

There Are Two of Them, and Only One Has a Ceiling

— Spin stays at **2** states and **1** bit whatever is done to it, while orbital reaches **101** states and **6.6582** bits at $|l| \leq 50$ — the same “angular momentum” splits into a finite and an infinite dimension — [Paper 418]

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— *From thinking to syncing.* —

Abstract

“The angular momentum of light” usually means polarisation. This paper shows that there are two of them—and that one has a ceiling while the other does not. No new theorem or law is claimed. First, the number of spin states is exactly **2**. There is nothing but left and right circular polarisation (Section 2). Second, the number of orbital states is $2L + 1$: **11** for $|l| \leq 5$ and **101** for $|l| \leq 50$ (Section 2). Third, this is the core of the paper. The information per photon is always **1** bit for spin and **6.6582** bits for orbital at $|l| \leq 50$ (Section 3). Fourth, multiplying L by ten adds only **3.3090** bits, because it is a logarithm (Section 3). Fifth, spin stays at **1** bit however far L is pushed. That is what having a ceiling means (Section 3). Sixth, the separator is whether the dimension is finite or infinite. One name, two types of state space (Section 4).

Scope of this paper (scope note): No new theorem or law is claimed—the separation of the spin and orbital angular momentum of light, and Laguerre–Gaussian modes, are both standard (since 1992). The legitimacy of the separation is not discussed—whether a gauge-invariant separation into spin and orbital is possible is not entered into. The standard paraxial treatment is used. No communication performance is claimed—in real multiplexing, atmospheric turbulence, crosstalk between modes and detector limits all count, and 6.6582 bits is not claimed to be deliverable. Generation is not treated—spiral phase plates, spatial light modulators and q -plates are not the subject. Quantum information is not discussed—high-dimensional entanglement and key-distribution security are not the subject. Electron and neutron vortices are not treated—only light. Relation to earlier papers: Paper 417 showed that the ceiling on chain length is not a fact about animals—there a ceiling came out of arithmetic; here one of the two has none. Paper 300 showed that common root and distinct root can be decided—spin and orbital are distinct roots by that criterion. Paper 411 showed that a transformation preserving nothing still has an invariant—there one invariant survived; here one invariant splits in two. Paper 416 showed that an equal product need not give an equal result—here too, two different properties sit under one name. What is added is tabulating the state counts at four values of L , giving the information in bits, giving the increment of 3.3090 bits for a tenfold L , placing spin as the control, and placing the separator at “finite or infinite dimension”.

1. Notation and the Question

Setting: The total angular momentum of light separates into a spin and an orbital part. Spin corresponds to the handedness of circular polarisation, taking $\pm\hbar$ per photon. Orbital corresponds to a helical wavefront, taking $l\hbar$ per photon for integer l .

$$N_{\text{spin}} = 2, \quad N_{\text{orb}}(L) = 2L + 1$$

A photon with N available states carries $\log_2 N$ bits.

Observation 1.1

“The angular momentum of light” usually means polarisation. Left-handed or right-handed—only two, so it is taken to be binary.

Observation 1.2

But there is another. The degree of freedom in which the wavefront traces a helix—and there any integer l will do.

Question: Both are angular momentum. Why does only one have no ceiling.

2. Two States, and $2L+1$

Computation 2.1: Set the state counts side by side.

Range	Spin states	Orbital states
$ l \leq 1$	2	3
$ l \leq 5$	2	11
$ l \leq 10$	2	21
$ l \leq 50$	2	101

Observation 2.2

The spin column does not move.It is **2** in every range—the operation of widening the range has no meaning there at all.

Observation 2.3

The orbital column grows as $2L + 1$.**101** states at $|l| \leq 50$ —until L is fixed, the number of states is not fixed.

Observation 2.4

The $+1$ in $2L + 1$ is $l = 0$.The flat wavefront, with no helix—so the count is always odd.

3. Bits per Photon

Computation 3.1: Convert the state counts into information.

L	Orbital states	Information [bit]	Excess over spin [bit]
1	3	1.5850	0.5850
5	11	3.4594	2.4594
50	101	6.6582	5.6582
500	1001	9.9672	8.9672

Observation 3.2

Spin is exactly **1.000000** bit. $\log_2 2 = 1$ —and this value does not change when L is moved.

Observation 3.3

Orbital reaches **6.6582** bits at $|l| \leq 50$. More than six times spin—over six bits carried on a single photon.

Computation 3.4: Take the increment when L goes from 50 to 500, a factor of ten.

$9.9672 - 6.6582 = \mathbf{3.3090}$ bits.

Observation 3.5

Ten times L buys only **3.3** bits. Because $\log_2 10 = 3.3219$ —information is the logarithm of the state count, so multiplying states by an order does not multiply bits by an order.

Observation 3.6

So “no ceiling” is not “unlimited gain”. There is no ceiling, and the growth is logarithmic—say only one of these and either becomes a misunderstanding.

4. The Separator

The separator: is the dimension of the state space finite or infinite.

Observation 4.1

The spin space is two-dimensional. The photon is a spin-1 particle and, being massless, has no longitudinal component—the number 2 is fixed there.

Observation 4.2

The orbital space is infinite-dimensional. l may be any integer—it is the winding number of the phase around the azimuth, and winding numbers have no ceiling.

Observation 4.3

These are distinct roots by the criterion of Paper 300. Both are angular momentum, but they come from different places—one an internal degree of freedom of the field, one the shape of space.

Observation 4.4

So “the angular momentum of light is binary” is half right. It is right about spin—about orbital, without a range even the number of states is undetermined.

Observation 4.5

And in practice the infinite side is cut finite. The aperture of the apparatus will not pass large l — L is set by the apparatus, not by physics, the same shape as the requirement setting the ceiling in Paper 417.

Observation 4.6

To be honest—6.6582 bits is a ceiling derived from the state count, not an amount deliverable in practice. Turbulence, crosstalk and detection error all subtract—no communication performance is claimed here.

Observation 4.7

One last thing—outside the paraxial regime the separation into spin and orbital becomes delicate. There is a discussion of whether a gauge-invariant separation is possible—the standard paraxial treatment is used here, and no step is taken beyond it.

5. Roll Call and Closing

1. Spin has exactly **2** states (Computation 2.1): **in every range**.
2. Orbital has **11** states at $|l| \leq 5$ (Computation 2.1): **that is $2L + 1$** .
3. Orbital has **101** states at $|l| \leq 50$ (Computation 2.1): **set by L** .
4. The count is always odd (Observation 2.4): **because $l = 0$ is included**.
5. Spin is exactly **1.000000** bit (Observation 3.2): **independent of L** .
6. Orbital is **6.6582** bits at $|l| \leq 50$ (Computation 3.1): **over six times**.
7. **9.9672** bits at $L = 500$ (Computation 3.1): **1001 states**.
8. Ten times L adds **3.3090** bits (Computation 3.4): **a logarithm**.
9. Spin does not grow with L (Observation 3.2): **the side with a ceiling**.
10. The separator is finite or infinite dimension (Observation 4.3): **distinct roots by Paper 300**.

Closing: “The angular momentum of light” usually means polarisation, and since there is only left and right it is taken to be binary—but there are two angular momenta, the other being the degree of freedom in which the wavefront traces a helix. And only one of them has a ceiling—the number of spin states is exactly **2** in every range, and widening the range has no meaning there at all. The orbital count grows as $2L + 1$, giving **11** at $|l| \leq 5$ and **101** at $|l| \leq 50$. The $+1$ is $l = 0$, the flat wavefront with no helix, so the count is always odd. In bits the difference is plain—spin is exactly **1.000000** bit and stays there however far L is pushed, while orbital reaches **6.6582** bits at $|l| \leq 50$, over six times as much on a single photon. But taking L from 50 to 500, a factor of ten, adds only **3.3090** bits—because $\log_2 10 = 3.3219$, and information is the logarithm of the state count, so multiplying states by an order does not multiply bits by an order. So “no ceiling” is not “unlimited gain”: there is no ceiling and the growth is logarithmic, and saying only one of these turns either into a misunderstanding. One thing separates them—whether the dimension of the state space is finite or infinite. The spin space is two-dimensional because the photon is a spin-1 particle and, being massless, has no longitudinal component—the number **2** is fixed there. The orbital space is infinite-dimensional because l is the winding number of the phase around the azimuth, and winding numbers have no ceiling. They are distinct roots by the criterion of Paper 300: one an internal degree of freedom of the field, one the shape of space. So “the angular momentum of light is binary” is half right—right about spin, while about orbital,

without a range even the number of states is undetermined. And in practice the infinite side is cut finite by the aperture, so L is set by the apparatus rather than by physics, the same shape as the requirement setting the ceiling in Paper 417. To be honest—6.6582 bits is a ceiling derived from the state count, not an amount deliverable in practice; turbulence, crosstalk and detection error all subtract, and no communication performance is claimed here. One last thing—outside the paraxial regime the separation into spin and orbital becomes delicate, and there is a discussion of whether a gauge-invariant separation is possible. The standard paraxial treatment is used here, and no step is taken beyond it.

References

1. Allen, L., Beijersbergen, M. W., Spreeuw, R. J. C., & Woerdman, J. P. (1992). Orbital Angular Momentum of Light and the Transformation of Laguerre-Gaussian Laser Modes. *Physical Review A*.
2. Beth, R. A. (1936). Mechanical Detection and Measurement of the Angular Momentum of Light. *Physical Review*.
3. Poynting, J. H. (1909). The Wave Motion of a Revolving Shaft, and a Suggestion as to the Angular Momentum in a Beam of Circularly Polarised Light. *Proceedings of the Royal Society A*.
4. Yao, A. M., & Padgett, M. J. (2011). Orbital Angular Momentum: Origins, Behavior and Applications. *Advances in Optics and Photonics*.
5. Yamagishi, Y. (2026). *The Three Hundredth Census: Whether Two Things Share a Root Is Decidable* [Paper 300]. Zenodo. 10.5281/zenodo.22171914
6. Yamagishi, Y. (2026). *A Transformation That Preserves Nothing Still Has an Invariant* [Paper 411]. Zenodo. 10.5281/zenodo.22699656
7. Yamagishi, Y. (2026). *The Product Is the Same and the Result Is Not* [Paper 416]. Zenodo. 10.5281/zenodo.22699831
8. Yamagishi, Y. (2026). *The Ceiling on Food-Chain Length Is Not a Fact About Animals* [Paper 417]. Zenodo. 10.5281/zenodo.22699882

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