

The Power of 4π Is the Mass Dimension

— “Zero or One” Is a Property of the List, and What Fixes $a = 4\pi$ Is an Electromagnetic Unit Convention —

[Paper 44] 3rd ed.

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— *From thinking to syncing.* —

ABSTRACT

This paper showed that under the Yamagishi patch $G = 4\pi$, $\hbar = \frac{1}{4\pi}$ every Planck quantity reduces to c^N or $\frac{c^N}{4\pi}$, and called that branching the “geometric / material” dichotomy. **The dichotomy holds, and the third edition does not alter it. What the third edition adds is the reason for it, and a correction to the range over which it may be asserted.** First, ϵ is the mass dimension. Paper 116 proved that under the one-parameter patch $G = a$, $\hbar = 1/a$ one has $X = \hbar^p G^q c^N = a^{-\mu} c^N$, that is, $\epsilon = \mu$ with μ the mass dimension. This was verified here for all nine quantities: length and time have $\mu = 0$ hence $\epsilon = 0$; mass, energy, temperature, force, power, density and pressure have $\mu = 1$ hence $\epsilon = 1$. **The words “geometric” and “material” were another name for $\mu = 0$ and $\mu = 1$.** The second edition's Theorem 3.1 ($\epsilon = 0$ iff the G -dependence runs only through $\hbar G$) is correct, but $\epsilon = \mu$ is stronger and derives it (Paper 116). **Second, and the most important correction here: “the power of 4π is confined to zero or one and never reaches two” is a property of the list, not of the patch. The counterexample sits inside this paper's own proof — the basic combination $\hbar/G = \frac{1}{16\pi^2}$ used in proving Theorem 2.1 becomes a^{-2} under the one-parameter form, that is $\epsilon = 2$ (verified here symbolically), because the mass dimension of $\hbar/G$ is 2. The correct statement is therefore “no quantity of mass dimension 2 or more occurs in this paper's list”, not “ 4π never reaches the second power”. The reason the second edition gave — the “cancellation chain” $\hbar G^2 = (\hbar G)G$ — is an explanation confined to that list. Third, the only thing fixing $a = 4\pi$ is item (x), the Planck charge, and the 4π there is an electromagnetic unit convention.** In SI (rationalised) units $q_P = \sqrt{4\pi\epsilon_0\hbar c} = 2\sqrt{\pi} c/\sqrt{a}$, a pure power of c precisely when $a = 4\pi$. But in Gaussian units the definition carries no 4π and $q_P = \sqrt{\hbar c} = \sqrt{c}/\sqrt{a}$, a pure power precisely when $a = 1$ (verified here). **Since which value looks “forced” depends on how electromagnetism is written, it is a convention and not a fact** (Papers 95, 116). Item (x) moreover rests on $\epsilon_0 = c$ (Papers 13, 14), and **Paper 14's third edition has retracted both ends of that switch. Item (x) is therefore held in abeyance here and removed from the grounds of the dichotomy.** Fourth, $\frac{8\pi G}{c^4} = \frac{32\pi^2}{c^4}$ is a product of a fact and a convention. The 8π is a number belonging to Einstein's equation (twice Poisson's 4π ; a fact) and the 4π is the patch (a convention); in general it reads $\frac{8\pi a}{c^4}$. Hence $32\pi^2$ cannot be called “the same geometric quantity” as the $32\pi^2$ of Papers 32 and 48 (Papers 35 and 43, 3rd eds.). **Fifth, the generation of the hierarchy is correct — $P_{P1} = E_P/l_P^3$, $F_P = E_P/l_P$ and $P_{\text{power}} = E_P/t_P$ were each verified symbolically. But that is dimensional analysis, not evidence for a hierarchy.** **Scope:** structural interpretation and self-audit of elementary established facts (Layer 2). No new theorem and no prediction. **The exponent of c is not read as a spatial dimension** (the second edition's self-limitation is kept), **4π is not treated as a forced value**, and the “cosmic OS” hierarchy reading is **quarantined under a non-scientific label**.

Standing of this paper (discipline): the mathematics used here is **entirely known and elementary** — the definition of the Planck units, dimensional analysis, symbolic computation. From within this series it uses **Paper 116's theorem $\epsilon = \mu$** and **the non-uniqueness of 4π** , **Paper 35's third-edition separation of fact from convention**, and **Paper 95 on convention and fact. Not one new theorem is proved.** The third edition's contribution is **a reason and a correction of range:** (i) identifying ϵ as the **mass dimension**, (ii) showing that “ $\epsilon \in \{0, 1\}$ ” is **a property of the list and not of the patch**, by way of a **counterexample inside this paper's own proof**, (iii) showing that what fixes $a = 4\pi$ is **an electromagnetic unit convention**, (iv) pointing out that **$32\pi^2$ is fact $\times$ convention**, and (v) restating the scope. **The most important restraints: the exponent of c is not read as a spatial dimension** (the second edition already self-limited here and this edition keeps it). **4π is not called a forced value** (it depends on how electromagnetism is written; Papers 95, 116). **A factual π and a conventional π are not gathered into one** (Paper 35, 3rd ed.). **Results resting on $\epsilon_0 = c$ are not treated as settled** (Paper 14's third edition retracted both ends). The cosmic-OS hierarchy reading (c^5 as a 5D bulk, c^6 as a 6D phase space, c^7 as a 7D stress capacity) is **Layer 3, and every one of those readings has been retracted in Papers 34, 42 and 43.** It is not placed in the body of this edition.

1. Introduction: The Paper Held a Dichotomy and Did Not Hold Its Reason

The second edition showed that under the Yamagishi patch every Planck quantity closes into two forms.

Class	Members	Form
Geometric	length, time	c^N (no 4π)
Material	mass, energy, temperature, force, power, density, pressure	$\frac{c^N}{4\pi}$

The dichotomy holds, and the third edition does not alter it.

But the second edition stated the reason for the branching as **a procedure** — “whether the G -dependence runs through $\hbar G$ ”. **Correct, but not a name.** And it stated “the power of 4π never reaches two” **as a theorem.**

The third edition supplies the name and corrects the range of that theorem.

2. ϵ Is the Mass Dimension

Established Fact 2.1 (Paper 116's theorem: $\epsilon = \mu$)

Under the one-parameter patch $G = a$, $\hbar = 1/a$ with arbitrary $a > 0$,

$$X = \hbar^p G^q c^N = a^{q-p} c^N = a^{-\mu} c^N.$$

That is, **the power of the patch constant carried by X is fixed by the mass dimension μ of X .** In this paper's notation, $\epsilon_X = \mu_X$.

Applied to the nine quantities (verified symbolically):

Quantity	One-parameter form	ϵ	μ	Agree
Planck length l_P	$c^{-3/2}$	0	0	yes
Planck time t_P	$c^{-5/2}$	0	0	yes
Planck mass m_P	$\sqrt{c}/a$	1	1	yes
Planck energy E_P	$c^{5/2}/a$	1	1	yes
Planck temperature T_P	$c^{5/2}/a$	1	1	yes
Planck force F_P	c^4/a	1	1	yes
Planck power	c^5/a	1	1	yes
Planck density ρ_P	c^5/a	1	1	yes
Planck pressure P_{P1}	c^7/a	1	1	yes

All nine agree.

Observation 2.1 (“Geometric / material” was another name for mass dimension)

What decides whether a 4π appears is **not whether the quantity feels geometric or material, but a number: the mass dimension**. Length and time have $\mu = 0$, so nothing appears. Mass through pressure have $\mu = 1$, so one factor appears. **The second edition's Theorem 3.1 is correct**, since “the G -dependence runs only through $\hbar G$ ” and “ $\mu = 0$ ” are equivalent. **But $\epsilon = \mu$ is stronger**: it states the matter as **a property of the quantity itself** rather than of a procedure, and it **derives** the second edition's dichotomy. **This paper held a dichotomy and did not hold its name**. The name is in Paper 116, written later.

3. “Zero or One” Is a Property of the List

The second edition's Theorem 2.1 stated: “**the power of 4π is confined to zero or one, and the second power never appears**”.

The counterexample sits inside this paper's own proof.

Theorem 3.1 (Quantities with $\epsilon \geq 2$ exist)

Writing the basic combinations used in proving Theorem 2.1 in one-parameter form gives, by symbolic computation,

$$\hbar G = 1 \ (\epsilon = 0), \quad \hbar G^2 = a \ (\epsilon = -1), \quad \boxed{\frac{\hbar}{G} = a^{-2} \ (\epsilon = 2)}$$

and at $a = 4\pi$ one has $\hbar/G = \frac{1}{16\pi^2}$. The mass dimension of $\hbar/G$ is 2, and $\epsilon = \mu$ gives $\epsilon = 2$ directly.

Observation 3.1 (Correction: narrowing the statement of Theorem 2.1)

That the power of 4π does not reach two is not a property of the patch. Form a quantity of mass dimension 2 and $(4\pi)^2$ appears — and the second edition's proof was using exactly such a quantity along the way. The correct statement is: No quantity of mass dimension 2 or more occurs in this paper's list. The reason the second edition gave — the cancellation chain $\hbar G^2 = (\hbar G)G$ — is correct for each item on that list but does not guarantee for a general quantity that the second power cannot occur. This is not a retraction but a statement of range. For the nine listed quantities the second edition's conclusion stands unchanged. What changed is the universal phrasing “never appears” (the discipline of Paper 17: saying “distinct roots” is only half; the address must be written before the claim becomes checkable).

4. What Fixes $a = 4\pi$ Is an Electromagnetic Unit Convention

Paper 116 showed that the patch constant may be any $a > 0$. What, then, inside this paper makes $a = 4\pi$ the choice?

Item (x), the Planck charge, and nothing else.

Theorem 4.1 (Which a looks “forced” depends on the electromagnetic units)

With $\epsilon_0 = c$ and $\hbar = 1/a$ (verified here symbolically),

$$\text{SI (rationalised): } q_P = \sqrt{4\pi\epsilon_0\hbar c} = \frac{2\sqrt{\pi}c}{\sqrt{a}} \implies \text{a pure power of } c \iff a = 4\pi,$$

$$\text{Gaussian: } q_P = \sqrt{\hbar c} = \frac{\sqrt{c}}{\sqrt{a}} \implies \text{a pure power of } c \iff a = 1.$$

Observation 4.1 (Hence it is a convention)

The 4π inside the SI definition of $\mathbf{q_P}$ is placed there by hand, **for the sake of rationalisation — a way of writing the units**. Gaussian units carry no such 4π in the definition, and **the same demand that the result be a pure power selects $a = 1$. Which value looks forced depends on how electromagnetism is written.** Then it is not a fact but a convention (Paper 95). Item (x) has a second problem as well. It rests on $\epsilon_0 = c$ (the electromagnetic patch of Papers 13 and 14), but **Paper 14's third edition concluded that the switch itself survives while its two ends do not, and $\epsilon_0 = c$ is one of those ends. The third edition therefore holds item (x) in abeyance and removes it from the grounds of the dichotomy.** That is why the table in Section 2 closes over nine quantities. **For those nine, $\epsilon = \mu$ has been verified, and the dichotomy stands without the charge.**

5. $32\pi^2$ Is a Product of a Fact and a Convention

The second edition wrote the coupling constant of Einstein's equation as $\frac{8\pi G}{c^4} = \frac{32\pi^2}{c^4}$ and stated that **this is “the same geometric quantity” as the $32\pi^2$ of Papers 32 and 48.**

Observation 5.1 (The identification does not hold)

In general it reads $\frac{8\pi G}{c^4} = \frac{8\pi a}{c^4}$. The breakdown:

8π	Fact (a number belonging to Einstein's equation; twice Poisson's 4π)
a	Convention (the patch constant; any positive value will do)

Only on setting $a = 4\pi$ does one get $8\pi \times 4\pi = 32\pi^2 = 2(4\pi)^2$ (verified here). **But that is a fact multiplied by a convention. In the resulting number no record survives of which π came from where.** This is the same shape Paper 43's third edition exhibited, and it violates the discipline established by Paper 35's third edition: **the 4π of Poisson's equation is a fact and the 4π of the G -patch a convention, and the two must not be gathered into one. The claim that this is the same geometric quantity as in Papers 32 and 48 is therefore retracted. Beyond the two coming out equal after substitution, this paper holds no ground for it.**

6. The Generation of the Hierarchy Is Correct — but It Is Dimensional Analysis

Theorem 6.1 (Generation from two seeds)

By symbolic computation, each of the following holds exactly (verified here):

$$F_P = \frac{E_P}{l_P} = \frac{c^4}{a}, \quad P_{\text{power}} = \frac{E_P}{t_P} = \frac{c^5}{a}, \quad P_{\text{PI}} = \frac{E_P}{l_P^3} = \frac{c^7}{a}.$$

Observation 6.1 (That it generates is not evidence of a hierarchy)

The second edition called this generation “**evidence that the table is not a mere juxtaposition of numbers but carries structure**”. **The algebra is right. But the strength of the evidence is limited.** E_P , l_P and t_P are all built from $\hbar$, G and c , and **the dimensions of force, power and pressure are fixed uniquely by products and quotients of them**. The generation therefore succeeds **because dimensional analysis is at work**, not because a hierarchy exists. **The second edition itself drew this distinction — “since the exponent of c is fixed uniquely by dimensional analysis, the value of the exponent cannot itself be evidence for the hierarchy reading”.** That self-limitation is correct and is kept unchanged.

7. The Scope, Restated

Item	What it is
Nine quantities close into c^N or c^N/a	Fact (verified; unchanged from the second edition)
The rule deciding $\epsilon = 0$ or 1	Mass dimension (Paper 116, $\epsilon = \mu$)
The words “geometric / material”	Another name for $\mu = 0$ / $\mu = 1$
That the power of 4π never reaches 2	A property of the list ($\hbar/G$ has $\epsilon = 2$)
The value $a = 4\pi$	Convention (depends on the electromagnetic units)
Item (x), the Planck charge	Held in abeyance ($\epsilon_0 = c$; Paper 14, 3rd ed.)
$32\pi^2$ identical with Papers 32 and 48	Retracted (fact $\times$ convention; Papers 35, 43)
Generation of the hierarchy (E_P/l_P^3 etc.)	Dimensional analysis (correct, but not evidence)
That the exponent of c means a spatial dimension	Not shown (the self-limitation is kept)
c^5 as a 5D bulk, c^6 as a 6D phase space	Retracted (Papers 34, 42, 43; removed from the body)

Of the ten rows, one is a fact proper to this paper — that the nine quantities close into two forms. The rest are consequences of other papers' theorems, conventions, items held in abeyance, retractions, or “not shown”.

This is not a shrinking but an addressing (Paper 17).

8. Conclusion

This paper held a dichotomy. **The dichotomy is correct.**

Three things changed. **Its name:** ϵ was the mass dimension. **Its range:** “never reaches the second power” was not a property of the patch but the property that no quantity of mass dimension 2 or more appears in the list. And **the standing of the 4π :** what made $a = 4\pi$ the choice was a single item, the charge, and the 4π there was placed by hand by a rationalised way of writing electromagnetism.

Write it in Gaussian units and the same demand selects $a = 1$.

What changes its answer when the writer changes is not a fact.

What is not claimed (restraint): the exponent of c is not read as a spatial dimension (the second edition's self-limitation is kept) / 4π is not called a forced value (it depends on the electromagnetic units; Papers 95, 116) / “ 4π never reaches the second power” is not asserted as a general proposition ($\hbar/G$ is a counterexample) / a factual π and a conventional π are not gathered into one (Papers 35, 43, 3rd eds.) / $32\pi^2$ is not identified with the quantity in Papers 32 and 48 / results resting on $\epsilon_0 = c$ are not treated as settled (Paper 14, 3rd ed.) / the generation of the hierarchy is not treated as evidence that the hierarchy is real (it is dimensional analysis) / the cosmic-OS hierarchy reading is not placed in the body (Layer 3, quarantined; the c^5 and c^6 readings were retracted in Papers 34, 42, 43) / no new mathematics and no prediction (Layer 2).

Revision Record

Third edition (August 2026): the dichotomy is unchanged. The content of Theorem 2.1 — that the nine quantities close into c^N or $c^N/(4\pi)$ — stands as before (re-verified symbolically). What is added is **a reason and a correction of range**. (i) ϵ was identified as **the mass dimension**, by Paper 116's theorem $\epsilon = \mu$, and confirmed for all nine quantities — “geometric / material” is another name for $\mu = 0$ / $\mu = 1$. The second edition's Theorem 3.1 (whether the G -dependence runs through $\hbar G$) is correct, but $\epsilon = \mu$ is stronger and derives it. (ii) **The statement of Theorem 2.1 was narrowed:** “the power of 4π never reaches two” is **a property of the list, not of the patch**. The counterexample lies inside the second edition's own proof, since $\hbar/G = a^{-2}$ has $\epsilon = 2$ (mass dimension 2). The correct statement is “no quantity of mass dimension 2 or more occurs in this paper's list”. (iii) It was shown that **the only thing fixing $a = 4\pi$ was item (x), the Planck charge, and that this is a convention:** in SI (rationalised) units $q_P = 2\sqrt{\pi}c/\sqrt{a}$ makes $a = 4\pi$ the pure-power choice, while in Gaussian units $q_P = \sqrt{c}/\sqrt{a}$ makes it $a = 1$. **Since which value looks forced depends on how electromagnetism is written, it is not a fact** (Papers 95, 116). Item (x) further rests on $\epsilon_0 = c$, and since **Paper 14's third edition retracted both ends of that switch**, it is **held in abeyance** here and removed from the grounds of the dichotomy. (iv) **The identification of $\frac{8\pi G}{c^4} = \frac{32\pi^2}{c^4}$ with the $32\pi^2$ of Papers 32 and 48 was retracted:** the 8π is a **fact** (a number belonging to Einstein's equation) and the a is **a convention**, the general form being $8\pi a/c^4$. Once multiplied, no record of origin survives (Papers 35, 43, 3rd eds.). (v) The generation of the hierarchy ($F_P = E_P/l_P$, $P_{\text{power}} = E_P/t_P$, $P_{\text{Pl}} = E_P/l_P^3$) was re-verified symbolically, and it was made explicit that **this is dimensional analysis and not evidence that the hierarchy is real** (keeping the second edition's self-limitation). (vi) The cosmic-OS hierarchy reading (c^5 as a 5D bulk, c^6 as a 6D phase space, c^7 as a 7D stress capacity) was **removed from the body**: it is Layer 3, and each of those readings has been retracted in Papers 34, 42 and 43. (vii) The bibliography was updated to current titles. **The second edition's correction of the fine-structure constant ($\alpha = e^2/c^2$) is kept, but since it too rests on $\epsilon_0 = c$ it is held in abeyance alongside item (x).**

References

1. Planck, M. (1899). *Über irreversible Strahlungsvorgänge*. Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften zu Berlin.
2. Einstein, A. (1915). *Die Feldgleichungen der Gravitation*. Sitzungsberichte der Preußischen Akademie der Wissenschaften.
3. Jackson, J. D. (1998). *Classical Electrodynamics*, 3rd ed. Wiley. (rationalised versus Gaussian units)
4. Bridgman, P. W. (1922). *Dimensional Analysis*. Yale University Press.
5. Yamagishi, Y. (2026). *What Does the Exponent of c Count?* [Paper 116]. Zenodo. <https://doi.org/10.5281/zenodo.22003794> (the theorem $\epsilon = \mu$; the non-uniqueness of 4π ; the self-audit of item (x) here)
6. Yamagishi, Y. (2026). *Convention and Fact* [Paper 95]. Zenodo. <https://doi.org/10.5281/zenodo.21758621>
7. Yamagishi, Y. (2026). *The Yamagishi Gravitational Constant Unpacking Theory* [Paper 35], 3rd ed. Zenodo. (the 4π of Poisson is a fact; the 4π of the G -patch is a convention). Zenodo. <https://doi.org/10.5281/zenodo.21430118>
8. Yamagishi, Y. (2026). *The Two Patches Are Not the Same Kind of Thing* [Paper 14], 3rd ed. Zenodo. <https://doi.org/10.5281/zenodo.21600268> / *A Convention Is a Number, Not a Function of the Configuration* [Paper 15], 3rd ed. Zenodo. <https://doi.org/10.5281/zenodo.21600352>
9. Yamagishi, Y. (2026). *The Four Planck Quantities Are Four Instances of One Rule* [Paper 40], 3rd ed. Zenodo. <https://doi.org/10.5281/zenodo.21435739>
10. Yamagishi, Y. (2026). *The G and the c^5 of the Quadrupole Formula Cannot Be Separated* [Paper 42], 3rd ed. Zenodo. <https://doi.org/10.5281/zenodo.21601988>
11. Yamagishi, Y. (2026). *Keep the Patch Constant as a Letter and the Ambiguity Dissolves* [Paper 43], 3rd ed. Zenodo. <https://doi.org/10.5281/zenodo.21602132>
12. Yamagishi, Y. (2026). *A Census of the c^5 s* [Paper 34], 3rd ed. Zenodo. <https://doi.org/10.5281/zenodo.21430340>
13. Yamagishi, Y. (2026). *The Yamagishi Planck Length Unpacking Theory ($l_p = c^{-3/2}$)* [Paper 39], 3rd ed. Zenodo. <https://doi.org/10.5281/zenodo.21430490> / *The Yamagishi Master Table of Classical-Physics Geometric Unpacking* [Paper 38], 3rd ed. Zenodo. <https://doi.org/10.5281/zenodo.21430234>
14. Yamagishi, Y. (2026). *The 8π Is an Artefact of the Units* [Paper 17], 3rd ed. Zenodo. <https://doi.org/10.5281/zenodo.21600813>

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