

Keep the Patch Constant as a Letter and the Ambiguity Dissolves

— 3rd ed.: in **the Hawking Radiation Formula under the Yamagishi Patch**, 15360π is physics, α^3 is convention, and the power of α is the mass dimension — [Paper 43]

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— *From thinking to syncing.* —

Abstract

This paper recorded the form taken by Hawking's radiated-power formula $P = \frac{\hbar c^6}{15360\pi G^2 M^2}$ under the substitution $G = 4\pi$, $\hbar = \frac{1}{4\pi}$, and its second edition retracted three geometric labels. **All three retractions were sound and are not reinstated here. But after retracting, the second edition went on asking the same question: how does $16384\pi^4$ decompose? The third edition shows that the question should not have been posed. First, the ambiguity was manufactured.** Instead of fixing the patch constant to a number, keep it as a letter: $G = a$, $\hbar = 1/a$. Symbolic computation then gives

$$T_H = \frac{c^3}{8\pi a^2 M}, \quad A = \frac{16\pi a^2 M^2}{c^4}, \quad \sigma = \frac{\pi^2 a^3}{60c^2}, \quad P = \frac{c^6}{15360\pi a^3 M^2}$$

(the step-by-step derivation and the direct substitution were verified here to agree exactly). **15360π belongs entirely to the physics** (the 60×256 of σAT_H^4), **and a^3 belongs entirely to the convention.** The number $983040\pi^4$ comes into being **only when the two are multiplied together.** The four decompositions the second edition listed, $64(16\pi^2)^2$, $64(4\pi)^4$, $4(8\pi)^4$, $1024(2\pi)^4$, are **all the same number** (verified here), and **of course no formula can choose between them: a fact had already been multiplied by a convention.** Kept as a letter, no number requiring decomposition ever appears. **Second, of the three $(4\pi)^3$ the second edition retained, only one is pure.** The $128\pi^3$ in the denominator of T_H can be written $2(4\pi)^3$, but the correct breakdown is $8\pi \times (4\pi)^2$: **the 8π belongs to Hawking's formula (a fact, from the surface gravity), the $(4\pi)^2$ to the choice of units (a convention).** Writing $2(4\pi)^3$ regroups across the boundary between **fact and convention.** The horizon area is the same: the correct breakdown of $4(4\pi)^3$ is **(the 4π of the sphere: fact) $\times 4 \times (a^2$: convention).** By contrast the $(4\pi)^3$ of σ comes from $\hbar^{-3}$ alone, with none of the physics coefficient $\pi^2/60$ mixed in: **the second edition was right to call this one well-sourced, and it is the only one of the three that is.** This distinction applies here the discipline established by Paper 35's third edition (the 4π of Poisson's equation is a fact; the 4π of the G -patch is a convention). **Third, the powers of a have a reason.** T_H carries a^{-2} , A carries a^2 , σ carries a^3 and P carries a^{-3} , and **the exponent count of $P = \sigma AT_H^4$ closes: $3 + 2 - 8 = -3$ (verified here).** And these powers are a consequence of Paper 116's theorem $\epsilon = \mu$: **the power of a is a mass dimension, not a geometric capacity.** Scope: structural interpretation and self-audit of established standard facts (Layer 2). **No prediction of Hawking radiation is altered.** **15360π is not explained, the information paradox is not touched, and greybody factors are not treated.**

Standing of this paper (discipline): the physics used here is **entirely standard**: Hawking (1974, 1975), the Stefan–Boltzmann law, $\zeta(4) = \pi^4/90$, the phase-space density of states. **No new theorem and no prediction.** The third edition’s contribution is **identifying where the ambiguity came from**: (i) keeping the patch constant **as a letter** so as to separate **15360 π** (physics) from **a^3** (convention), (ii) showing that the non-uniqueness of the decomposition that troubled the second edition **arose from multiplying the two together**, (iii) judging that of the three retained **$(4\pi)^3$ only the one in σ is pure**, and (iv) attributing the powers of **a** to **mass dimension** via Paper 116. **The most important restraints: $G = 4\pi$, $\hbar = 1/(4\pi)$ is a choice of unit system, not a new fact about nature** (Paper 95). **No claim to have explained 15360 π** : it follows completely from the standard derivation. **The exponent of c is not read as a spatial dimension** (Paper 44; retracted in the second edition). **Powers of 4π are not read as geometric capacities** (they are mass dimensions; Paper 116). **The information paradox is not touched. Greybody factors are not treated** (Page 1976). Layer-3 lenses (the picture of an OS memory limit and the like) are **quarantined under a non-scientific label**.

1. Introduction: After Retracting, the Second Edition Kept Asking the Same Question

The second edition retracted three labels: (a) reading **256 π^4** as “the square of the double-torus cocoon” (indistinguishable, since **$(16\pi^2)^2 = (4\pi)^4$**), (b) reading a bare **64** as “the numerical part of the three-dimensional torus volume” (stripping **π^3** off **$(4\pi)^3$**), and (c) reading **c^6** as “six-dimensional phase space” (it follows from dimensional analysis).

All three are sound and are not reinstated here.

But after retracting, the second edition went on asking one question: how does 16384 π^4 decompose? It listed the non-uniqueness (**$64(16\pi^2)^2$** , **$4(8\pi)^4$** , **$1024(2\pi)^4$** all hold) and concluded that “**which module one divides by is chosen by the reader, not decided by the formula**”.

Correct, and one step short. Why did the reader come to be in a position of having to choose? The second edition does not ask.

The third edition asks. The answer is: because that number was formed.

2. Keeping the Patch Constant as a Letter

The second edition substituted the **numbers $G = 4\pi$, $\hbar = \frac{1}{4\pi}$** from the outset. **Put a letter there instead**: as Paper 116 showed, the family preserving **$\hbar G = 1$** is **$G = a$, $\hbar = 1/a$** for arbitrary **$a > 0$** , and nothing requires **$a = 4\pi$** .

Theorem 2.1 (The forms with the patch constant retained)

With $G = a$, $\hbar = 1/a$ and $k_B = 1$,

$$T_H = \frac{c^3}{8\pi a^2 M}, \quad r_s = \frac{2aM}{c^2}, \quad A = \frac{16\pi a^2 M^2}{c^4}, \quad \sigma = \frac{\pi^2 a^3}{60c^2},$$
$$P = \sigma A T_H^4 = \frac{c^6}{15360\pi a^3 M^2}.$$

Symbolic verification: the step-by-step derivation (computing σ , A and T_H separately and multiplying) and the direct substitution into the standard formula $P = \frac{\hbar c^6}{15360\pi G^2 M^2}$ were confirmed to agree **exactly**. Setting $a = 4\pi$ gives $P = \frac{c^6}{983040\pi^4 M^2}$, reproducing the second edition's value.

Observation 2.1 (This one line separates everything)

$$P = \frac{c^6}{15360\pi \cdot a^3 \cdot M^2}$$

The denominator falls into two parts: 15360π on the side of the physics, a^3 on the side of the convention.

15360π belongs entirely to the physics: it is the 60×256 obtained on computing $\sigma A T_H^4$, where the **60** descends from $\zeta(4) = \pi^4/90$ and the **256** from tidying $8^4/16$. **It has nothing to do with the patch.**

a^3 belongs entirely to the convention: it comes only from what is substituted for G and $\hbar$, a choice of units. **It has nothing to do with the physics.**

And the number $983040\pi^4$ comes into being only when these two are multiplied:

$$15360\pi \times (4\pi)^3 = 15360\pi \times 64\pi^3 = 983040\pi^4.$$

3. The Ambiguity Was Born at the Moment of Multiplication

Observation 3.1 (The second edition’s non-uniqueness was manufactured)

The second edition correctly noted that the decomposition of $16384\pi^4$ is not unique. Verified here:

Decomposition	Value	Equals $16384\pi^4$?
$64(16\pi^2)^2$	$16384\pi^4$	yes
$64(4\pi)^4$	$16384\pi^4$	yes
$4(8\pi)^4$	$16384\pi^4$	yes
$1024(2\pi)^4$	$16384\pi^4$	yes

All four are the same number. Of course no formula can choose among them.

But the question to ask is not “why can it not choose” but “why did a choice become necessary”. There is one answer: because a fact (15360π) had already been multiplied by a convention (a^3). In the resulting number, no record survives of which π came from where.

Kept as a letter, no number requiring decomposition ever appears.

In $P = \frac{c^6}{15360\pi a^3 M^2}$ there is no room for decomposition: 15360π cannot be split, and neither can a^3 .

Observation 3.2 (The second edition’s retractions were consequences of this)

The second edition retracted its identification because $(16\pi^2)^2$ and $(4\pi)^4$ are indistinguishable. That judgement is right. The third edition goes one step further back: they are indistinguishable because the expression was processed into a form in which they cannot be distinguished.

The same holds for the “module chopping” the second edition forbade. A bare **64** can be extracted from $(4\pi)^3 = 64\pi^3$ only because π^4 had already been absorbed into another factor, leaving no π^3 to accompany the **64**. Kept as a letter, a^3 remains one block to the end, and the stripping operation is not even definable.

4. Of the Three Retained $(4\pi)^3$, Only One Is Pure

Having retracted, the second edition wrote that “the places where $(4\pi)^3$ appears with its π intact do remain”, and listed three: the Hawking temperature, the horizon area, and the Stefan–Boltzmann constant. **The third edition inspects all three.**

The Hawking temperature: the 8π is a fact, the a^2 a convention

The second edition wrote $T_H = \frac{c^3}{2(4\pi)^3 M}$. As a number this is correct. But look at the breakdown:

Writing	Value	Meaning
$8\pi \times (4\pi)^2$	$128\pi^3$	(fact) $\times$ (convention)
$2 \times (4\pi)^3$	$128\pi^3$	regrouped across the boundary

The same number, a different meaning. The 8π in $T_H = \frac{\hbar c^3}{8\pi G M k_B}$ is a number belonging to Hawking’s formula, arising from the surface gravity $\kappa = c^4/(4GM)$ and $T = \hbar\kappa/(2\pi c k_B)$: a fact. The $a^2 = \hbar/G$ side is a choice of units.

Writing $2(4\pi)^3$ borrows one π from the factual 8π and hands it to the conventional side.

This violates, in this setting, the discipline established by Paper 35’s third edition: **the 4π of Poisson’s equation is a fact and the 4π of the G -patch a convention, and the two must not be gathered into one.**

The horizon area: the 4π of the sphere is a fact

The same thing happens. The 4π of $A = 4\pi r_s^2$ is the total solid angle of a sphere, a fact; the a in $r_s = 2aM/c^2$ is a convention. Hence

$$A = 4\pi \times 4 \times a^2 \cdot \frac{M^2}{c^4} = \frac{16\pi a^2 M^2}{c^4},$$

where the 4π is the total solid angle of the sphere, a fact, and the a^2 is the convention. At $a = 4\pi$ this is $\frac{256\pi^3 M^2}{c^4}$ (verified here). **The second edition’s $4(4\pi)^3$ crosses the boundary here too.**

The Stefan–Boltzmann constant: this one alone is pure

Observation 4.1 (Only the $(4\pi)^3$ of σ is unmixed)

Substituting the patch into $\sigma = \frac{\pi^2 k_B^4}{60 \hbar^3 c^2}$ gives $\sigma = \frac{\pi^2 a^3}{60 c^2}$. The a^3 comes from $\hbar^{-3}$ alone, with none of the physics coefficient $\pi^2/60$ mixed in.

The second edition was therefore right to judge the $(4\pi)^3$ of σ well-sourced. That $\hbar$ appears cubed descends from the phase-space density of states $1/(2\pi\hbar)^3$, and that cube is the three-dimensionality of space itself.

Of the three listed, two were mixtures of fact and convention; exactly one was pure.

But being pure and being a discovery are two different things. Once $\hbar = 1/a$ has been set, an a^3 appearing in $\hbar^{-3}$ is a consequence of the substitution.

5. The Powers of a Have a Reason: They Are Mass Dimensions

Collecting the powers of a carried by the four quantities:

Quantity	Power of a	Source
T_H	a^{-2}	$\hbar/G = 1/a^2$
A	a^2	$r_s^2 \propto G^2 = a^2$
σ	a^3	$\hbar^{-3} = a^3$
$P = \sigma A T_H^4$	a^{-3}	$3 + 2 - 4 \times 2 = -3$

The exponent count closes: $3 + 2 - 8 = -3$ (verified here).

Observation 5.1 (Powers of 4π are not geometric capacities)

Paper 116 showed that under the one-parameter patch $G = a$, $\hbar = 1/a$ one has $X = \hbar^p G^q c^N = a^{-\mu} c^N$, that is, the power of a is fixed by the mass dimension μ . The four quantities here are instances of that.

Hence the content of the observation “a $(4\pi)^3$ appears” is not geometry but dimensional counting. The 3 is neither the dimension of a torus nor the dimension of space: it is the mass dimension of the quantity.

This supports the second edition’s retraction (b) from another direction: a bare 64 cannot be called a three-dimensional torus volume not only because the π was stripped, but because what the exponent counts was never a geometric capacity in the first place.

6. The Scope, Restated

Item	What it is
$P = \frac{\hbar c^6}{15360\pi G^2 M^2}$	Established fact (Hawking 1974; not altered here)
$15360 = 60 \times 256$	Consequence of the standard derivation (no mystery)
$P = \frac{c^6}{15360\pi a^3 M^2}$	Fact $\times$ convention, kept apart
$983040\pi^4$	The product of the two (the source of the non-uniqueness)
The a^3 of σ	Pure convention (from $\hbar^{-3}$ alone)
The $(4\pi)^3$ of T_H and A	Mixtures (the 8π and 4π belong to the formulas)
The powers of a	Mass dimensions (Paper 116, $\epsilon = \mu$)
That c^6 is six-dimensional phase space	Not shown (retracted; not reinstated)
An explanation of 15360π	Not shown (a known quantity)
A resolution of the information paradox	Not shown

Of the ten rows, not one carries a claim proper to this paper. One established formula, one consequence of the standard derivation, one separated form; the rest are conventions, consequences of other papers’ theorems, or “not shown”.

7. Conclusion

The second edition retracted three labels. **It was right.** And it stopped by writing that “**which module one divides by is chosen by the reader**”. **That too was right.**

But the cause of the reader’s having to choose lay on the side of the expression. Multiply a fact by a convention and the π s no longer carry a record of where they came from.

Keep the patch constant as a letter and no number requiring decomposition appears.
And the powers of a that remain are not geometric capacities but mass dimensions.

What is not claimed (restraint): $G = 4\pi$, $\hbar = 1/(4\pi)$ is not a fact about nature (a choice of unit system; Papers 95, 116) / **no claim to have explained 15360π** (it follows completely from the standard derivation) / **the exponent of c is not read as a spatial dimension** (Paper 44; retracted in the second edition) / **powers of 4π are not read as geometric capacities** (they are mass dimensions; Paper 116) / **a factual π and a conventional π are not gathered into one** (Paper 35, 3rd ed.) / **the $(4\pi)^3$ of T_H , A and σ are not unified as “the same geometric quantity”** (only one of the three is pure) / **the information paradox is not touched / greybody factors are not treated** (Page 1976; the ideal blackbody approximation only) / **Layer-3 lenses (the picture of an OS memory limit and the like) are kept out of the body** (quarantined under a non-scientific label) / no new mathematics and no prediction (Layer 2).

Revision Record

Third edition (August 2026): the algebra is unchanged. The values of T_H , A , σ and P all stand as in the second edition ($P = \frac{c^6}{983040\pi^4 M^2}$ at $a = 4\pi$). The three items the second edition retracted, $(16\pi^2)^2$ as the square of the double torus, a bare 64 as a three-dimensional torus volume, and c^6 as six-dimensional phase space, are not reinstated. What is added is an identification of where the ambiguity came from. (i) Rather than fixing the patch constant to a number, it was kept as a letter, $G = a$, $\hbar = 1/a$, giving $P = \frac{c^6}{15360\pi a^3 M^2}$ (the step-by-step derivation and the direct substitution were verified symbolically to agree). **15360π belongs entirely to the physics and a^3 entirely to the convention.** (ii) The non-uniqueness of the decomposition of $16384\pi^4$ that troubled the second edition (with $64(16\pi^2)^2$, $64(4\pi)^4$, $4(8\pi)^4$ and $1024(2\pi)^4$ verified here to be the same number) was shown to arise from multiplying the fact by the convention: kept as a letter, no number requiring decomposition appears at all. (iii) The three $(4\pi)^3$ retained by the second edition were inspected, and only the one in σ was judged pure. The correct breakdown of the $128\pi^3$ in T_H is $8\pi \times (4\pi)^2$ (fact $\times$ convention), so writing $2(4\pi)^3$ regroups across the boundary (the discipline of Paper 35, 3rd ed.); the horizon area behaves the same way. **The second edition’s judgement that the σ case is well-sourced was correct.** (iv) The powers of a (-2 for T_H , $+2$ for A , $+3$ for σ , -3 for P , with the count $3 + 2 - 8 = -3$ closing) were shown to be a consequence of Paper 116’s theorem $\epsilon = \mu$, hence mass dimensions and not geometric capacities. (v) The bibliography was updated to

current titles and the Layer-3 picture (an OS memory limit) was removed from the body.

The dashes used as separators in the text were replaced by punctuation (October 2026). No claim, number or reference was changed.

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