

Both Improved **7.2446**-Fold and the Ratio Never Moved

— Improving at two per cent a year, capability rises **7.2446**-fold over a century while the ratio stays at exactly **1.000000000** — let one side stop and the odds move from **0.5** to **0.878803** — one runs not to overtake but not to fall — [Paper 438]

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— *From thinking to syncing.* —

Abstract

Improving is expected to bring advantage. This paper shows that if the other improves as much, nothing changes—and that stopping nonetheless costs a great deal. No new theorem or law is claimed. First, improving at two per cent a year raises capability **7.2446**-fold over a century (Section 2). Second, this is the core of the paper. If both improve equally, the ratio after a century is exactly **1.000000000** (Section 2). Third, the odds stay at exactly **0.500000000** as well (Section 2). Fourth, raising the rate fivefold, from two to ten per cent, still leaves the ratio at **1** so long as both do it (Section 2). Fifth, let one side stop and the odds move from **0.5** to **0.878803** in a century, a shift of **0.378803** (Section 3). Sixth, the separator is absolute capability against the ratio to the other (Section 4). **Keywords:** Red Queen hypothesis, coevolution, arms race, relative fitness.

Scope of this paper (scope note): No new theorem or law is claimed—the Red Queen hypothesis, coevolution and relative fitness are all standard (since 1973). No biological claim is made—the origin of sex, host-parasite coevolution and the constancy of extinction rates are not the subject. No rate is asserted—two per cent a year is representative for exposition and is not the value of any field. One model of winning is fixed—the odds are written as a ratio of capabilities. Other forms change the numbers but not the conclusion that the ratio does not move. Optimisation is not discussed—how much one should invest is not the subject. Asymmetries in time are not treated—the case where catching up and staying ahead differ in difficulty is not treated. Relation to earlier papers: Paper 437 showed that each choosing best lands both on the worst—there the choices affected each other; here the improvements cancel each other. Paper 300 showed that common root and distinct root can be decided—“capability rose” and “advantage was gained” are distinct roots by that criterion. Paper 424 showed that dividing turns a move in the denominator into an event—there the denominator moved; here numerator and denominator move together and the quotient does not. Paper 434 showed that what makes things cheaper is the count, not the calendar—there too, what moved and what acted were different. What is added is tabulating capability and ratio at four times, showing to nine digits that the ratio and the odds do not move, confirming that a fivefold rate leaves the ratio unchanged, giving the shift when one side stops as 0.378803, giving the absolute rise as 7.2446-fold to establish that running does happen, and placing the separator at “absolute or ratio”.

1. Notation and the Question

Setting: Let two parties have capabilities $a(t)$ and $b(t)$, improving at an annual rate r .

$$a(t) = a_0(1 + r)^t, \quad b(t) = b_0(1 + r)^t$$

Take the odds of winning to follow the ratio of capabilities,

$$p = \frac{a}{a + b}$$

Below, $a_0 = b_0 = 1$ and $r = 0.02$.

Observation 1.1

Improving is expected to bring advantage. Faster, stronger, better—the shape in which effort is rewarded.

Observation 1.2

But advantage is set by the ratio to the other, not by absolute capability. If the other is improving too, the account changes—and usually it is.

Question: If both improve equally, what is left.

2. A 7.2446-Fold Rise and a Ratio of One

Computation 2.1: Take capability and ratio with both improving at two per cent a year.

Year	First	Second	Ratio	Odds
0	1.0000	1.0000	1.000000	0.500000
10	1.2190	1.2190	1.000000	0.500000
50	2.6916	2.6916	1.000000	0.500000
100	7.2446	7.2446	1.000000	0.500000

Observation 2.2

Capability has risen a great deal. **7.2446**-fold in a century—the running is real.

Observation 2.3

And the ratio is exactly **1.000000000**. Unmoved to nine digits—the same factor multiplies both, so it cancels in the quotient.

Observation 2.4

The odds are exactly **0.500000000** too. The ratio does not move, so the odds do not—a century has passed and nothing has changed.

Computation 2.5: Raise the rate fivefold, to ten per cent.

Capability after a century rises 13780.6-fold, and the ratio is still exactly **1.000000000**.

Observation 2.6

Changing the speed changes nothing so long as both change it. However fast one runs, the relative position holds—which is what “running to stay in the same place” means.

3. Stopping Costs a Great Deal

Computation 3.1: Take the odds when only the second party stops improving.

Year	First	Second	Odds for the first
10	1.2190	1.0000	0.549291
50	2.6916	1.0000	0.729116
100	7.2446	1.0000	0.878803

Observation 3.2

The one that stopped falls from 0.5 to 0.121197. Seen from the other side, **0.878803**—a shift of **0.378803**.

Observation 3.3

So running does mean something. Running brings no advantage; stopping brings disadvantage—two different statements.

Observation 3.4

At ten years it is still 0.549291. A small gap—the cost of stopping opens with compounding over time.

Observation 3.5

So in the short run stopping does not look costly. Only 0.05 in ten years—and that is where judgement goes wrong.

4. The Separator

The separator: absolute capability, or the ratio to the other.

Observation 4.1

Absolute capability has risen. 7.2446-fold—beyond doubt.

Observation 4.2

The ratio has not moved. Exactly 1—also beyond doubt.

Observation 4.3

These are distinct roots by the criterion of Paper 300. Both are called “progress”—one looks at oneself alone, one at the pair.

Observation 4.4

Different quantities, different answers. Both are correct—“progress was made and nothing changed” is not a contradiction but two instruments reported together.

Observation 4.5

The same shape as Paper 424. There the denominator alone moved and the quotient moved—here numerator and denominator move together and the quotient does not. In both, the divided quantity is a function of two independent ones.

Observation 4.6

To be honest—one model of the odds has been fixed. Other forms change the numbers—what does not change is that a common factor leaves the ratio unmoved.

Observation 4.7

One last thing—it is not claimed that effort is wasted. Running brings no advantage, and stopping costs **0.378803**—one runs not to overtake but not to fall.

5. Roll Call and Closing

1. **7.2446-fold in a century** (Computation 2.1): **the running is real.**
2. **The ratio is exactly 1.000000000** (Observation 2.3): **to nine digits.**
3. **The odds are exactly 0.500000000** (Observation 2.4): **nothing changed.**
4. **Fivefold rate, ratio still 1** (Computation 2.5): **speed does not act.**
5. **Capability then 13780.6-fold** (Computation 2.5): **only the absolute grows.**
6. **Stopping gives odds of 0.878803** (Computation 3.1): **over a century.**
7. **A shift of 0.378803** (Observation 3.2): **large.**
8. **At ten years, 0.549291** (Computation 3.1): **still small.**
9. **The cost compounds** (Observation 3.4): **invisible in the short run.**
10. **The separator is absolute or ratio** (Observation 4.3): **distinct roots by Paper 300.**

Closing: Improving is expected to bring advantage—faster, stronger, better—but advantage is set by the ratio to the other rather than by absolute capability, and the other is usually improving too. If both improve equally, nothing changes—running at two per cent a year for a century raises capability **7.2446-fold**, and yet the ratio is exactly **1.000000000** and the odds exactly **0.500000000**. Unmoved to nine digits, because the same factor multiplies both and cancels in the quotient. Raise the rate fivefold to ten per cent and capability rises 13780.6-fold while the ratio is still exactly **1**—however fast one runs, the relative position holds so long as both run. And still, stopping costs—let one side stop and the odds move from 0.5 to **0.878803** over a century, a shift of **0.378803**. Running brings no advantage; stopping brings disadvantage—two different statements. At ten years it is still 0.549291, a small gap, because the cost opens with compounding; so in the short run stopping does not look costly, and that is where judgement goes wrong. One thing separates them—absolute capability, or the ratio to the other. Absolute capability

has risen 7.2446-fold and the ratio has not moved from 1; both are beyond doubt. They are distinct roots by the criterion of Paper 300: both are called progress, but one looks at oneself alone and one at the pair. Different quantities give different answers, and both are correct—“progress was made and nothing changed” is not a contradiction but two instruments reported together. It is the same shape as Paper 424, where the denominator alone moved and the quotient moved; here they move together and it does not, because in both cases the divided quantity is a function of two independent ones. To be honest—one model of the odds has been fixed. Other forms change the numbers, but not that a common factor leaves the ratio unmoved. One last thing—it is not claimed that effort is wasted. Running brings no advantage and stopping costs **0.378803**: one runs not to overtake but not to fall.

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