

What Makes It Judder Is Not the Size of the Friction but the Gap Between Two

— With static **30 N**, kinetic **20 N** and a spring of **1000 N/m**, each slip jumps **20.0000 mm** — make the two equal and the amplitude is exactly **0** — stiffen the spring tenfold and it only falls to a tenth; it does not go —

[Paper 439]

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— *From thinking to syncing.* —

Abstract

Heavy friction is expected to make motion jerky. This paper shows that the cause is not the size but the gap—make the two frictions equal and the motion is smooth however large they are. No new theorem or law is claimed. First, this is the core of the paper. The amplitude of one slip is $2(F_s - F_k)/k$, giving **20.0000 mm** for static **30 N**, kinetic **20 N** and a spring of **1000 N/m** (Section 2). Second, if the two frictions are equal, the amplitude is exactly **0** (Section 3). Third, raising the ratio to 1.1, 1.5 and 2.0 gives 5.4545, 20.0000 and **30.0000 mm**, increasing monotonically (Section 3). Fourth, stiffening the spring tenfold takes the amplitude from 20.0000 to **2.0000 mm**, exactly a tenth (Section 3). Fifth, but it does not go: so long as the ratio is not 1, the amplitude remains (Section 3). Sixth, the separator is the size of the friction against the gap between two frictions (Section 4). **Keywords:** stick-slip, static friction, kinetic friction, friction-induced vibration.

Scope of this paper (scope note): No new theorem or law is claimed—the distinction between static and kinetic friction, and the mechanism of stick-slip, are both standard. The origin of friction is not discussed—why static friction is the larger is not the subject. Velocity dependence is not treated—the detailed treatment where kinetic friction varies with sliding speed is not the subject; two constants are used. Damping is not included—in reality the oscillation damps and settles. Only the size of one slip is examined. No material values are asserted—30 N, 20 N and 1000 N/m are representative for exposition. Sound is not treated—the frequency and loudness of squeal are not the subject. Relation to earlier papers: Paper 430 showed that widening the contact does not change the friction—there too, a quantity that was moved did not move the result. Paper 300 showed that common root and distinct root can be decided—“the friction is large” and “the two frictions differ” are distinct roots by that criterion. Paper 438 showed that equal improvement leaves the ratio unmoved—there too, a difference of zero meant nothing happened. Paper 422 showed that the condition named as a rule’s core does not move the result—here too, the conspicuous quantity is not the one acting. What is added is giving the amplitude of one slip as 20.0000 mm, placing the equal-friction case at exactly 0 as the other side of the boundary, tabulating the amplitude at four ratios, showing the inverse relation with spring stiffness, showing that stiffening does not remove it, and placing the separator at “size or gap”.

1. Notation and the Question

Setting: Pull an object through a spring of stiffness k . Let F_s be the static and F_k the kinetic friction. When the spring force reaches F_s the object breaks away, overshoots the position where the force equals F_k , and stops again.

$$\Delta x = \frac{2(F_s - F_k)}{k}$$

Below, $F_s = 30$ N, $F_k = 20$ N and $k = 1000$ N/m.

Observation 1.1

Heavy friction is expected to make motion jerky. Drag something heavy and it judders—so the size looks like the cause.

Observation 1.2

But what appears in the expression is the gap. $F_s - F_k$ —neither F_s nor F_k appears on its own.

Question: If the gap is zero, is the motion smooth however large the friction.

2. Each Slip Jumps Twenty Millimetres

Computation 2.1: Take the amplitude at the given values.

$\Delta x = 2(30 - 20)/1000 = 0.020000 \text{ m}$, that is **20.0000 mm**.

Observation 2.2

Twenty millimetres is visible. Pulling by hand, the jump is unmistakable—the object is not moving smoothly; it stops and jumps.

Observation 2.3

Neither load nor area appears in the expression. Only the two frictions and the spring—the same vanishing as when shape left the friction coefficient in Paper 430.

Observation 2.4

Raise F_s and F_k together and the amplitude does not move. The gap is unchanged—press down harder and it jumps just the same.

3. Zero Gap, Zero Amplitude

Computation 3.1: Hold the static friction at 30 N and vary the static-to-kinetic ratio.

Static / kinetic	Amplitude [mm]
1.00	0.0000
1.10	5.4545
1.50	20.0000
2.00	30.0000

Observation 3.2

At a ratio of exactly 1, the amplitude is exactly 0. The friction is still 30 N and there is no jump at all—the other side of the boundary, showing that size is not the cause.

Observation 3.3

Even a ratio of 1.1 jumps 5.4545 mm. A tenth of a gap, five millimetres—the gap need not be large.

Computation 3.4: Stiffen the spring.

Spring [N/m]	Amplitude [mm]
1000	20.0000
10000	2.0000

Observation 3.5

Ten times stiffer gives exactly a tenth. An inverse relation—so a stiffer machine judders less.

Observation 3.6

But it does not go. So long as the ratio is not 1, the amplitude stays positive however stiff the spring—stiffness dilutes the symptom without removing the cause.

4. The Separator

The separator: the size of the friction, or the gap between two frictions.

Observation 4.1

Move the size and, with the gap unchanged, the amplitude does not move. Raise F_s and F_k together—press harder and it jumps the same.

Observation 4.2

Move the gap and the amplitude moves. From 0 to 30 mm as the ratio goes from 1.0 to 2.0—this alone acts.

Observation 4.3

These are distinct roots by the criterion of Paper 300. Both are blamed on friction—one is a level, the other the gap between two levels.

Observation 4.4

So there are two remedies. Reduce the gap (change the lubrication) or stiffen the spring—the first treats the cause, the second the symptom.

Observation 4.5

And “switch to a slipperier material” need not help. Small friction with a large gap still jumps—what to choose is not small friction but friction with a small gap.

Observation 4.6

To be honest—kinetic friction has been treated as a constant here. In reality it varies with sliding speed, and that dependence shapes the oscillation—what does not change is that a zero gap gives a zero amplitude.

Observation 4.7

One last thing—damping is not included either. Real oscillations settle—what is given is the size of one slip, not the history that follows.

5. Roll Call and Closing

1. One slip jumps 20.0000 mm (Computation 2.1): visible.

2. Neither load nor area appears (Observation 2.3): **two frictions and a spring.**
3. Raising both together changes nothing (Observation 2.4): **the gap is unchanged.**
4. A ratio of 1 gives exactly 0 (Computation 3.1): **the other side of the boundary.**
5. A ratio of 1.1 gives 5.4545 mm (Computation 3.1): **a tenth of a gap.**
6. A ratio of 2.0 gives 30.0000 mm (Computation 3.1): **monotone.**
7. Tenfold stiffness gives 2.0000 mm (Computation 3.4): **exactly a tenth.**
8. Stiffening does not remove it (Observation 3.6): **only dilutes.**
9. Choose a small gap, not small friction (Observation 4.5): **the criterion.**
10. The separator is size or gap (Observation 4.3): **distinct roots by Paper 300.**

Closing: Heavy friction is expected to make motion jerky—drag something heavy and it judders, so the size looks like the cause—but what appears in the expression is the gap $F_s - F_k$, and neither friction appears on its own. Each slip jumps unmistakably—with static 30 N, kinetic 20 N and a spring of 1000 N/m, the amplitude is $2(30 - 20)/1000 = 20.0000$ mm. Pulling by hand the jump is plain: the object is not moving smoothly but stopping and jumping. Neither load nor area appears in the expression—the same vanishing as when shape left the friction coefficient in Paper 430—so raising both frictions together leaves the amplitude unmoved, and pressing harder changes nothing. Zero gap, zero amplitude—at a static-to-kinetic ratio of exactly 1, the friction is still 30 N and yet the amplitude is exactly 0. Raising the ratio to 1.1, 1.5 and 2.0 gives 5.4545, 20.0000 and 30.0000 mm, and even a tenth of a gap jumps five millimetres. Stiffening the spring tenfold takes the amplitude to 2.0000 mm, exactly a tenth, so a stiffer machine judders less. But it does not go—so long as the ratio is not 1 the amplitude stays positive however stiff the spring, and stiffness dilutes the symptom without removing the cause. One thing separates them—the size of the friction, or the gap between two frictions. Move the size with the gap unchanged and nothing moves; move the gap and the amplitude runs from 0 to 30 mm. They are distinct roots by the criterion of Paper 300: both are blamed on friction, but one is a level and one the gap between two levels. So there are two remedies—reduce the gap by changing the lubrication, or stiffen the spring—and the first treats the cause while the second treats the symptom. And “switch to a slipperier material” need not help, since small friction with a large gap still jumps: what to choose is friction with a small gap. To be honest—kinetic friction has been treated as a constant. In reality it varies with sliding speed and that dependence shapes the oscillation. What does not change is that a zero gap gives a zero amplitude. One last thing—damping is not included either. Real oscillations settle. What is given is the size of one slip, not the history that follows.

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