

There Is No Undistorted Map, and the Projection Is Not to Blame

— Mercator gives exactly **2** times the length and **4** times the area at latitude **60**, and **33.1634** times the area at **80** — but the cause is not the projection: the Gaussian curvature of a sphere differs from that of a plane — a million-square-kilometre triangle has **1.4113** degrees of angular excess — [Paper 409]

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— *From thinking to syncing.* —

Abstract

The Mercator projection is criticised for enlarging high latitudes. This paper shows that every projection distorts — the cause is not the projection but the impossibility of flattening a curved surface. No new theorem or law is claimed. First, Mercator gives exactly **2** times the length and **4** times the area at latitude **60** (Section 2). Second, at latitude **80** the area factor is **33.1634** (Section 2). Third, and this is the core of the paper. The cause is not the projection: the Gaussian curvature is $1/R^2$ for a sphere and 0 for a plane, and the values differ (Section 3). Fourth, a million-square-kilometre spherical triangle has **1.4113** degrees of angular excess, and a plane has exactly **0** (Section 3). Fifth, at a 100 km side the excess shrinks to **0.0071** degrees: the distortion is set by scale (Section 4). Sixth, the separator is whether another projection removes it or nothing but a change of scale will (Section 5).

Scope of this paper (scope note): No new theorem or law is claimed——Gauss’s Theorema Egregium, Girard’s theorem and the Mercator scale factors are all known.No theorem is proved——both the Theorema Egregium and Girard’s theorem are used as known. Only numbers are put in.No projection is designed——which projection is best is not the subject. Only that distortion is unavoidable.The Earth is taken as a sphere——the flattening ($1/298$) is not the subject; the qualitative conclusion is unchanged.Politics is not discussed——arguments about which projection ought to be used are not the subject.Geodesics are not treated——great circles and rhumb lines are not the subject.Relation to earlier papers: Paper 397 showed that the failure was not because the theoretical strength was reached——here the distortion is not because the projection is poor.Paper 389 showed that length, not thickness, is what binds——here curvature, not the projection.Paper 326 showed that the square–cube “law” is an identity——here too, “it distorts” is a consequence of geometry rather than a result of choice.Paper 300 showed that same-root and different-root can be decided——distortion from the projection and distortion from curvature are different-root by that criterion.What is added is putting numbers on the Mercator factors at five latitudes, giving the true area ratio of Africa to Greenland, computing the spherical excess at a million square kilometres and at a 100 km side, setting the plane’s exactly zero excess as a control, and placing the separator at “projection or scale”.

1. Notation and the Question

Setting: on the Mercator projection the length factor at latitude φ is $\sec \varphi$ and the area factor $\sec^2 \varphi$. On a sphere, the amount by which the angles of a triangle exceed π is proportional to its area (Girard’s theorem).

$$E = \frac{A}{R^2}$$

Take $R = 6371$ km.

Observation 1.1

the Mercator factor depends only on latitude. $\sec \varphi$ ——one at the equator, divergent at the poles.

Observation 1.2

another projection removes that factor.An equal-area projection gets the areas right——and distorts the angles instead.

Observation 1.3

repair one and another breaks. All one chooses is where to put the distortion—and the reason lies in the curvature.

Question: does a map distort because of its projection?

2. Exactly Four Times the Area at Latitude Sixty

Computation 2.1: Mercator scale factors (computed in this paper).

Latitude [deg]	Length factor $\sec \varphi$	Area factor $\sec^2 \varphi$
0	1.0000	1.0000
30	1.1547	1.3333
45	1.4142	2.0000
60	2.0000	4.0000
80	5.7588	33.1634

Observation 2.2

at latitude 60 the length factor is exactly 2 and the area factor exactly 4. $\sec 60^\circ = 2$ —the round numbers appear because $\cos 60^\circ = 1/2$.

Observation 2.3

at latitude 45 the area factor is exactly 2. $\sec^2 45^\circ = 2$ —already doubled in the middle latitudes.

Observation 2.4

at latitude 80 it is 33.1634. Most of Greenland lies in that band—Africa, on the equator, stays at one.

Observation 2.5

in truth Africa is 14.0212 times Greenland. 3.037×10^7 against $2.166 \times 10^6 \text{ km}^2$ ——on a Mercator map they look about the same size.

Observation 2.6

this distortion vanishes under an equal-area projection.It can be chosen away——which is why it looks like the projection’s fault.

3. The Cause Is Curvature: On a Plane the Excess Is Exactly Zero

Computation 3.1: angular excess of a triangle (computed in this paper).

Figure	Gaussian curvature	Angular excess
Sphere (10^6 km^2)	$2.4636 \times 10^{-14} \text{ /m}^2$	1.4113 degrees
Plane	0	0 degrees
Cylinder, cone	0	0 degrees

Observation 3.2

this is the core of the paper.On the sphere the angles exceed by 1.4113 degrees; on a plane, by exactly 0——no mapping can close that difference, because the Gaussian curvature does not depend on how the surface is bent (Theorema Egregium).

Observation 3.3

a cylinder and a cone have curvature 0.Though they look curved——which is why they can be made from paper and cut open flat.

Observation 3.4

a sphere has $1/R^2 > 0$. Not zero——so it cannot be made from paper and cannot be flattened.

Observation 3.5

so an “undistorted map” is not a question of projection but of existence. Searching does not find one——no ingenuity turns $1/R^2$ into 0.

Observation 3.6

the shape is that of Paper 326. There the square--cube “law” was an identity——here “it distorts” is a consequence of geometry rather than a result of choice.

4. The Size of the Distortion Is Set by Scale

Computation 4.1: spherical excess against scale (computed in this paper).

Area of the triangle	Angular excess
10^6 km^2	1.4113 degrees
A 100 km side (5000 km^2)	0.0071 degrees

Observation 4.2

the excess is proportional to the area. $E = A/R^2$ ——a two-hundredth of the area is a two-hundredth of the excess.

Observation 4.3

a 100 km side gives 0.0071 degrees. Twenty-five arcseconds——visible to a surveyor, invisible on a map.

Observation 4.4

so at small extent a map may be treated as flat. A municipal map—that the curvature is nonzero and that it is not negligible are different statements.

Observation 4.5

the shape is that of Paper 396. There a number was put on “a small angle”—here on “a small region”. In both, how small counts as small is fixed by the accuracy demanded.

Observation 4.6

they are different-root by the criterion of Paper 300. Projection distortion is set by choice, curvature distortion by geometry—one phrase, “map distortion”, with two roots.

5. Separator: Does Another Projection Remove It

Observation 5.1

separator—does it vanish under a different projection or not.

Distortion	Origin	Under another projection	Example
Area enlargement	choice of projection	vanishes	Mercator’s 33.1634
Angles and areas together	Gaussian curvature	does not vanish	excess of 1.4113 degrees

Observation 5.2

the third column is the test. Does changing the projection remove it—if not, blaming the projection repairs nothing.

Observation 5.3

criticism of Mercator is correct about the first row only. Areas do distort—but it does preserve angles, which is what a navigation chart requires.

Observation 5.4

the separator can be moved. Shrink the region and the second distortion effectively disappears—the curvature is unchanged; the area is smaller.

Observation 5.5

to be honest—both 1.4113 degrees and 33.1634 take the Earth as a perfect sphere. Including the flattening ($1/298$) moves them slightly—what does not move is that the sphere’s curvature is positive and the plane’s is zero, and that no mapping closes the gap.

Observation 5.6

one last thing—“design a better projection” is right only when a better one exists. A projection that fixes areas exists—a projection that fixes everything does not. Its absence is not a matter of how hard one looks.

6. Roll Call and Closing

1. Exactly 2 and 4 at latitude 60 (Computation 2.1): because $\cos 60^\circ = 1/2$.
2. Exactly 2 in area at latitude 45 (Observation 2.3): already in the middle latitudes.
3. 33.1634 at latitude 80 (Computation 2.1): Greenland’s latitude band.
4. Africa is truly 14.0212 times (Observation 2.5): they look alike on the map.
5. Sphere curvature $2.4636 \times 10^{-14} / \text{m}^2$ (Computation 3.1): the plane is 0.
6. 1.4113 degrees at 10^6 km^2 (Computation 3.1): exactly 0 on a plane.
7. Cylinder and cone have curvature 0 (Observation 3.3): so paper makes them.
8. 0.0071 degrees at a 100 km side (Computation 4.1): twenty-five arcseconds.

9. The excess is proportional to area (Observation 4.2): set by scale.

10. No projection fixes everything (Observation 5.6): not a matter of searching.

Closing: the Mercator projection is criticised for enlarging high latitudes—but the cause is not the projection. The Mercator factor depends only on latitude—exactly **2** in length and **4** in area at latitude 60, the round numbers appearing because $\cos 60^\circ = 1/2$. Already **2** in area at latitude 45, and **33.1634** at 80. In truth Africa is **14.0212** times Greenland (3.037×10^7 against $2.166 \times 10^6 \text{ km}^2$), and on a Mercator map they look about the same size. And this distortion vanishes under an equal-area projection, which is why it looks like the projection's fault. But the cause is curvature—on a sphere a 10^6 km^2 triangle has **1.4113** degrees of angular excess; on a plane, exactly **0**. No mapping can close that difference, because Gaussian curvature does not depend on how a surface is bent (Theorema Egregium). A cylinder and a cone look curved and have curvature 0, which is why paper makes them and they cut open flat; a sphere has $1/R^2 > 0$ and does neither. So an “undistorted map” is not a question of projection but of existence: no ingenuity turns $1/R^2$ into 0. As in Paper 326, where the square–cube “law” was an identity, “it distorts” is a consequence of geometry rather than a result of choice. The size of the distortion is set by scale—since $E = A/R^2$ the excess is proportional to area, so a 100 km side (5000 km^2) gives **0.0071** degrees, twenty-five arcseconds. Visible to a surveyor, invisible on a map. So a municipal map may be treated as flat: that the curvature is nonzero and that it is not negligible are different statements. As in Paper 396, how small counts as small is fixed by the accuracy demanded. One thing does the separating—does it vanish under a different projection or not. Area enlargement comes from the choice of projection and vanishes; preserving angles and areas together comes from the curvature and does not. Criticism of Mercator is correct about the first only—it does preserve angles, which is what a navigation chart requires. By the criterion of Paper 300 they are different-root: choice on one side, geometry on the other. Shrink the region and the second effectively disappears: the curvature is unchanged and the area is smaller. To be honest—both **1.4113** degrees and **33.1634** take the Earth as a perfect sphere, and including the flattening moves them slightly. What does not move is that the sphere's curvature is positive and the plane's is zero, and that no mapping closes the gap. One last thing—“design a better projection” is right only when a better one exists. A projection that fixes areas exists; one that fixes everything does not. Its absence is not a matter of how hard one looks.

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