

The Product Is the Same and the Result Is Not

— Divide the illuminance by **100** and multiply the time by **100**: the product ***It*** stays at **1.00** while the effective exposure falls to **0.524807** — multiplication holds only inside the range where it was assumed to hold —

[Paper 416]

Yuuki YAMAGISHI (山岸 勇輝)

Yamagishi Complexity Torus Lab, Japan

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— *From thinking to syncing.* —

Abstract

Exposure is taught as “illuminance times time”. This paper shows that the multiplication fails—holding the product fixed, the result moves from **0.524807** to **1.905461**. No new theorem or law is claimed. First, the product ***It*** is exactly **1.00** in every pairing (Section 2). Second, this is the core of the paper. The effective exposure still moves: dividing the illuminance by **100** and multiplying the time by **100** gives **0.524807** (Section 2). Third, in photographic stops that is a loss of **0.9301** (Section 2). Fourth, pushed the other way, to short times and high illuminance, it rises to **1.905461**. The sign changes across the boundary (Section 3). Fifth, setting the exponent to $p = 1$ makes the effective exposure agree with the product exactly. The failure comes only from the exponent (Section 3). Sixth, the separator is whether it can be written as a product or carries an exponent. Writing ***It*** is itself a hypothesis (Section 4).

Scope of this paper (scope note): No new theorem or law is claimed—the reciprocity law, the Schwarzschild exponent, and the existence of reciprocity failure are all standard (since 1899). The mechanism is not explained—latent-image formation and recombination are not the subject; the exponent p is taken as given. No value of p is asserted— $p = 0.86$ is a representative value for exposition and differs between emulsions. No claim is made about any particular material. Digital sensors are not treated—whether reciprocity holds in semiconductor detectors is not the subject. Astrophotographic practice is not treated—correction tables for long exposures are not the subject. High-illuminance failure is not separated—it is known that the mechanisms differ at short and long times; a single exponent is used throughout. Relation to earlier papers: Paper 415 showed that the same glass became 5000 times more transparent—there a name did not fix a value; here a formula does not. Paper 300 showed that common root and distinct root can be decided—“settled by the product” and “settled by an exponent” are distinct roots by that criterion. Paper 414 showed that the same hour varies by 1.5 with the season—there too one name did not point at one quantity. Paper 413 showed that adding a peg destroys the closed form—here the moment the exponent leaves 1, the closed form of a product stops working. What is added is showing the product constant across six pairings, giving the effective exposure that nonetheless moves, converting the loss into photographic stops, showing the sign reversal at short times, placing the $p = 1$ control, and placing the separator at “product or exponent”.

1. Notation and the Question

Setting: The reciprocity law says that equal products of illuminance I and time t give equal results.

$$H = I t$$

Where reciprocity fails, the effective exposure is written (Schwarzschild) as

$$H_{\text{eff}} = I t^p, \quad p < 1$$

Below, $p = 0.86$ is used as a representative value.

Observation 1.1

$H = I t$ has the shape of a multiplication. So “stop down one and double the time” works—the basics of photography stand on it.

Observation 1.2

But it is a hypothesis with the name of a law. It is called the reciprocity law—and once something has a name, holding starts to look like a premise.

Question: Hold the product fixed and push I and t as far as they go. What happens.

2. The Product Holds and the Result Moves

Computation 2.1: Holding $It = 1$, take six pairings of I and t .

Illuminance I	Time t	Product It	Effective It^p
1.000	1.0	1.00	1.000000
0.100	10.0	1.00	0.724436
0.010	100.0	1.00	0.524807
0.001	1000.0	1.00	0.380189
10.000	0.1	1.00	1.380384
100.000	0.01	1.00	1.905461

Observation 2.2

The product column is fixed at exactly 1.00. That column is what the reciprocity law asserts—and looking only at it, the six pairings are indistinguishable.

Observation 2.3

The right-hand column moves. From 0.380189 to 1.905461, a range of **5.01**—a factor of five at one and the same “exposure”.

Computation 2.4: Convert the case of one hundredth the illuminance and a hundred times the time into photographic stops.

A factor of 0.524807 is a loss of **0.9301** stops. Almost a full stop—the amount made up by opening one stop or doubling the time.

Observation 2.5

At a thousand times the time it is **1.3952** stops. The loss is proportional to the logarithm of the time—taking $\log_2$ gives $(1 - p) \log_2 t$.

3. The Other Side of the Boundary, and a Control

Observation 3.1

Pushed to short times and high illuminance, the effective exposure rises. A hundred times the illuminance and a hundredth of the time gives **1.905461**—a gain, not a loss.

Observation 3.2

So $t = 1$ is the boundary. For $t > 1$, $t^p < t$ and one loses; for $t < 1$, $t^p > t$ and one gains—the sign changes across it.

Computation 3.3: As a control, set $p = 1$.

All six pairings then give the effective exposure equal to the product, exactly 1.000000.

Observation 3.4

The control does work. It shows the failure comes from $p \neq 1$ alone—had anything else been mixed in, $p = 1$ would not restore agreement.

Observation 3.5

And p is close to 1. It is 0.86—a departure of only 14 per cent, yet three decades of time turn it into 1.4 stops. A shift in an exponent acts as a power.

4. The Separator

The separator: can it be written as a product, or does it carry an exponent.

Observation 4.1

Writing It says that I and t are on equal terms. Doubling either does the same—that is a claim, not a definition.

Observation 4.2

Writing It^p says they are not. Time counts for less—whether a given amount is bought with I or with t changes the result.

Observation 4.3

These are distinct roots by the criterion of Paper 300. “Settled by the product” claims that two quantities are interchangeable—“settled by an exponent” claims that the exchange has a price.

Observation 4.4

And the name helps the confusion. Calling it the reciprocity law says it is a law—the reverse of Paper 326, where a “law” turned out to be an identity: here a thing with the name of a law is an empirical rule.

Observation 4.5

The failure was found in practice first. In astrophotography long exposures do not deliver what the formula promises—the formula did not come first with the phenomenon following; it was the other way round.

Observation 4.6

To be honest— $p = 0.86$ is a representative value for exposition and differs between emulsions. No claim is made about any particular material—what does not move is that unless $p = 1$, no product will do.

Observation 4.7

One last thing—that It fails and that It is useless are different. In ordinary photography the range of t is narrow and the product suffices—the failure becomes visible when t is swung over three decades.

5. Roll Call and Closing

1. The product is exactly **1.00** in all six (Computation 2.1): **what reciprocity asserts.**
2. The effective exposure spans **0.380189--1.905461** (Computation 2.1): **a range of 5.01.**
3. **0.524807** at one hundredth the illuminance (Computation 2.1): **same product.**
4. That is a loss of **0.9301** stops (Computation 2.4): **almost a full stop.**
5. **1.3952** stops at a thousand times the time (Observation 2.5): **logarithmic.**
6. **1.905461** on the short side (Observation 3.1): **the sign reverses.**
7. The boundary is $t = 1$ (Observation 3.2): **opposite directions across it.**
8. $p = 1$ restores **1.000000** in all six (Computation 3.3): **the control works.**
9. $p = 0.86$ departs by 14 % (Observation 3.5): **a power amplifies it.**
10. The separator is product or exponent (Observation 4.3): **interchangeable or not.**

Closing: Exposure is taught as “illuminance times time”, which is why “stop down one and double the time” works—but holding the product $It = 1$ across six pairings, the product column stays fixed at exactly **1.00** while the effective exposure runs from **0.380189** to **1.905461**, a range of **5.01**. One hundredth the illuminance with a hundred times the time gives **0.524807**, a loss of **0.9301** stops—almost a full stop short. At a thousand times the time it is **1.3952** stops, the loss being proportional to the logarithm of the time. And on the other side of the boundary the sign changes—a hundred times the illuminance with a hundredth of the time gives **1.905461**, a gain rather than a loss. The boundary is $t = 1$: for $t > 1$, $t^p < t$; for $t < 1$, $t^p > t$. That the cause is the exponent alone is shown by the control: at $p = 1$ all six pairings return to exactly **1.000000**, which they would not do had anything else been mixed in. And p is close to 1—it is 0.86, a departure of only 14 per cent, and yet three decades of time turn it into 1.4 stops, because a shift in an exponent acts as a power. One thing separates them—can it be written as a product, or does it carry an exponent. Writing It says that I and t are on equal terms, which is a claim and not a definition; writing It^p says they are not, and whether a given amount is bought with illuminance or with time changes the result. They are distinct roots by the criterion of Paper 300: one claims the quantities are interchangeable, the other that the exchange has a price. And the name helps the confusion—the reverse of Paper 326, where a “law” turned out to be an identity, since here a thing with the name of a law is an empirical rule. The failure was found in practice first: in astrophotography long exposures do not deliver what the formula promises. To

be honest— $p = 0.86$ is a representative value for exposition and differs between emulsions; no claim is made about any particular material. What does not move is that unless $p = 1$, no product will do. One last thing—that *It* fails and that *It* is useless are different. In ordinary photography the range of t is narrow and the product suffices; the failure becomes visible when t is swung over three decades.

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