

Add Too Many Workers and the Workers Start Leaving

— Raising the sludge concentration from **1000** to **8000** mg/L raises removal monotonically from **0.4000** to **0.8421**, yet effective performance peaks at **5000** mg/L with **66.92** and falls to **62.21** at **8000** — washout begins at **2500** mg/L — [Paper 447]

Yuuki YAMAGISHI (山岸 勇輝)

Yamagishi Complexity Torus Lab, Japan

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— *From thinking to syncing.* —

Abstract

To treat better, add more workers—so it is thought. This paper shows that the added workers flow out of the exit—so performance is not monotone but has an interior peak. No new theorem or law is claimed. First, removal rises monotonically from **0.4000** to **0.8421** between **1000** and **8000** mg/L (Section 2). Second, but washout begins above **2500** mg/L (Section 3). Third, this is the core of the paper. Effective performance peaks at **5000** mg/L with **66.92** and falls to **62.21** at **8000** (Section 3). Fourth, the peak is not at the point of maximum removal (Section 3). Fifth, the boundary is the **2500** mg/L at which washout starts (Section 3). Sixth, the separator is whether the exit passes the workers (Section 4). **Keywords:** activated sludge process, wastewater treatment, sludge concentration, settling tank, solid-liquid separation.

Scope of this paper (scope note): No new theorem or law is claimed—the activated sludge process, mixed liquor concentration and solid-liquid separation in a clarifier are all standard. No plant is designed—eration rate, return ratio and clarifier area are not the subject. No microbiology is argued—community composition, growth rate and sludge age are not the subject. The numbers are a model—both the removal curve and the settling limit are simple forms chosen to bring out the trend, not measurements. No regulation is discussed—effluent standards and statutes are not the subject. Relation to earlier papers: Paper 446 showed that a theoretical floor moves with the demand—there too, a condition hid behind an apparently monotone quantity. Paper 300 showed that common root and distinct root can be decided—“removal” and “effective performance” are distinct roots by that criterion. Paper 417 showed that a ceiling moves with the requirement—there too, the operation meant to improve things ran into a different limit. What is added is tabulating removal and washout at five concentrations, showing the effective peak at **5000** mg/L, giving the onset of washout as **2500** mg/L, showing that the peak does not coincide with maximum removal, and placing the separator at whether the exit passes the workers.

1. Notation and the Question

Setting: Let the sludge concentration in the aeration tank be X [mg/L]. Removal η rises with saturation.

$$\eta(X) = \frac{X}{X + X_0}$$

The clarifier can hold only up to X_c ; the excess is washed out.

$$L(X) = \max(0, a(X - X_c))$$

Effective performance subtracts what is lost from what is removed.

$$P(X) = 100\eta(X) - L(X)$$

Below, $X_0 = 1500$ mg/L, $X_c = 2500$ mg/L and $a = 0.004$.

Observation 1.1

That more workers means better treatment is true. η is increasing in X —there is no lie in that part.

Observation 1.2

But the workers can also leave through the exit. Once the clarifier cannot hold them, the sludge itself washes out—what was added turns straight into a loss.

Question: Put the two together, and where is the best point.

2. Removal Rises Monotonically

Computation 2.1: Take removal at five concentrations.

Concentration [mg/L]	Removal
1000	0.4000
2000	0.5714
3000	0.6667
5000	0.7692
8000	0.8421

Observation 2.2

It rises monotonically. From 0.4000 to 0.8421—an eightfold concentration buys a factor of 2.11.

Observation 2.3

But the gain slows. +0.1714 from 1000 to 2000, +0.0729 from 5000 to 8000—saturation.

Observation 2.4

On this table alone, thicker is always better. There is no peak—so the conclusion is “thicker still”.

3. Effective Performance Peaks at 5000 mg/L

Computation 3.1: Add the washout and take the effective performance.

Concentration [mg/L]	Washout	Effective
1000	0	40.0000
2000	0	57.1429
3000	2	64.6667
5000	10	66.9231
8000	22	62.2105

Observation 3.2

The peak is at 5000 mg/L.66.9231—beyond it, the curve falls.

Observation 3.3

At 8000 it drops to 62.2105.Even though removal is at its maximum 0.8421—the operating point that removes best is not the best operating point.

Observation 3.4

64.6667 at 3000 beats 62.2105 at 8000.A 2.67-fold concentration, and performance falls—the direction of increase is not always right.

Observation 3.5

The boundary is 2500 mg/L.No washout up to there, washout beyond—a line where the character changes lies between 2000 and 3000.

Observation 3.6

Note that the peak is at 5000, not at 2500.For a while after washout begins, the gain in removal still wins—the boundary and the optimum are different places.

4. The Separator

The separator: does the exit pass the workers or hold them.

Observation 4.1

If it holds them, more is always better. Because η is monotone—there is no peak, and the ceiling is set by something else.

Observation 4.2

If it passes them, a peak appears. Because added workers turn into losses—the optimum is interior.

Observation 4.3

These are distinct roots by the criterion of Paper 300. Both are called “treatment performance”—one a property of the tank, one of the tank paired with the clarifier.

Observation 4.4

The same shape as Paper 446. There, raising the recovery raised the theoretical floor—here, raising the concentration defeats the exit. In both, the improving operation rebounded on the other side.

Observation 4.5

So when reinforcement is discussed, look at the exit first. Investing in more workers pays only within what the exit can hold—miss where the bottleneck moved to, and only the cost rises.

Observation 4.6

To be honest—the numbers are a model. X_0 , X_c and a are chosen, not measured—the location of the peak differs by plant. What does not differ is that a peak exists.

Observation 4.7

One last thing—enlarge the clarifier and X_c rises, moving the peak to the right. The peak is not at a fixed place—it is a number for deciding which side to fix.

5. Roll Call and Closing

1. Removal from 0.4000 to 0.8421 (Computation 2.1): **monotone.**
2. The gain slows (Observation 2.3): **saturation.**
3. The table alone says “thicker” (Observation 2.4): **no peak in view.**
4. Washout from 2500 mg/L (Observation 3.5): **the boundary.**
5. The effective peak is 66.92 at 5000 (Computation 3.1): **the core.**
6. 62.21 at 8000 (Observation 3.3): **best removal is not best.**
7. 3000 beats 8000 (Observation 3.4): **increase can be wrong.**
8. Boundary and optimum differ (Observation 3.6): **2500 and 5000.**
9. The separator is the exit (Observation 4.3): **distinct roots by Paper 300.**
10. A bigger clarifier moves the peak right (Observation 4.7): **which side to fix.**

Closing: To treat better, add more workers—so it is thought—and on removal alone that is right. Raising the sludge concentration from 1000 to 8000 mg/L raises removal monotonically from 0.4000 to 0.8421. The gain slows, but no peak appears—+0.1714 from 1000 to 2000 against +0.0729 from 5000 to 8000 is saturation, not decline. On this table alone the conclusion is “thicker still”. But the workers can also leave through the exit—the clarifier holds only so much, and above 2500 mg/L the sludge itself washes out, so what was added turns straight into a loss.

Subtracting the loss gives 40.00, 57.14, 64.67, 66.92 and 62.21, with the peak at 5000 mg/L. At 8000 removal is at its maximum 0.8421 and performance is down at 62.21—the operating point that removes best is not the best operating point. 64.67 at 3000 beats 8000 outright: a 2.67-fold concentration, and performance falls. And the boundary at 2500 is not the optimum at 5000—for a while after washout begins, the gain in removal still wins. One thing separates them—does the exit pass the workers or hold them. If it holds them, η is monotone and more is always better, with no peak. If it passes them, added workers turn into losses and the optimum is interior. They are distinct roots by the criterion of Paper 300, one a property of the tank and one of the tank paired with the clarifier. It is the same shape as Paper 446, where raising the recovery raised the theoretical floor: in both, the improving operation rebounded on the other side. So when reinforcement is discussed, look at the exit first—miss where the bottleneck moved to, and only the cost rises. To be honest—the numbers are a model, with X_0 , X_c and a chosen rather than measured. The location of the peak differs by plant; what does not differ is that a peak exists. One last

thing—enlarge the clarifier and the holding limit rises, moving the peak to the right. The peak is not at a fixed place: it is a number for deciding which side to fix.

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Yuuki YAMAGISHI (山岸 勇輝)

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