

The Horizon Is Not the Limit of What Can Be Seen

— the horizon of a person **1.7** m tall is only **4.6542** km away, yet a **100** m tower is visible from **40.3503** km, which is **8.6697** times as far — Mount Fuji from **224.0352** km — the distance goes as the square root of height, so doubling one's height buys **1.4142** — [Paper 404]

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— *From thinking to syncing.* —

Abstract

Nothing beyond the horizon can be seen, it would seem. This paper shows that tall things are seen there routinely — a tower **8.6697** times one's own horizon away. No new theorem or law is claimed. First, the distance to the horizon is $\sqrt{2Rh + h^2}$, or **4.6542** km at a height of **1.7** m (Section 2). Second, being a square root, doubling one's height buys only **1.4142** (Section 2). Third, and this is the core of the paper. Tall things are visible past the horizon: a **100** m tower from **40.3503** km, **8.6697** times one's own horizon (Section 3). Fourth, Mount Fuji (**3776** m) is visible from **224.0352** km (Section 3). Fifth, refraction multiplies the distance by **1.063015**, the square root of the **1.13** effective-radius factor (Section 4). Sixth, the separator is whether one is looking at the ground or at an object (Section 5).

Scope of this paper (scope note): No new theorem or law is claimed—the horizon distance, the sum formula for two objects and the effective-radius factor for atmospheric refraction are all known. No surveying is done—real visibility is decided by terrain and buildings. Only a sphere and straight lines of sight. No atmosphere is built—the vertical profile of refractive index, mirages and looming are not the subject. One representative value $k = 1.13$ is used. The Earth is taken as a sphere—oblateness, swell and tide are not the subject. Whether a thing is actually seen is not claimed—eyesight, atmospheric clarity and contrast are not the subject. Only whether the sight line is geometrically blocked. Horizon over land and sea are not distinguished—an unobstructed spherical horizon is treated. Relation to earlier papers: Paper 395 showed that sound travels far because it bends—here light bends too and gains 1.063. Paper 389 showed that length, not thickness, is what binds—here the other object’s height, not one’s own. Paper 391 showed that a representative value does not represent—here “distance to the horizon” does not represent visibility. Paper 300 showed that same-root and different-root can be decided—one’s own height and the object’s height are different-root by that criterion. What is added is putting numbers on the horizon distance at four heights, confirming the square root at factors of two and four, obtaining the visibility distances of a tower and of a mountain, giving the refraction factor 1.063015, and placing the separator at “ground or object”.

1. Notation and the Question

Setting: take the Earth as a sphere of radius $R = 6371$ km and find the distance from an eye at height h to the horizon.

$$d(h) = \sqrt{2Rh + h^2} \approx \sqrt{2Rh}$$

An object of height h_2 is visible from an eye at height h_1 out to $d(h_1) + d(h_2)$.

Observation 1.1

d is the square root of h . The h^2 term is negligible for $h \ll R$ —raising the height buys little.

Observation 1.2

the two-object formula is a sum. $d(h_1) + d(h_2)$ —one’s own height and the object’s act separately.

Observation 1.3

if the object is tall, a low eye still reaches. One term of the sum is large——“distance to the horizon” is only one half of that sum.

Question: is nothing beyond the horizon visible?

2. It Grows Only as the Square Root of Height

Computation 2.1: horizon distance against height (computed in this paper).

Eye height [m]	To the horizon [km]	With refraction [km]
1.7 (a person)	4.6542	4.9475
10	11.2881	11.9994
100	35.6961	37.9455
10000 (an airliner)	357.0994	379.6019

Observation 2.2

the horizon of a person 1.7 m tall is only 4.6542 km. Under five kilometres——an hour’s walk.

Observation 2.3

four times the height gives exactly twice the distance. $1.7 \rightarrow 6.8$ m gives $4.6542 \rightarrow 9.3084$ km——the square root confirmed numerically.

Observation 2.4

twice the height buys only $1.414214\sqrt{2}$ ——standing on tiptoe is inefficient.

Observation 2.5

at an airliner's 10 km it is 357.0994 km. Here the h^2 term acts— $\sqrt{2Rh}$ alone gives 356.9 km, off by 0.06 per cent.

Observation 2.6

at a person's height the h^2 term acts at under 10^{-6} . The range where the approximation holds—where it starts to matter is settled by putting in the numbers.

3. A 100 m Tower Is Visible 8.6697 Horizons Away

Computation 3.1: visibility from an eye at 1.7 m (computed in this paper).

Object	Visible from [km]	In units of one's own horizon
Sea surface (the horizon)	4.6542	1.0000
A 10 m boat	15.9423	3.4253
A 100 m tower	40.3503	8.6697
Mount Fuji, 3776 m	224.0352	48.1361

Observation 3.2

this is the core of the paper. A 100 m tower is visible from 40.3503 km—8.6697 times one's own horizon of 4.6542 km. Seeing past the horizon is not an exception.

Observation 3.3

Mount Fuji from 224.0352 km, 48.1361 times. Nearly fifty horizons—consistent with its being seen from far away.

Observation 3.4

what is seen is only the summit. The base is hidden by the Earth——“visible” and “visible entire” are different.

Observation 3.5

the object’s height matters more than one’s own. The larger term dominates the sum——the tower’s 35.6961 km is 7.67 times one’s own 4.6542 km.

Observation 3.6

the shape is that of Paper 389. There length rather than thickness was what bound——here the object’s height rather than one’s own. In both, the quantity attended to is not the one that acts.

4. Light Bends a Little Too

Observation 4.1

air thins with height, so rays bend downward. The same sense as the Earth’s curvature——putting the horizon a little further off.

Observation 4.2

by convention the Earth’s radius is multiplied by 1.13 and the rays treated as straight. The effective radius——the distance grows by $\sqrt{1.13} = 1.063015$.

Observation 4.3

at a person’s height 4.6542 km becomes 4.9475 km. 293 m further——a 6.3 per cent effect.

Observation 4.4

the shape is that of Paper 395. There sound travelled far because it bent——here light is seen a little further because it bends. In both, not travelling straight extends the reach.

Observation 4.5

but the magnitudes differ by orders. Sound is trapped and carries thousands of kilometres; light gains 6.3 per cent——the same word “bends”, with gradients that differ by orders of magnitude.

Observation 4.6

1.13 is representative, not constant. A temperature inversion raises it——and a ship beyond the horizon appears to float.

5. Separator: The Ground, or an Object

Observation 5.1

separator——does the thing being looked at have height or not.

Object	Height	Visible from	At an eye height of 1.7 m
Sea or ground	0	$d(h_1)$	4.6542 km
Anything with height	$h_2 > 0$	$d(h_1) + d(h_2)$	unbounded

Observation 5.2

only the first row is “the distance to the horizon”. An object of height zero——confuse it with visibility in general and the error is orders of magnitude.

Observation 5.3

the second row has no ceiling. As far as the object is tall—the Moon is 380,000 km away and is seen.

Observation 5.4

the separator can be moved. Let the object's height go to zero and the visibility falls to one's own horizon—the two are connected continuously.

Observation 5.5

to be honest—both 40.3503 km and 224.0352 km are pure geometry without refraction. In practice clarity and contrast decide—what does not move is that the distance goes as the square root of height and that two objects add.

Observation 5.6

one last thing—"the distance to the horizon" is a quantity about oneself. It says nothing about the other object—call it visibility and one attends only to one's own height.

6. Roll Call and Closing

1. $d = \sqrt{2Rh + h^2}$ (Section 1): **the square root of height.**
2. **4.6542 km at 1.7 m** (Computation 2.1): **under five kilometres.**
3. **Four times the height, twice the distance** (Observation 2.3): **the square root confirmed.**
4. **Twice the height buys 1.414214** (Observation 2.4): **tiptoe is inefficient.**
5. **357.0994 km from an airliner** (Computation 2.1): **the h^2 term starts to act.**
6. **A 100 m tower from 40.3503 km** (Computation 3.1): **8.6697 horizons.**
7. **Mount Fuji from 224.0352 km** (Computation 3.1): **48.1361 horizons.**
8. **Only the summit is seen** (Observation 3.4): **the base is hidden.**
9. **Refraction gives 1.063015** (Observation 4.2): **the root of 1.13.**

10. Only a zero-height object is the horizon (Observation 5.2): confusing them costs orders.

Closing: nothing beyond the horizon can be seen, it would seem—but tall things are seen there routinely. The distance to the horizon is $d(h) = \sqrt{2Rh + h^2}$, the square root of the height—only **4.6542** km at 1.7 m, an hour’s walk. Four times the height gives exactly twice the distance, and twice the height buys only 1.414214—standing on tiptoe is inefficient. At an airliner’s 10 km it is 357.0994 km, where the h^2 term acts and $\sqrt{2Rh}$ alone is off by 0.06 per cent; at a person’s height that term acts at under 10^{-6} . But visibility between two objects is a sum— $d(h_1) + d(h_2)$, so a 100 m tower is visible from **40.3503** km, which is **8.6697** times one’s own horizon. Mount Fuji (3776 m) from **224.0352** km, 48.1361 times, consistent with its being seen from far away. What is seen is only the summit, the base being hidden by the Earth—“visible” and “visible entire” are different. The larger term dominates: the tower’s 35.6961 km is 7.67 times one’s own 4.6542 km, so the object’s height matters more than one’s own. As in Paper 389, the quantity attended to is not the one that acts. Light bends a little too—air thins with height so rays bend in the same sense as the Earth’s curvature, and by convention the radius is multiplied by 1.13, giving a distance factor of $\sqrt{1.13} = \mathbf{1.063015}$. At a person’s height 4.6542 km becomes 4.9475 km, 293 m further, a 6.3 per cent effect. As in Paper 395, not travelling straight extends the reach—but the magnitudes differ by orders: sound is trapped and carries thousands of kilometres, light gains 6.3 per cent. And 1.13 is representative rather than constant: a temperature inversion raises it, and a ship beyond the horizon appears to float. One thing does the separating—does the thing being looked at have height or not. Sea or ground, of height zero, is visible out to $d(h_1) = 4.6542$ km; anything with height, out to $d(h_1) + d(h_2)$, without a ceiling. Only the first is “the distance to the horizon”, and confusing it with visibility in general costs orders of magnitude. Let the object’s height go to zero and the visibility falls to one’s own horizon: the two are connected continuously. To be honest—both 40.3503 km and 224.0352 km are pure geometry without refraction, and in practice clarity and contrast decide. What does not move is that the distance goes as the square root of height and that two objects add. One last thing—“the distance to the horizon” is a quantity about oneself. It says nothing about the other object. Call it visibility and one attends only to one’s own height.

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