.3

four unknowns need four equations. One satellite gives one—hence four.

Question: space is three-dimensional; why are four satellites needed?

2. A Clock That Drifts 1079.25 km in an Hour

Computation 2.1: error of a quartz clock good to 10^{-6} (computed in this paper).

Elapsed	Clock error	As a distance
1 second	1.000×10^{-6} s	299.7925 m
1 minute	6.000×10^{-5} s	17.9875 km
1 hour	3.600×10^{-3} s	1079.2528 km

Observation 2.2

this is the core of the paper. With an ordinary clock the receiver is off by 299.79 m after one second—so the clock has to be treated as unknown: it is nowhere near the accuracy at which it could be assumed known.

Observation 2.3

left alone for an hour it is 1079.25 km. The length of the Japanese archipelago—without calibration, positioning does not exist.

Observation 2.4

so the unknowns number four. Three of position, one of clock—not “three because space is three-dimensional”.

Observation 2.5

with a known clock three suffice. Differencing three spheres gives two linear equations, which fix a line meeting the sphere at two points—one of which lies off the Earth or below it and is discarded.

Observation 2.6

the fourth is therefore not for the geometry. The geometry closes with three—the fourth removes an unknown on the receiver's side.

3. Solving for the Clock Amounts to Distributing the Clock

Observation 3.1

solving the four equations yields b along with the position. Not a by-product but one component of the same solution—the receiver learns its own clock bias.

Observation 3.2

so it obtains atomic-clock time without carrying an atomic clock. The satellites' atomic clocks are broadcast—a positioning system that is simultaneously a time-distribution system.

Observation 3.3

that time requires a relativistic correction. $+45.7185 \mu\text{s/day}$ from general relativity, -7.2136 from special— $+38.5048 \mu\text{s/day}$ in total.

Observation 3.4

the two have opposite signs and partly cancel. Gravity speeds the clock, speed slows it—but they differ by a factor of 6.3 and do not cancel.

Observation 3.5

neglect the correction and the error is 11.54 km a day. $38.5048 \mu s \times c$ ——unusable within one day.

Observation 3.6

so in this system relativity is not a theory but a design value. The satellite clocks are offset in frequency before launch——if it did not act, the positions would not agree.

4. Three Ranges Alone Do Not Fix the Position

Observation 4.1

“three distances fix a position” is correct if the distances really are distances. Three spheres meeting——what is measured here is not a distance.

Observation 4.2

what is measured is a time of arrival. Converting it to a distance needs a clock——and an unknown clock leaves an unknown inside the distance.

Observation 4.3

that unknown rides on all three in common. The same cb in every equation——so one unknown suffices and a fourth equation solves it.

Observation 4.4

were it not common, each satellite would add an unknown and nothing would solve. n equations with $3 + n$ unknowns——commonality is the condition on which the method rests.

Observation 4.5

they are different-root by the criterion of Paper 300. The position unknowns are set by geometry, the clock unknown by the receiver's crystal—one word, “unknown”, with two roots.

Observation 4.6

the shape is that of Paper 384. There the frequency term came from the receiving antenna—here the fourth satellite exists for an unknown on the receiving side. In both, a demand thought to be external comes from the receiver.

5. Separator: Three Unknowns or Four

Observation 5.1

separator—is the receiver's time known or unknown.

Receiver clock	Unknowns	Satellites needed	What is obtained
An atomic clock	3	3	position only
Ordinary quartz	4	4	position and time

Observation 5.2

the second row needs one more satellite and gets more back. The time comes with it—receiving one extra signal substitutes for owning an atomic clock.

Observation 5.3

the prices differ by orders of magnitude. An atomic clock against a fourth receive channel—the choice of method is decided by cost, not by geometry.

Observation 5.4

the separator can be moved. At a fixed station of known position the only unknown is the clock, and one satellite suffices——time-only receivers are used exactly that way.

Observation 5.5

to be honest——both $38.5048 \mu\text{s/day}$ and 11.54 km come from an estimate with a spherical Earth and a circular orbit. Eccentricity and the Earth's oblateness are not included——what does not move is that there are four unknowns and that the two relativistic effects have opposite signs.

Observation 5.6

one last thing——counting “three because space is three-dimensional” mistakes what is being measured. What is measured is not distance but time——recount what is measured and the required number changes.

6. Roll Call and Closing

1. **0.299792 m per nanosecond** (Observation 1.1): **time error is distance error.**
2. **299.7925 m after a second** (Computation 2.1): **quartz at 10^{-6} .**
3. **1079.2528 km after an hour** (Computation 2.1): **no positioning without calibration.**
4. **Unknowns: 3 position + 1 clock** (Observation 2.4): **not three for three dimensions.**
5. **Three satellites give two points if the clock is known** (Observation 2.5): **geometry closes at three.**
6. **Solving yields b as well** (Observation 3.1): **the time comes with it.**
7. **+45.7185 and -7.2136** (Observation 3.3): **+38.5048 $\mu\text{s/day}$.**
8. **11.54 km a day if neglected** (Observation 3.5): **a design value.**
9. **cb is common to all three** (Observation 4.3): **hence a single unknown.**
10. **One satellite at a fixed station** (Observation 5.4): **the separator moves.**

Closing: position is three-dimensional, so three satellites ought to be enough——but the fourth is not there for the position. Light travels **0.299792 m** in a nanosecond, so a time error is directly a distance error——a quartz clock good to 10^{-6} is off by **299.7925 m** after a second, **17.9875 km** after a minute and **1079.2528 km** after an hour.

The receiver's clock is nowhere near the accuracy at which it could be assumed known, so it must be treated as unknown. The unknowns therefore number four—three of position and one of clock; not “three because space is three-dimensional”. With a known clock three suffice: differencing three spheres gives two linear equations fixing a line that meets the sphere at two points, one of which lies off the Earth and is discarded. The geometry closes with three, and the fourth removes an unknown on the receiver's side. And solving for the clock amounts to distributing the clock—the four equations yield b along with the position, so the receiver obtains atomic-clock time without carrying an atomic clock. The satellites' atomic clocks are broadcast, and a positioning system is simultaneously a time-distribution system. That time requires a relativistic correction— $+45.7185\ \mu\text{s/day}$ from general relativity and -7.2136 from special, $+38.5048$ in total. The two have opposite signs and partly cancel, but differ by a factor of 6.3 and do not. Neglect it and the error is **11.54** km a day: in this system relativity is not a theory but a design value, and the satellite clocks are offset in frequency before launch. Three ranges alone do not fix the position, because what is measured is not a distance—it is a time of arrival, and converting it needs a clock. The unknown rides on all three as the same cb , so one unknown suffices and a fourth equation solves it. Were it not common, n equations would carry $3 + n$ unknowns and nothing would solve: commonality is the condition on which the method rests. By the criterion of Paper 300 they are different-root—geometry on one side, the receiver's crystal on the other, the shape of Paper 384. One thing does the separating—is the receiver's time known or unknown. An atomic clock means three unknowns, three satellites and position only; ordinary quartz means four, four, and position with time. Receiving one extra signal substitutes for owning an atomic clock, and the prices differ by orders of magnitude: the choice is decided by cost, not by geometry. At a fixed station of known position the only unknown is the clock and one satellite suffices, which is how time-only receivers are used. To be honest—both $38.5048\ \mu\text{s/day}$ and **11.54** km come from an estimate with a spherical Earth and a circular orbit, without eccentricity or oblateness. What does not move is that there are four unknowns and that the two relativistic effects have opposite signs. One last thing—counting “three because space is three-dimensional” mistakes what is being measured. What is measured is not distance but time. Recount what is measured and the required number changes.

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Yuuki YAMAGISHI (山岸 勇輝)

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