

The Ceiling on Food-Chain Length Is Not a Fact About Animals

— Each step keeps a tenth, so the fifth step is at 10^{-6} of the sunlight — raising the requirement from **1** to **10** alone drops the ceiling from **6** steps to **5** — what sets it is the multiplication — [Paper 417]

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September 2026

— *From thinking to syncing.* —

Abstract

Food chains usually stop at four or five steps. This paper shows that what stops them is arithmetic rather than ecology—each step costs an order of magnitude, so a step that cannot be met always arrives. No new theorem or law is claimed. First, primary production is **1** per cent of the sunlight, and each step keeps exactly one tenth (Section 2). Second, this is the core of the paper. The fifth step is at 10^{-6} of the sunlight, one part in a million (Section 2). Third, that each step costs one order is confirmed by the inverse computation as exactly **5.000000** decades (Section 2). Fourth, with a requirement of **1** the ceiling is step **6**, and step **7** falls to 0.1 and does not reach it (Section 3). Fifth, raising the requirement to **10** alone drops the ceiling to step **5**. Nothing about the organisms changed (Section 3). Sixth, the separator is efficiency against requirement. At 20 per cent transfer, step **5** becomes **32** times larger (Section 4).

Scope of this paper (scope note): No new theorem or law is claimed—Lindeman’s ten per cent rule, the ecological pyramid, and the limit on trophic levels are all standard (since 1942). No side is taken in the ecological debate—whether chain length is set by energy or by dynamical stability is not entered into. Only the arithmetic on the energy side is examined. Ten per cent is not asserted as a measurement—transfer efficiency varies widely between ecosystems, and ten per cent is a textbook representative value. Real food webs are not treated—omnivory, detrital chains and parasitism are not the subject; a single linear chain is used. No units are carried—“a requirement of 1” is in arbitrary units and is not a biomass. Conservation is not discussed—the fragility of top predators is only touched. Relation to earlier papers: Paper 416 showed that an equal product need not give an equal result—there the multiplication failed; here it holds all the way down. Paper 300 showed that common root and distinct root can be decided—“short because the efficiency is low” and “short because the requirement is high” are distinct roots by that criterion. Paper 415 showed that the same glass became 5000 times more transparent—there attenuation set the distance; here transfer efficiency sets the number of steps. Paper 413 showed that adding a peg destroys the closed form—here adding a step destroys an order of magnitude. What is added is tabulating seven steps, confirming the decade per step by the inverse computation, showing the ceiling from both sides of the boundary, showing that moving the requirement moves the ceiling, giving the factor of 32 when the efficiency is changed, and placing the separator at “efficiency or requirement”.

1. Notation and the Question

Setting: Let E_0 be the incident solar energy, ε_p the efficiency of primary production, and ε_t the transfer efficiency per step.

$$E_k = E_0 \varepsilon_p \varepsilon_t^{k-1}$$

Below, $E_0 = 10^7$ (arbitrary units), $\varepsilon_p = 0.01$ and $\varepsilon_t = 0.10$.

Observation 1.1

Food chains usually stop at four or five steps. Grass, grazer, carnivore, larger carnivore—and then no further.

Observation 1.2

One sometimes hears “because large predators are rare”. But that is the consequence, not the reason—they are rare because there is no energy to support them.

Question: What decides where it stops.

2. One Order of Magnitude per Step

Computation 2.1: Give the energy through seven steps.

Step	Energy	Fraction of sunlight
1	100,000.0000	1.0000×10^{-2}
2	10,000.0000	1.0000×10^{-3}
3	1,000.0000	1.0000×10^{-4}
4	100.0000	1.0000×10^{-5}
5	10.0000	1.0000×10^{-6}
6	1.0000	1.0000×10^{-7}
7	0.1000	1.0000×10^{-8}

Observation 2.2

Primary production is exactly 1 per cent of the sunlight.0.010000—that is all photosynthesis extracts.

Observation 2.3

The fifth step is at 10^{-6} .One part in a million of the sunlight—after only five multiplications.

Computation 2.4: Take the common logarithm of the ratio between steps 1 and 6.

It is **5.000000** decades.Five steps, five orders—that the inverse computation returns the same number is a self-test.

Observation 2.5

The ratio between steps is perfectly constant.Any adjacent pair gives 0.1—it is a geometric sequence, with no exceptions.

3. The Ceiling Is Set by the Requirement

Computation 3.1: Taking 1 as the amount needed to support an individual, how far does it reach?

The top is step **6** (value 1.0000). Step 7 is 0.1000 and does not reach the requirement of 1.0.

Observation 3.2

Both sides of the boundary are shown. Step 6 reaches it and step 7 does not—looking at one side only, one cannot tell that it is the boundary.

Computation 3.3: Raise the requirement to 10.

The top drops to step **5**.

Observation 3.4

Nothing about the organisms changed. The efficiency, the sunlight and the structure of the chain are the same—only “what one individual needs” changed, and a step disappeared.

Observation 3.5

So “the ceiling on chain length” is not a property of the chain. It is set by the chain together with the requirement—the same shape as “nearest” demanding a distance in Paper 412.

4. The Separator

The separator: is it set by the efficiency or by the requirement.

Computation 4.1: Raise the transfer efficiency from 10 to 20 per cent.

The energy at step 5 rises from 1.0000 to **32.0000**, a factor of 32.

Observation 4.2

And $32 = 2^5$. Double the efficiency and step k rises by 2^k —efficiency acts as a power, so the deeper the step the wider the gap.

Observation 4.3

These are distinct roots by the criterion of Paper 300. Efficiency changes the base of the multiplication—the requirement changes where it is cut. Both shorten the chain, from different places.

Observation 4.4

Marine chains are known to be longer. It is explained by a higher transfer efficiency for phytoplankton than on land—consistent with the shape here, that changing the base changes the number of steps.

Observation 4.5

The fragility of top predators comes from the same arithmetic. Halve the primary production and step 6 falls by $2^5 = 32$ —a change below is amplified above.

Observation 4.6

To be honest—ten per cent is a textbook representative value and real ecosystems vary widely. This paper does not assert it as a measurement—what does not move is that so long as the ratio is below 1, the chain is always cut off somewhere.

Observation 4.7

One last thing—it is not claimed that chain length is set by energy alone. Ecology also explains it through dynamical stability—what is shown here is that the energy side alone already produces a ceiling.

5. Roll Call and Closing

1. **Primary production is 1 % of sunlight** (Observation 2.2): **0.010000**.
2. **Exactly 0.1 per step** (Observation 2.5): **a geometric sequence**.
3. **Step 5 at 10^{-6}** (Observation 2.3): **one in a million**.
4. **5.000000 decades from step 1 to 6** (Computation 2.4): **the inverse agrees**.
5. **Ceiling at step 6 for a requirement of 1** (Computation 3.1): **inside the boundary**.
6. **Step 7 at 0.1000 falls short** (Computation 3.1): **outside the boundary**.
7. **Ceiling at step 5 for a requirement of 10** (Computation 3.3): **organisms unchanged**.
8. **Step 5 at 32.0000 with 20 % transfer** (Computation 4.1): **$32 = 2^5$** .
9. **Efficiency acts as a power** (Observation 4.2): **wider the deeper it goes**.
10. **The separator is efficiency or requirement** (Observation 4.3): **the base, or where it is cut**.

Closing: Food chains usually stop at four or five steps, and one sometimes hears “because large predators are rare”—but that is the consequence, not the reason. They are rare because there is no energy to support them. What stops them is arithmetic—primary production is exactly **1** per cent of the sunlight, each step keeps exactly **0.1**, and the fifth step is at **10^{-6}** , one part in a million. The common logarithm of the ratio between steps 1 and 6 is exactly **5.000000** decades—five steps, five orders—and that the inverse computation returns the same number is a self-test. Any adjacent pair gives **0.1**; it is a geometric sequence with no exceptions. And the ceiling is not set by the chain alone—taking **1** as what an individual needs, the top is step **6** at a value of **1.0000**, while step **7** falls to **0.1000** and does not reach it. But raising the requirement to **10** alone drops the top to step **5**—the efficiency, the sunlight and the structure of the chain all unchanged, with only “what one individual needs” moved. So “the ceiling on chain length” is not a property of the chain but of the chain together with the requirement, the same shape as “nearest” demanding a distance in Paper 412. One thing separates them—the efficiency or the requirement. Raising the transfer efficiency from **10** to **20** per cent lifts step **5** from **1.0000** to **32.0000**, and **$32 = 2^5$** —efficiency acts as a power, so the deeper the step the wider the gap. They are distinct roots by the criterion of Paper 300: efficiency changes the base of the multiplication, the requirement changes where it is cut. Marine chains are known to be longer, explained by a higher transfer efficiency for phytoplankton, which is consistent with changing the base changing the number of steps. The fragility of top predators comes from the same arithmetic: halve the primary production and step **6** falls by **$2^5 = 32$** , a change below amplified above. To be honest—ten per cent is a textbook representative value and real ecosystems vary widely; this paper does not assert it as a measurement. What does not move is that so long as the ratio is below **1**, the chain is always cut off somewhere. One last thing—it is not claimed that chain length is set by energy alone. Ecology also explains it through dynamical stability. What is shown here is that the energy side alone already produces a ceiling.

References

- 1. Lindeman, R. L. (1942). The Trophic-Dynamic Aspect of Ecology. *Ecology*.
- 2. Elton, C. (1927). *Animal Ecology*. London: Sidgwick & Jackson.
- 3. Pimm, S. L. (1982). *Food Webs*. London: Chapman & Hall.
- 4. Post, D. M. (2002). The Long and Short of Food-Chain Length. *Trends in Ecology & Evolution*.
- 5. Yamagishi, Y. (2026). *The Three Hundredth Census: Whether Two Things Share a Root Is Decidable* [Paper 300]. Zenodo. 10.5281/zenodo.22171914
- 6. Yamagishi, Y. (2026). *“Nearest” Is Not Defined Until the Distance Is* [Paper 412]. Zenodo. 10.5281/zenodo.22699661
- 7. Yamagishi, Y. (2026). *Adding One Peg Destroys the Closed Form* [Paper 413]. Zenodo. 10.5281/zenodo.22699750
- 8. Yamagishi, Y. (2026). *The Same Glass Became 5000 Times More Transparent* [Paper 415]. Zenodo. 10.5281/zenodo.22699790
- 9. Yamagishi, Y. (2026). *The Product Is the Same and the Result Is Not* [Paper 416]. Zenodo. 10.5281/zenodo.22699831

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