

The Fringes Do Not Remember the Exponent

— 3rd ed.: in the **Yamagishi Double-Slit Geometric Wavefront Restoration Theory**, flux conservation forces

$n = d - 1$ while the fringe spacing comes from phase: two distinct roots — [Paper 41]

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— *From thinking to syncing.* —

Abstract

This paper held two results: **(A)** that the condition for the flux of a spherical wave to be conserved in r fixes the exponent uniquely at $n = 2$, and **(B)** that the superposition of two spherical waves reproduces the standard fringe spacing $\Delta y = \lambda D/a$. Both pieces of algebra are correct, and the third edition changes neither. What the third edition shows is that the two do not share a root. First, the fringes do not remember the exponent. Relaxing the $|\psi| \propto r^{-1}$ that (A) forces to a general $|\psi| \propto r^{-m}$, the maxima were located numerically. Between $m = 1$ (what (A) forces) and $m = 3$ (an entirely different value), the first maximum sits at $5.00004983966 \times 10^{-3}$ m and $5.00002450778 \times 10^{-3}$ m: a difference of 2.53×10^{-8} m, five parts per million of one fringe. Sweeping m from 0 to 3 changes the mean fringe spacing by a relative 7.6×10^{-6} only. The exponent acts on the envelope and not on the fringe spacing. What fixes the fringes is the phase difference $k(r_1 - r_2)$ alone, not the falloff of the amplitude. Second, 4π appears in neither. The intensity of Theorem 3.1, $I = \frac{2|A|^2}{R^2} [1 + \cos(kay/R)]$, contains no 4π . Setting the amplitude A to 1, to $1/\sqrt{4\pi}$ and to 17 leaves the first maximum at 1.000045×10^{-2} m in all three cases: 4π enters only the normalisation of the flux and cancels out of the shape of the pattern (the overall-factor check of Paper 11, 3rd ed.). Third, (A) does not select three dimensions. In d spatial dimensions, $|\psi|^2 \propto r^{-n}$ gives a flux proportional to r^{d-1-n} , so the conservation condition is $n = d - 1$ (checked numerically for $d = 2, 3, 4, 5$). $n = 2$ is merely the $d = 3$ case; the condition itself holds in the same form in every dimension (Papers 114, 116). The Flatland framing of the introduction is therefore not to be read as explaining the number of dimensions. Fourth, two internal citations are updated. (i) The second edition's Section 4 described the detector picture as "based on the Lazy Evaluation protocol of Paper 11". Paper 11's third edition has since failed that metaphor on the machinery-transfer test: the core of lazy evaluation is a structure in which the value is already determined and only the computation is deferred (a thunk), and that is precisely what the Kochen–Specker theorem forbids (confirmed there via the Peres–Mermin magic square). This edition therefore removes the appeal to Paper 11. What remains is only the vocabulary of a drop in dimension, which the second edition's Note 4.2 had already limited as "a notational convenience, not a derivation". That limitation was correct and is kept. (ii) the second edition's Note 4.1 assigned the Born exponent 2 to Paper 53 alone, but Paper 11's third edition holds a roll-call of three addresses: Gleason's theorem (Paper 71: a theorem, not a postulate), unitarity (Paper 53: Banach–Lamperti, weaker, since it assumes the density is a power of $|\psi|$), and the 2 of the Hopf map (Paper 68: same root or rhyme, undecided). This paper was pointing only at the weakest of the three. Scope: structural interpretation and self-audit of elementary established facts (Layer 2). No prediction of quantum mechanics is altered. The measurement problem is not solved, $\mathcal{P}_{2D}$ is not called a projection operator, and wave–particle duality is not denied.

Standing of this paper (discipline): the physics used here is **entirely standard**: spherical waves, double-slit interference in the Fraunhofer limit, flux conservation. **No new theorem and no prediction.** The third edition’s contribution is **a separation and an update of citations**: (i) showing by perturbation that this paper’s two results are **distinct roots**, (ii) generalising $n = 2$ to $n = d - 1$ and making explicit that **it does not select a dimension**, (iii) confirming that 4π acts on neither, (iv) **removing Section 4’s appeal to Paper 11** in the light of that paper’s third edition, and (v) writing all three addresses of the Born exponent **2**. **The most important restraints: no claim to have solved the measurement problem** ($\mathcal{P}_{2D}$ is notation; the second edition’s Note 4.2). **Lazy evaluation is not used as a mechanism** (it fails in Paper 11, 3rd ed.). $n = d - 1$ is not read as “the reason space is three-dimensional” (it holds in every dimension; Paper 114). 4π is not treated as a **mechanism** (an identity of any integral over a sphere). **Only known predictions are reproduced; nothing new is predicted.** Layer-3 lenses are quarantined under a non-scientific label.

1. Introduction: The Paper Held Two Results, and Its Title Welded Them Together

The second edition of this paper held two correct results.

(A) Section 2	Flux conservation fixes the exponent uniquely at $n = 2$
(B) Theorem 3.1	Two spherical waves reproduce the fringe spacing $\Delta y = \lambda D/a$

Both pieces of algebra are correct, and the third edition alters neither.

What it alters is the tacit assumption about their relation. The second edition’s title, “3D Spherical Solid Angle 4π Wavefield and Forced 2D Downscaling Collapse via Boundary Measurement”, named the 4π wavefield and the interference picture as **one mechanism**.

This edition tests that. The answer was: they are separate.

2. (A) What Flux Conservation Forces

Write the spherical wave as $\Psi(\vec{r}) = Ae^{ik|\vec{r}|}/|\vec{r}|$. Integrating with the **area element** $dS = r^2 d\Omega$, the flux through the sphere is

$$\Phi(r) = \int_{S^2} |\Psi|^2 dS = \int_{S^2} \frac{|A|^2}{r^2} r^2 d\Omega = 4\pi |A|^2,$$

independent of r . More generally, with $|\Psi|^2 \propto r^{-n}$ we get $\Phi(r) = 4\pi|A|^2 r^{2-n}$, and the only real value that conserves the flux is $n = 2$.

Theorem 2.1 (Generalisation to d dimensions)

In d spatial dimensions with $|\psi|^2 \propto r^{-n}$, the area of the sphere S^{d-1} of radius r scales as r^{d-1} , so

$$\Phi(r) \propto r^{d-1-n}.$$

Φ is independent of r precisely when $n = d - 1$.

Numerical check: setting $n = d - 1$ for each of $d = 2, 3, 4, 5$ and evaluating Φ at $r = 0.5, 1, 2, 5, 100$ gave a spread of 0 (exactly constant) in every case.

Observation 2.1 (The condition does not select a dimension)

$n = 2$ is nothing but the $d = 3$ case of $n = d - 1$. Since the condition itself holds in the same form in every dimension, one cannot say “space is three-dimensional because the flux is conserved”. This is exactly how Paper 114 classified it, “ $n = d - 1$ stands on the side that does not select”, and it is the same quantity Paper 116 addressed by noting that d enters only through the denominator $n = d - 1$.

The Flatland analogy of the introduction must therefore not be read as explaining the number of dimensions. It is a picture for the fact that the observing surface is of lower dimension than the field, and it says nothing about why space is three-dimensional.

Observation 2.2 (On 4π (the second edition’s verdict is kept))

4π is the total solid angle of a sphere and appears in any integral over a sphere. Its appearance is an identity, not a mechanism. The substance of Section 2 lies on the side of the exponent $n = d - 1$; the second edition reached this verdict, and the third edition keeps it.

3. (B) The Fringe Spacing (Unchanged from the Second Edition)

Theorem 3.1 (The interference term)

With slit separation a (at $y = \pm a/2$), screen at $z = D$, and $R \equiv \sqrt{D^2 + x^2 + y^2}$, for $R \gg a$

$$I(x, y) = |\Psi_{\text{total}}|^2 \simeq \frac{2|A|^2}{R^2} \left[1 + \cos \left(\frac{kay}{R} \right) \right],$$

and in the small-angle limit $R \simeq D$ the phase kay/D gives the fringe spacing $\Delta y = 2\pi D/(ka) = \lambda D/a$.

Proof.

Write the spherical wave from each slit as $\psi_{1,2} = Ae^{ikr_{1,2}}/r_{1,2}$ with $r_{1,2} = \sqrt{D^2 + x^2 + (y \mp a/2)^2}$. For $R \gg a$, $r_{1,2} \simeq R \mp ay/(2R)$, hence $r_1 - r_2 \simeq -ay/R$. Approximating both amplitude denominators by R ,

$$|\psi_1 + \psi_2|^2 = \frac{|A|^2}{R^2} |e^{ikr_1} + e^{ikr_2}|^2 = \frac{2|A|^2}{R^2} [1 + \cos(k(r_1 - r_2))],$$

and since $\cos$ is even the stated form follows.

Numerical verification (carried over from the second edition): with $a = 10^{-4}$ m, $D = 1$ m and $\lambda = 5 \times 10^{-7}$ m, comparison against the exact superposition without approximation agreed to a **relative error below 10^{-5}** at $y = 0, 0.002, 0.005, 0.01$. The maxima were spaced 4.99999×10^{-3} m apart (theoretical value 5×10^{-3} m).

This agreement confirms the paper's soundness; it is not a new prediction.

4. The Core of the Third Edition: (A) and (B) Are Distinct Roots

Does (B) use the exponent that (A) forces?

The test is simple: **deliberately violate (A)'s conclusion and see whether (B) moves**. Writing the amplitude as $\psi = Ae^{ikr}/r^m$, the maxima were located numerically for various m (no approximation; a dense scan over 4×10^6 points with parabolic vertex refinement).

Maximum	$m = 1$ ((A) forces this)	$m = 3$ (a different value)	Difference
1st	$5.00004983966 \times 10^{-3}$	$5.00002450778 \times 10^{-3}$	2.53×10^{-8}
2nd	$1.0000474715 \times 10^{-2}$	$1.00004240467 \times 10^{-2}$	5.07×10^{-8}
3rd	$1.50016497876 \times 10^{-2}$	$1.50015737497 \times 10^{-2}$	7.60×10^{-8}

The difference is five parts per million of one fringe (5×10^{-3} m). Sweeping m over 0, 0.5, 1, 1.5, 2, 3 changed the mean fringe spacing by a relative 7.6×10^{-6} .

Observation 4.1 (The fringes do not remember the exponent)

Changing the exponent that (A) forces, from $|\psi|^2 \propto r^{-2}$ all the way to r^{-6} , does not move the fringes. The formula says why: **what fixes the fringes is the phase difference $k(r_1 - r_2)$ alone**, into which the falloff of the amplitude does not enter. The exponent acts on **the envelope $2|A|^2/R^{2m}$** , that is, on **how the brightness of the fringes decays outward**, and nowhere else.

The residual 10^{-8} is not of phase origin either: it is **the slight imbalance of the two amplitudes** arising from $r_1 \neq r_2$, and it grows with m precisely because the imbalance does.

**Hence (A) and (B) are not two appearances of one mechanism.
One fixes how the brightness falls; the other fixes the spacing of the fringes.**

Observation 4.2 (4π acts on neither)

The intensity of Theorem 3.1 contains **no occurrence of 4π** . Setting the amplitude A to 1, to $1/\sqrt{4\pi} = 0.2820947918$ and to 17, the maxima were found at 1.000045×10^{-2} m in all three cases.

4π enters only the normalisation of the flux and cancels out of the shape of the pattern. This is an instance of the general check Paper 11’s third edition wrote down: **a quantity that enters a normalised probability only as an overall factor is, by definition, invisible to observation.**

The “ **4π wavefield**” of the second edition’s title therefore contributes nothing to the description of the interference.

5. On the Description of Measurement (the Appeal in Section 4 Is Removed)

The second edition’s Section 4 read the placement of a detector as a projection from the three-dimensional field onto a two-dimensional surface, wrote it $\mathcal{P}_{2D}$, and added: **“based on the Lazy Evaluation protocol of Paper 11”**.

Established Fact 5.1 (Paper 11, 3rd ed.: the metaphor fails at its engine)

Lazy evaluation in computer science is a structure in which **the value is already determined and only the computation is deferred** (a thunk). **That core is precisely what the Kochen–Specker theorem forbids:** a non-contextual assignment of values is impossible, independently of the state. Paper 11’s third edition computed the Peres–Mermin magic square itself and confirmed that **the product of the rows and the product of the columns come out +1 and −1. The metaphor crosses over as a picture; the machinery does not.**

This edition therefore removes the appeal to Paper 11. Lazy evaluation does not support Section 4 of this paper.

What **survives** and what does not:

Survives	the vocabulary of a drop in dimension (3D field, 2D observing surface)
Does not	the grounding in lazy evaluation (failed in Paper 11, 3rd ed.)

Observation 5.1 (the second edition’s Note 4.2’s limitation was right)

The second edition had already written that “ $\mathcal{P}_{2D}$ is not a projection operator in the mathematical sense”: it fails idempotence $\mathcal{P}^2 = \mathcal{P}$, and a δ function is not even an element of the state space, so the expression is **neither a theorem nor a derivation but a notational convenience**.

That limitation was correct and is kept unchanged. What the third edition removed is only **the grounding that stood outside it**: the clause “based on the protocol of Paper 11”.

This paper does not solve the measurement problem. What happens upon measurement remains open. What this paper supplies is **a vocabulary for describing it**, and no more.

6. The Born Exponent 2 Has Three Addresses

The second edition’s the second edition’s Note 4.1 deferred the probability distribution to Paper 53, writing that “**the usable exponent is fixed at 2 by unitarity**”. That is correct, but it is the weakest of three.

Address	Ground	Strength
Gleason’s theorem (Paper 71)	in dimension ≥ 3 any measure is $\text{tr}(\rho P)$	a theorem, not a postulate
Unitarity (Paper 53)	isometries of ℓ^p for $p \neq 2$ are phases and permutations	weaker: assumes a power of $ \psi $
The 2 of the Hopf map (Paper 68)	the same 2 appears	same root or rhyme: undecided

Paper 53 itself states that it is **weaker than Gleason’s theorem** (its Note 2.3). **This paper was pointing only there. The third edition writes all three.**

7. The Scope, Restated

Item	What it is
Flux conservation forces $n = d - 1$	Fact (Section 2, Theorem 2.1)
That $n = 2$ means three dimensions	Not shown (holds in every dimension; Paper 114)
Reproduction of $\Delta y = \lambda D/a$	Fact (Theorem 3.1, numerically verified)
That (A) and (B) are one mechanism	Not shown (separated by perturbation; Section 5)
4π	Identity (always present in a spherical integral; absent from the pattern)
$\mathcal{P}_{2D}$	Notation (not a projection operator; the second edition's Note 4.2)
A solution of the measurement problem	Not shown
Any change to quantum predictions	None (known predictions are reproduced)

Of the eight rows, two are facts proper to this paper: the flux condition of Section 2 and the fringe spacing of Theorem 3.1. **And what the third edition adds is the single line that those two are independent of each other.**

8. Conclusion

This paper began from a discomfort felt thirty-three years ago and arrived at two correct pieces of algebra. **That algebra stands unchanged.**

What changed is **the line believed to run between them**. Flux conservation fixes **how the brightness falls**; the phase difference fixes **the spacing of the fringes**. **Tripling the exponent moved the fringes by five parts per million of a single fringe.**

The fringes do not remember the exponent.

What is not claimed (restraint): no claim to have solved the measurement problem ($\mathcal{P}_{2D}$ is notation; the second edition's Note 4.2) / **lazy evaluation is not used as a mechanism** (it fails in Paper 11, 3rd ed.) / $n = d - 1$ is not read as “the reason space is three-dimensional” (it holds in every dimension; Papers 114, 116) / 4π is not treated as a **mechanism** (an identity of spherical integration, absent from the pattern; the overall-factor check of Paper 11, 3rd ed.) / **(A)**

and (B) are not merged into one mechanism (perturbation separates them) / **wave–particle duality is not denied** (the terminology functions well as a summary of experimental fact) / **no prediction of quantum mechanics is altered** / **no claim to have derived the Born rule** (there are three addresses, and the strongest is Gleason’s theorem) / no new mathematics and no prediction (Layer 2).

Acknowledgements

The origin of this work is a discomfort the author felt in high school at the standard account of the double-slit experiment. That question, held for thirty-three years, led to the ideas here. The author thanks the writers of the textbooks and popular expositions that gave him access to a foundational problem in physics without formal training.

Revision Record

Third edition (August 2026): the algebra is unchanged. The flux condition of Section 2 and Theorem 3.1 both stand. **What is added is a separation and an update of internal citations.** (i) The paper’s two results were shown to be **distinct roots** by perturbation: relaxing the amplitude to r^{-m} and comparing $m = 1$ with $m = 3$, the maxima moved by 2.53×10^{-8} m (five parts per million of one fringe), and sweeping m from 0 to 3 changed the mean spacing by a relative 7.6×10^{-6} . **The fringes are fixed by the phase difference alone; the exponent acts only on the envelope.** (ii) $n = 2$ was generalised to $n = d - 1$ (checked numerically for $d = 2, 3, 4, 5$), making explicit that **the condition does not select a dimension** (Papers 114, 116); the Flatland analogy in the introduction was correspondingly limited so as **not to be read as explaining the number of dimensions.** (iii) 4π was confirmed to act nowhere in the description of the interference (setting A to 1, $1/\sqrt{4\pi}$ and 17 leaves the maxima fixed), confirming the second edition’s verdict that **4π is an identity and not a mechanism, now on the side of the pattern as well.** (iv) Section 4’s clause “**based on the Lazy Evaluation protocol of Paper 11**” was removed, because Paper 11’s third edition established, via the Peres–Mermin magic square, that the core of lazy evaluation (a value already determined with only the computation deferred) is **what the Kochen–Specker theorem forbids. Only the vocabulary of a drop in dimension was retained**, and the second edition’s Note 4.2’s limitation ($\mathcal{P}_{2D}$ is notation, not a projection operator) **was correct and is kept.** (v) For the Born exponent 2 in the second edition’s Note 4.1, **three addresses** are now written (Gleason’s theorem, Paper 71; unitarity, Paper 53; the Hopf map, Paper 68); the second edition pointed only at **the weakest one. The item retracted in the second edition (the first edition’s $\int_{S^2} |\Psi|^2 d\Omega = 4\pi|A|^2$, a confusion of integration elements) is not reinstated.**

The dashes used as separators in the text were replaced by punctuation (October 2026). No claim, number or reference was changed.

References

1. Abbott, E. A. (1884). *Flatland: A Romance of Many Dimensions*. Seeley & Co.
2. Born, M. (1926). *Zur Quantenmechanik der Stoßvorgänge*. Zeitschrift für Physik 37, 863–867.
3. Feynman, R. P., Leighton, R. B. & Sands, M. (1965). *The Feynman Lectures on Physics*, Vol. III, Ch. 1. Addison-Wesley.
4. Gleason, A. M. (1957). *Measures on the Closed Subspaces of a Hilbert Space*. Journal of Mathematics and Mechanics 6, 885–893.
5. Kochen, S. & Specker, E. P. (1967). *The Problem of Hidden Variables in Quantum Mechanics*. Journal of Mathematics and Mechanics 17, 59–87.
6. Mermin, N. D. (1990). *Simple Unified Form for the Major No-Hidden-Variables Theorems*. Physical Review Letters 65, 3373–3376.
7. Yamagishi, Y. (2026). *Factors That Cancel in Normalisation, and the Exponent $\mathbf{2}$ of Born's Rule* [Paper 11], 3rd ed. Zenodo. 10.5281/zenodo.21404636 (the failure of the lazy-evaluation metaphor on the machinery-transfer test; the overall-factor check)
8. Yamagishi, Y. (2026). *The Uniqueness Theorem of $\mathbf{n} = \mathbf{2}$ for Spherically Symmetric Density Distributions* [Paper 01]. Zenodo. 10.5281/zenodo.21403186
9. Yamagishi, Y. (2026). *Only Dimensionless Numbers Can Be Fine-Tuned* [Paper 10], 3rd ed. Zenodo. 10.5281/zenodo.21404556
10. Yamagishi, Y. (2026). *The Yamagishi Born-Rule Geometric Structure Theory* [Paper 53]. Zenodo. 10.5281/zenodo.21603193 (uniqueness via unitarity; itself stated to be weaker than Gleason)
11. Yamagishi, Y. (2026). *Born's Exponent and Gleason's Theorem* [Paper 71]. Zenodo. 10.5281/zenodo.21650577 / *The Qubit as a Hopf Fibration* [Paper 68]. Zenodo. 10.5281/zenodo.21642238
12. Yamagishi, Y. (2026). *Which Objects Know They Are in Four Dimensions?* [Paper 114]. Zenodo. 10.5281/zenodo.21963544 ($\mathbf{n} = \mathbf{d} - \mathbf{1}$ stands on the side that does not select)
13. Yamagishi, Y. (2026). *What Does the Exponent of $\mathbf{c}$ Count?* [Paper 116]. Zenodo. 10.5281/zenodo.22003794 ($\mathbf{d}$ enters only through the denominator $\mathbf{n} = \mathbf{d} - \mathbf{1}$)

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