

The Magnetic Field Does No Work, and Still It Confines

— The dot product of the Lorentz force with the velocity is exactly **0** — yet at $L = 4$ the dipole field produces a mirror ratio of **115.3776** and traps **99.5657** per cent of the solid angle — what confines is not work but solid angle — [Paper 410]

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— *From thinking to syncing.* —

Abstract

A magnetic field does no work on a charged particle. Yet the Van Allen belts hold particles for years. This paper shows that the two statements do not conflict—what confines is not work but the space of velocity directions. No new theorem or law is claimed. First, the dot product of the Lorentz force with the velocity is exactly **0**, and numerically **0.0000000000000000** comes out (Section 2). Second, in a dipole field the mirror ratio at $L = 4$ is **115.3776**, and this is not assumed but follows from the geometry (Section 3). Third, this is the core of the paper. At $L = 4$, **99.5657** per cent of the solid angle is trapped. Not one unit of energy has changed (Section 3). Fourth, the loss-cone half-angle shrinks from **16.3301** degrees at $L = 2$ to **2.8514** degrees at $L = 6$ (Section 3). Fifth, a particle with a 30-degree pitch angle mirrors where the field is **4.000000** times stronger, and the inverse computation returns the same value (Section 4). Sixth, the separator is whether the energy changes or the direction changes (Section 5).

Scope of this paper (scope note): No new theorem or law is claimed—that the Lorentz force does no work, the adiabatic invariance of the magnetic moment, and the loss-cone formula are all standard. The relation to Paper 310 is stated first—the loss-cone formula itself already appears as Check 7.5 of Paper 310 (mirror ratios 2, 5, 10 giving 45.0000, 26.5651, 18.4349 degrees). What this paper adds is two things: deriving the mirror ratio from dipole geometry instead of assuming it, and measuring confinement as a fraction of solid angle rather than as an angle. The figure 99.5657 per cent does not appear in Paper 310. Adiabatic invariance is not proved—that μ is conserved is used as known. How slowly it must change is the subject of Paper 310. No forecast of the real radiation belts is made—the solar wind, magnetic storms, pitch-angle scattering and wave-particle interaction are not the subject. Only an idealised dipole is treated. No particle lifetime is given—how many years a particle stays is set by scattering, which is absent from these computations. The origin of the field is not discussed—the dynamo is not the subject. Relativistic corrections are not included—corrections for high-energy particles are not the subject, and the qualitative conclusion is unchanged. Relation to earlier papers: Paper 310 showed that conserved has two distinct roots—here confinement occurs although the work is zero. Paper 300 showed that common root and distinct root can be decided—“confining by energy” and “confining by direction” are distinct roots by that criterion. Paper 404 showed that the horizon is not the limit of what can be seen—here zero work is not the limit of what can confine. What is added is checking numerically that the dot product vanishes, obtaining the mirror ratio from geometry at four L shells, measuring confinement as a fraction of solid angle, placing the inverse computation from pitch angle as a control, and placing the separator at “energy or direction”.

1. Notation and the Question

Setting: The Lorentz force on a charged particle is $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$. The magnetic moment $\mu = mv_{\perp}^2/2B$ is conserved as long as the field varies slowly.

$$\sin^2 \alpha_c = \frac{B_{\min}}{B_{\max}} = \frac{1}{R}$$

Here R is the mirror ratio and α_c the loss-cone half-angle.

Observation 1.1

A magnetic field does no work. The force is perpendicular to the velocity—so the speed of the particle never changes.

Observation 1.2

And yet particles are confined. The Van Allen belts have been there since they were found in 1958—how, if the speed never changes.

Question: How does something that does no work confine anything.

2. The Dot Product Is Exactly Zero

Computation 2.1: Take an arbitrary velocity and field and evaluate $\mathbf{v} \cdot (\mathbf{v} \times \mathbf{B})$ numerically.

The result is 0.0000000000000000. This is not an approximation—by the definition of the cross product, $\mathbf{v} \times \mathbf{B}$ is orthogonal to $\mathbf{v}$.

Observation 2.2

Hence the kinetic energy is exactly constant. $dE/dt = \mathbf{F} \cdot \mathbf{v} = 0$ —with no electric field, the speed of the particle never changes.

Observation 2.3

This is where ordinary intuition stops. One thinks confinement needs a well—a well is potential energy, and potential energy does work.

Observation 2.4

But confinement does not require lowering the energy. It requires blocking the way out—and the ways out are counted not by position but by velocity direction.

3. The Geometry of the Dipole Sets the Mirror Ratio

Computation 3.1: In a dipole field, following a field line from the equatorial plane the field strength increases. For each L shell, take the ratio of the equatorial value to the value at the foot of the line.

L shell	Mirror ratio R	Loss-cone half-angle	Solid angle lost	Trapped
2.0	12.6491	16.3301 deg	4.0342 %	95.9658 %
3.0	46.7654	8.4085 deg	1.0749 %	98.9251 %
4.0	115.3776	5.3418 deg	0.4343 %	99.5657 %
6.0	404.0990	2.8514 deg	0.1238 %	99.8762 %

Observation 3.2

The mirror ratio is not assumed. It comes out of the dipole expression—Paper 310 set $R = 2, 5, 10$ and produced the angles. Here R is itself a consequence of the geometry.

Observation 3.3

Confinement is measured in solid angle, not in angle. The loss cone is two cones, up and down, so the fraction lost is $1 - \cos \alpha_c$ —at $L = 4$ that is **0.4343** per cent, leaving **99.5657** per cent.

Observation 3.4

The angle 5.3418 degrees looks small, and in solid angle it is smaller still. As a fraction of angle it is $5.3418/90 = 5.9$ per cent, but as a fraction of solid angle it is **0.4343** per cent—more than an order of magnitude apart, because the solid angle of a cone goes as the square of the angle.

Observation 3.5

The farther out, the tighter the confinement. From 16.3301 degrees at $L = 2$ down to 2.8514 degrees at $L = 6$ —because farther out the equatorial field is weaker, so the ratio to the foot is larger.

4. The Inverse Computation

Computation 4.1: At what field strength does a particle with a 30-degree pitch angle mirror?

$$\frac{B}{B_{\text{eq}}} = \frac{1}{\sin^2 \alpha} = \frac{1}{\sin^2 30^\circ} = 4$$

Numerically **4.000000** comes out.

Observation 4.2

This is only Section 3 run backwards. The same relation takes an angle to a ratio and a ratio to an angle—that both directions give the same number is evidence that the relation is implemented correctly.

Observation 4.3

At the mirror point the speed is still unchanged. What changed is the split between $v_{\parallel}$ and $v_{\perp}$ —since μ is constant, a larger B means a larger $v_{\perp}$, and since the speed is constant $v_{\parallel}$ falls. The point where it reaches zero is the mirror point.

5. The Separator

The separator: does the energy change, or does the direction change.

Observation 5.1

A potential well confines by energy. A particle with less energy than the wall cannot leave—whether it can leave is decided by its speed.

Observation 5.2

A magnetic mirror confines by direction. However large the speed, a particle outside the loss cone cannot leave—whether it can leave is decided by its direction.

Observation 5.3

These are distinct roots by the criterion of Paper 300. Both are called confinement, but the quantity used to decide differs—speed in one case, pitch angle in the other.

Observation 5.4

So “it does no work yet it confines” is not a contradiction. It looks like one only if one believes confinement must happen through energy—drop that belief and what remains is geometry.

Observation 5.5

To be honest—in the real radiation belts particles do not stay forever. Pitch-angle scattering feeds particles into the loss cone—scattering is absent from these computations. Include it and the lifetime becomes finite. The mechanism of confinement itself is unchanged.

Observation 5.6

One last thing—“does no work” is not “does nothing”. Work measures only the flow of energy—changing a direction does not register on that instrument.

6. Roll Call and Closing

1. $\mathbf{v} \cdot \mathbf{F} = 0.0000000000000000$ (Computation 2.1): **exactly zero.**
2. Mirror ratio **12.6491** at $L = 2$ (Computation 3.1): **from geometry.**
3. Mirror ratio **115.3776** at $L = 4$ (Computation 3.1): **not assumed.**
4. Mirror ratio **404.0990** at $L = 6$ (Computation 3.1): **larger farther out.**
5. Loss cone **16.3301** $\rightarrow$ **2.8514** degrees (Computation 3.1): **tighter farther out.**
6. **99.5657 %** trapped at $L = 4$ (Computation 3.1): **in solid angle.**
7. **5.9 %** in angle, **0.4343 %** in solid angle (Observation 3.4): **an order apart.**
8. Even $L = 2$ holds **95.9658 %** (Computation 3.1): **the loosest shell.**
9. A **30-degree** pitch angle mirrors at $B/B_{\text{eq}} = 4.000000$ (Computation 4.1): **the inverse agrees.**
10. The separator is speed or direction (Observation 5.3): **distinct roots by Paper 300.**

Closing: A magnetic field does no work on a charged particle—the Lorentz force is perpendicular to the velocity, so the dot product is exactly **0.0000000000000000**. This is not an approximation but a consequence of the definition of the cross product, so with no electric field the speed of the particle never changes. And yet particles are confined—in a dipole field the mirror ratio at the $L = 4$ shell is **115.3776**, and this is not assumed but follows from the

geometry. Check 7.5 of Paper 310 set mirror ratios of 2, 5, 10 and produced 45.0000, 26.5651, 18.4349 degrees. This paper derives R itself from the dipole and measures confinement in solid angle rather than in angle. At $L = 4$ the loss-cone half-angle is **5.3418** degrees, the solid angle lost is **0.4343** per cent, and what remains is **99.5657** per cent. What looks like 5.9 per cent as a fraction of angle becomes **0.4343** per cent as a fraction of solid angle—more than an order of magnitude apart, because the solid angle of a cone goes as the square of the angle. Even the loosest shell, $L = 2$, retains **95.9658** per cent, and $L = 6$ retains **99.8762** per cent. And through all of this not one unit of energy has changed—a particle with a 30-degree pitch angle mirrors at $B/B_{\text{eq}} = 4.000000$, and at the instant it mirrors its speed is the same as before. What changed is only the split between $v_{\parallel}$ and $v_{\perp}$: since μ is constant a larger B means a larger $v_{\perp}$, and since the speed is constant $v_{\parallel}$ falls, and the point where it reaches zero is the mirror point. One thing separates them—confining by energy or confining by direction. In a potential well whether a particle can leave is decided by its speed; in a magnetic mirror it is decided by its pitch angle. However large the speed, a particle outside the loss cone cannot leave. They are distinct roots by the criterion of Paper 300, and the very quantity used to decide is different. To be honest—in the real radiation belts particles do not stay forever, because pitch-angle scattering feeds them into the loss cone, and scattering is absent from these computations. Include it and the lifetime becomes finite. The mechanism of confinement itself is unchanged. One last thing—“does no work” is not “does nothing”. Work measures only the flow of energy, and changing a direction does not register on that instrument.

References

1. Van Allen, J. A., & Frank, L. A. (1959). Radiation Around the Earth to a Radial Distance of 107,400 km. *Nature*.
 2. Alfvén, H. (1950). *Cosmical Electrodynamics*. Oxford: Clarendon Press.
 3. Northrop, T. G. (1963). *The Adiabatic Motion of Charged Particles*. New York: Interscience.
 4. Walt, M. (1994). *Introduction to Geomagnetically Trapped Radiation*. Cambridge: Cambridge University Press.
 5. Yamagishi, Y. (2026). *The Three Hundredth Census: Whether Two Things Share a Root Is Decidable* [Paper 300]. Zenodo. 10.5281/zenodo.22171914
 6. Yamagishi, Y. (2026). “Conserved” Has Two Distinct Roots, and Noether Explains Only One [Paper 310]. Zenodo. 10.5281/zenodo.22172372
 7. Yamagishi, Y. (2026). *The Horizon Is Not the Limit of What Can Be Seen* [Paper 404]. Zenodo. 10.5281/zenodo.22699418
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