

For X-Rays the Refractive Index Is Less Than One and the Phase Velocity Exceeds c

— in silicon $n = \mathbf{0.999992558}$ and the phase velocity is **2231.11** m/s above c — and still no information travels faster — the index is small not because the medium is thinner than vacuum but because the frequency is above the binding resonances — [Paper 407]

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— *From thinking to syncing.* —

Abstract

We are taught that the refractive index exceeds one and that light slows inside matter. This paper shows that for X-rays it is the other way round— $n < 1$, and the phase velocity exceeds c . No new theorem or law is claimed. First, for Cu $K\alpha$ in silicon, $\delta = \mathbf{7.442142} \times 10^{-6}$ and $n = \mathbf{0.999992558}$ (Section 2). Second, and this is the core of the paper. The phase velocity is **2231.11** m/s above c (Section 2). Third, and still no information travels faster: the group velocity does not exceed c (Section 4). Fourth, the critical angle is **0.221048** degrees, matching the known 0.223 for silicon (Section 3). Fifth, since $n < 1$, total reflection occurs on entry from vacuum: on the outside, not the inside (Section 3). Sixth, the separator is phase velocity against signal velocity (Section 5).

Scope of this paper (scope note): No new theorem or law is claimed—the X-ray index $n = 1 - \delta$, total external reflection and the critical angle $\sqrt{2\delta}$ are all known in X-ray optics. No dispersion theory is built—why $n < 1$ above the resonances is not derived. Numbers are put into the free-electron formula. Absorption is not treated—the imaginary part of $n = 1 - \delta + i\beta$ is not the subject. Only the real part. Relativity is not discussed—that a phase velocity above c violates no causality is known; this paper only names the group velocity. No instrument is designed—the design of X-ray mirrors and multilayers is not the subject. Absorption edges are not treated—the free-electron approximation fails near an edge; a wavelength away from edges is used. Relation to earlier papers: Paper 403 showed that the order of temperature is not the order of hotness—here the order of refractive index is not the order of “density”. Paper 395 showed that sound travels far because it bends—here the sign of the index is reversed and total reflection happens on the outside. Paper 384 showed that free-space loss is not a loss—in both, a name disagrees with the substance. Paper 300 showed that same-root and different-root can be decided—phase velocity and signal velocity are different-root by that criterion. What is added is computing δ from silicon’s electron density, checking the critical angle against the known value, putting the phase-velocity excess at 2231.11 m/s, confirming that doubling the wavelength quadruples δ , and placing the separator at “phase or signal velocity”.

1. Notation and the Question

Setting: in the X-ray region, treating the electrons as free, the refractive index is written

$$n = 1 - \delta, \quad \delta = \frac{r_e \lambda^2 n_e}{2\pi}$$

with $r_e = 2.8179 \times 10^{-15}$ m the classical electron radius and n_e the electron density. Take silicon (density 2.329 g/cm³, $Z = 14$) and Cu K α ($\lambda = 0.15406$ nm).

Observation 1.1

δ is positive. r_e , λ^2 and n_e are all positive—so n is necessarily less than one.

Observation 1.2

if $n < 1$ then the phase velocity c/n exceeds c . By definition—not an exception; for X-rays it is always so.

Observation 1.3

δ goes as λ^2 .The shorter the wavelength the closer to one——this is not the formula for visible light.

Question: does light slow down inside matter?

2. $n = 0.999992558$, and the Phase Velocity Is $c + 2231.11$ m/s

Computation 2.1: Cu $K\alpha$ in silicon (computed in this paper).

Quantity	Value
Electron density n_e	6.991434×10^{29} /m ³
δ	7.442142×10^{-6}
Refractive index $n = 1 - \delta$	0.999992558
Phase velocity c/n	$c + \mathbf{2231.11}$ m/s
Critical angle $\sqrt{2\delta}$	0.221048 degrees

Observation 2.2

this is the core of the paper.The phase velocity is 2231.11 m/s above c ——a direct consequence of $n < 1$, requiring no special conditions.

Observation 2.3

n departs from one only slightly.By 7.4×10^{-6} ——five orders of magnitude smaller than for glass in visible light ($n \approx 1.5$).

Observation 2.4

doubling the wavelength quadruples δ exactly. λ^2 ——the structure of the formula confirmed numerically.

Observation 2.5

the critical angle is exactly proportional to the wavelength. $\sqrt{2\delta} \propto \lambda$ —double the wavelength and double the angle.

Observation 2.6

apply the same formula to visible light (600 nm) and $\delta > 1$. The free-electron approximation fails—a formula has a range.

3. Total Reflection Happens on the Outside

Observation 3.1

in visible light, total reflection happens on leaving a dense medium for a thin one. Glass to air—from larger n to smaller.

Observation 3.2

for X-rays the vacuum has the larger n . $1 > 0.999992558$ —so total reflection happens on entering matter from vacuum.

Observation 3.3

it is called total external reflection. The word “external” is in the name—because it is the reverse of the visible-light intuition.

Observation 3.4

the critical angle is a grazing 0.221048 degrees. Measured from the surface—agreeing with silicon’s known 0.223 degrees to within 0.01.

Observation 3.5

so X-ray mirrors are used at grazing incidence. Grazing-incidence optics——X-ray telescopes are nested tubes because of this one angle.

Observation 3.6

the shape is the inverse of Paper 395. There sound bent toward a minimum and was trapped——here the index runs the other way and, unable to enter, the ray reflects.

4. A Phase Velocity Above c Carries No Information

Observation 4.1

the phase velocity is the speed of surfaces of constant phase. A property of an infinite monochromatic wave——on which nothing is marked.

Observation 4.2

information is carried by changes in the waveform. An envelope or a discontinuity——whose speed is the group or signal velocity.

Observation 4.3

since δ goes as λ^2 there is dispersion. n depends on wavelength——so phase and group velocity do not coincide.

Observation 4.4

they are different-root by the criterion of Paper 300. The phase velocity is the motion of a phase surface, the signal velocity the motion of a front——one word, “speed”, with two roots.

Observation 4.5

the shape is that of Paper 403. There the order of temperature was not the order of hotness—here the order of speed is not the order of information speed. In both, a familiar quantity does not carry the ordering.

5. Separator: Phase Velocity or Signal Velocity

Observation 5.1

separator—is the wave being watched marked or unmarked.

Quantity	Speed of what	Exceeds c	Bears on causality
Phase velocity c/n	surfaces of constant phase	yes	no
Group / signal velocity	changes in the waveform	no	yes

Observation 5.2

the last column is the test. Does it bear on causality—a quantity that does not may exceed c and break nothing.

Observation 5.3

the phrase “faster than light” names two things. One happens routinely—the other does not happen.

Observation 5.4

the separator can be moved. Put an amplitude variation on the monochromatic wave and a signal velocity becomes definable—the moment a mark is made, the speed changes.

Observation 5.5

to be honest—both 0.221048 degrees and 2231.11 m/s are free-electron values. They fail near an absorption edge—what does not move is that δ is positive and that $n < 1$ follows.

Observation 5.6

one last thing—“the refractive index exceeds one” is an empirical rule about visible light. Not a law—change the wavelength and the sign changes.

6. Roll Call and Closing

1. $\delta = 7.442142 \times 10^{-6}$ (Computation 2.1): **positive**.
2. $n = 0.999992558$ (Computation 2.1): **less than one**.
3. Phase velocity $c + 2231.11$ m/s (Observation 2.2): **a direct consequence**.
4. Double the wavelength, quadruple δ (Observation 2.4): λ^2 .
5. $\delta > 1$ for visible light (Observation 2.6): **a formula has a range**.
6. Critical angle 0.221048 degrees (Computation 2.1): **matching the known 0.223**.
7. Total reflection on the outside (Observation 3.2): **vacuum has the larger n** .
8. X-ray mirrors graze (Observation 3.5): **this one angle**.
9. The group velocity does not exceed c (Observation 4.2): **no information gain**.
10. “Exceeds one” is empirical (Observation 5.6): **not a law**.

Closing: we are taught that the refractive index exceeds one and that light slows inside matter—for X-rays it is the other way round. In the X-ray region $n = 1 - \delta$ with $\delta = r_e \lambda^2 n_e / (2\pi)$, and δ is necessarily positive—since r_e , λ^2 and n_e are all positive, n is necessarily less than one. And $n < 1$ makes the phase velocity c/n exceed c —not as an exception but always, for X-rays. For Cu K α in silicon, $\delta = 7.442142 \times 10^{-6}$ and $n = 0.999992558$ —the phase velocity is 2231.11 m/s above c . The index departs from one by only 7.4×10^{-6} , five orders of magnitude less than glass in visible light. Doubling the wavelength quadruples δ exactly and doubles the critical angle, confirming the structure numerically. Apply the same formula at 600 nm and $\delta > 1$: the free-electron approximation fails, and a formula has a range. And total reflection happens on the outside—in visible light it happens leaving glass for air, from larger n to smaller; for X-rays the vacuum has the larger index ($1 > 0.999992558$), so it happens on entering matter from vacuum. It is called total external reflection, and the word “external” is there because it reverses the

visible-light intuition. The critical angle is a grazing **0.221048** degrees, agreeing with silicon’s known **0.223** to within **0.01**. X-ray mirrors are therefore used at grazing incidence, and X-ray telescopes are nested tubes because of this one angle. A phase velocity above c carries no information—the phase velocity is the speed of surfaces of constant phase on an infinite monochromatic wave, on which nothing is marked. Information is carried by changes in the waveform, whose group or signal velocity does not exceed c . Since δ goes as λ^2 there is dispersion, and the two velocities do not coincide. By the criterion of Paper 300 they are different-root, and as in Paper 403 a familiar quantity does not carry the ordering. One thing does the separating—is the wave being watched marked or unmarked. The test is whether the quantity bears on causality: one that does not may exceed c and break nothing. The phrase “faster than light” names two things, one of which happens routinely and one of which does not. Put an amplitude variation on the wave and a signal velocity becomes definable: the moment a mark is made, the speed changes. To be honest—both **0.221048** degrees and **2231.11** m/s are free-electron values and fail near an absorption edge. What does not move is that δ is positive and that $n < 1$ follows. One last thing—“the refractive index exceeds one” is an empirical rule about visible light, not a law. Change the wavelength and the sign changes.

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