

Constant at Every Rank, and the Total Is Still Undetermined

— Rank times frequency is exactly **1.000000** at rank **1** and at rank **10000** alike — yet the total is **9.787606** and keeps growing with N — moving the exponent from **0.8** to **1.2** changes the share of the top one per cent by **2.5022** — [Paper 435]

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— *From thinking to syncing.* —

Abstract

One hears that “rank times frequency is constant”. This paper shows that that constancy does not settle the whole— constant at every rank, and the total still grows with N . No new theorem or law is claimed. First, at an exponent of **1**, rank times frequency is exactly **1.000000** at rank **1** and at rank **10000** (Section 2). Second, this is the core of the paper. Constant at every rank, the total is **9.787606** and does not level off. Increase N and it keeps growing (Section 2). Third, that total agrees with $\ln N + \gamma = \mathbf{9.787556}$ to within **0.000050** (Section 2). Fourth, at an exponent of **1.0** the top one per cent holds **52.9995** per cent of the whole (Section 3). Fifth, moving the exponent from **0.8** to **1.2** takes that share from **0.300046** to **0.750766**, a factor of **2.5022** (Section 3). Sixth, the separator is whether one looks at a single rank or at the whole (Section 4). **Keywords:** Zipf’s law, power law, harmonic series, rank-frequency, heavy-tailed distribution.

Scope of this paper (scope note): No new theorem or law is claimed—Zipf’s law, the divergence of the harmonic series and power-law tails are all standard. No data is fitted—surveys of which field has which exponent are not the subject; the exponent is treated as given. No mechanism is explained—why power laws appear (preferential attachment, optimisation, maximum entropy) is not the subject. Methods of fitting are not discussed—the choice of estimator, the cut on the tail and testing are not the subject. A finite system is not treated as infinite—real collections are finite, so the total is always finite. What is claimed is that increasing N keeps increasing it, not that it becomes infinite. Discrete and continuous are not mixed—ranks are integers; the integral approximation is used only to check against $\ln N + \gamma$. Relation to earlier papers: Paper 434 showed that what makes things cheaper is the count, not the calendar—there too a power law was the actor; here the value of the exponent settles the whole picture. Paper 300 showed that common root and distinct root can be decided—“each term is constant” and “the total is determined” are distinct roots by that criterion. Paper 417 showed that the ceiling on chain length is not a fact about animals—there a geometric series converged; here it does not. Paper 424 showed that dividing turns a move in the denominator into an event—here too a share is the quotient of two sums. What is added is showing the product constant at five ranks, checking the total against $\ln N + \gamma$, confirming that increasing N increases it, giving the share of the top one per cent at three exponents, giving the ratio of shares as 2.5022, and placing the separator at “a single rank or the whole”.

1. Notation and the Question

Setting: Write the frequency of the element at rank k as

$$f(k) \propto \frac{1}{k^\alpha}$$

At $\alpha = 1$ the product $kf(k)$ is independent of k . Below, $N = 10000$ and the constant of proportionality is 1.

Observation 1.1

“Rank times frequency is constant” sounds like a strong regularity. Whatever rank one looks at, the same number appears—it feels as though the whole has been grasped.

Observation 1.2

But “constant” is a statement about each term. It is about the value at a chosen k —and says nothing about the value of the sum.

Question: If each term is constant, is the total determined.

2. Constant at Every Rank, and the Total Keeps Growing

Computation 2.1: At $\alpha = 1$, take the frequency and the product at five ranks.

Rank	Frequency	Rank $\times$ frequency
1	1.00000000	1.000000
10	0.10000000	1.000000
100	0.01000000	1.000000
1000	0.00100000	1.000000
10000	0.00010000	1.000000

Observation 2.2

The same value across four orders of magnitude. Zero spread—as the definition requires, confirming the implementation.

Computation 2.3: Sum over all ranks.

The total is **9.787606**.

Observation 2.4

It agrees with $\ln N + \gamma \ln 10000 + 0.5772156649 = \mathbf{9.787556}$, differing by **0.000050**—a check against an independent known approximation.

Observation 2.5

Take N to 100000 and the total grows further. There is no ceiling—each term constant, and the total undetermined.

Observation 2.6

The growth is logarithmic. Tenfold N adds only $\ln 10 = 2.30$ —so “diverges” and “grows fast” are also different statements.

3. Moving the Exponent Remakes the Picture

Computation 3.1: Vary α and take the share held by the top one per cent.

Exponent α	Share of the top 1 %
0.8	0.300046
1.0	0.529995
1.2	0.750766

Observation 3.2

At 1.0 the top one per cent holds more than half. **52.9995** per cent—a hundredth of the ranks holding half the mass.

Observation 3.3

From 0.8 to 1.2 the share changes by **2.5022**. From 0.30 to 0.75—a difference of 0.4 in the exponent remakes the picture.

Observation 3.4

All three are equally “a power law”. The form is the same; only the exponent differs—so “a power law was found” says nothing on its own.

Observation 3.5

An exponent of 1 sits at a special place. For $\alpha > 1$ the total converges; for $\alpha \leq 1$ it diverges—exactly on the boundary.

4. The Separator

The separator: a single rank, or the whole.

Observation 4.1

Look at a single rank and the product is constant. Choose k and compute $kf(k)$ —whatever is chosen, the same number appears.

Observation 4.2

Look at the whole and the total depends on N . It changes with how far one counted—one rule yielding two quantities of different character.

Observation 4.3

These are distinct roots by the criterion of Paper 300. Both are spoken of as “constant”—one a property of each term, one of the sum.

Observation 4.4

So “it is stable because rank times frequency is constant” does not follow. Each term can be stable while the total moves with the counting—without deciding where to cut, there is no total.

Observation 4.5

The same holds for shares. The share of the top one per cent depends on both the exponent and N —to say “it is concentrated” with one number, both must be printed.

Observation 4.6

To be honest—real collections are finite, so the total is always finite. What is claimed is that increasing N keeps increasing it—not that it becomes infinite.

Observation 4.7

One last thing—that the growth is logarithmic also matters. Tenfold N adds only 2.30—“diverges” is not “large”. Mixing the two makes both a misunderstanding.

5. Roll Call and Closing

1. The product is **1.000000** across four orders (Computation 2.1): **zero spread**.
2. The total is **9.787606** (Computation 2.3): **constant terms, undetermined sum**.
3. $\ln N + \gamma = 9.787556$ (Observation 2.4): **differing by 0.000050**.
4. Increasing N keeps increasing it (Observation 2.5): **no ceiling**.
5. The growth is logarithmic (Observation 2.6): **tenfold adds 2.30**.
6. At $\alpha = 1.0$ the top 1 % holds **52.9995 %** (Computation 3.1): **over half**.
7. At 0.8 it is **30.0046 %** (Computation 3.1): **thin**.
8. At 1.2 it is **75.0766 %** (Computation 3.1): **thick**.
9. A ratio of **2.5022** (Observation 3.3): **from 0.4 in the exponent**.
10. The separator is term or sum (Observation 4.3): **distinct roots by Paper 300**.

Closing: “Rank times frequency is constant” sounds like a strong regularity—whatever rank one looks at, the same number appears—but “constant” is a statement about each term, and says nothing about the value of the sum. In fact each term is constant and the total is not determined—at an exponent of 1 the product is exactly **1.000000** at rank 1 and at rank 10000, with zero spread across four orders of magnitude. Yet the total is **9.787606**, agreeing with $\ln N + \gamma = 9.787556$ to within **0.000050**, and increasing N keeps increasing it with no ceiling. The growth is

logarithmic, tenfold N adding only 2.30—so “diverges” and “grows fast” are also different statements. And moving the exponent a little remakes the picture—the share of the top one per cent is 0.300046 at $\alpha = 0.8$, **0.529995** at 1.0 and 0.750766 at 1.2. At 1.0 a hundredth of the ranks holds more than half the mass, and a difference of 0.4 in the exponent moves the share by **2.5022**. All three are equally “a power law”, so “a power law was found” says nothing on its own. An exponent of 1 sits exactly on the boundary: above it the total converges, at or below it the total diverges. One thing separates them—a single rank, or the whole. Look at a single rank and the product is constant whatever is chosen; look at the whole and the total depends on how far one counted. They are distinct roots by the criterion of Paper 300: both are spoken of as “constant”, but one is a property of each term and one of the sum. So “it is stable because rank times frequency is constant” does not follow—each term can be stable while the total moves with the counting, and without deciding where to cut there is no total. The same holds for shares, which depend on both the exponent and N , so saying “it is concentrated” with one number requires printing both. To be honest—real collections are finite, so the total is always finite. What is claimed is that increasing N keeps increasing it, not that it becomes infinite. One last thing—that the growth is logarithmic also matters. Tenfold N adds only 2.30, so “diverges” is not “large”, and mixing the two makes both a misunderstanding.

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