

On Ice, Driving Slowly Can Be More Dangerous Than Driving Fast

— A curve of radius **300** m with **6** per cent superelevation admits **0** to **89.89** km/h when dry, but **33.79** to **58.63** when iced — the lower bound rises from zero to **33.79** and the admissible band narrows to **24.84** km/h

— [Paper 453]

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September 2026

— *From thinking to syncing.* —

Abstract

On ice, going slowly is taken to be safe. This paper shows that once the friction falls below the superelevation, a lower bound on speed appears—so going too slowly slides one down the inside. No new theorem or law is claimed. First, the frictionless balance speed is **47.83** km/h (Section 2). Second, on dry road (friction 0.15) anything from **0** to **89.89** km/h is admissible (Section 2). Third, this is the core of the paper. On ice (friction 0.03) the lower bound rises to **33.79** km/h (Section 3). Fourth, the upper bound is then **58.63** km/h, leaving a band of only **24.84** (Section 3). Fifth, the boundary is where friction exactly equals the superelevation, and there the lower bound returns to zero (Section 3). Sixth, the separator is whether there is only a ceiling or a band on both sides (Section 4). **Keywords:** banked curve, cornering, icy road, friction coefficient, vehicle dynamics.

Scope of this paper (scope note): No new theorem or law is claimed—the force balance on a superelevated curve is entirely standard. No road is designed—design standards, transition curves and sight distance are not the subject. No driving instruction is given—nothing here recommends a manoeuvre; a hazardous situation calls for judgement, stopping included. Friction is not measured—0.15 and 0.03 are representative, and real surfaces vary greatly with place and hour. Vehicles are not distinguished—centre of gravity, load transfer, rollover and tyre behaviour are not the subject; a point mass is used. Wind and grade are excluded—crosswind, longitudinal grade and surface irregularity are not the subject. Relation to earlier papers: Paper 452 showed that what lifts is a difference in surface tension—there too, changing the condition removed the phenomenon entirely. Paper 300 showed that common root and distinct root can be decided—“a band with only a ceiling” and “a band bounded on both sides” are distinct roots by that criterion. Paper 447 showed that the optimum is interior—there too, looking at one side alone misled. What is added is giving the frictionless balance as 47.83 km/h, giving the dry ceiling as 89.89, giving the iced floor as 33.79, giving the band width as 24.84, placing friction equal to superelevation as the boundary, and placing the separator at one-sided versus two-sided.

1. Notation and the Question

Setting: Traverse a curve of radius R and superelevation e (the tangent of the cross slope) with a lateral friction coefficient f . The ceiling above which the vehicle slides outward is

$$v_{\max} = \sqrt{\frac{gR(e+f)}{1-ef}}$$

and the floor below which it slides down the inside is

$$v_{\min} = \sqrt{\frac{gR(e-f)}{1+ef}}$$

with the radicand negative—so no floor exists, that is, the floor is zero—whenever $e \leq f$. Below, $R = 300$ m and $e = 0.06$.

Observation 1.1

On ice, going slowly is taken to be safe. Lower speed means less centrifugal force—and looking at the ceiling alone, that is right.

Observation 1.2

But the curve is banked. Standing still, the vehicle slides down that bank towards the inside—what holds it is friction.

Question: When the friction falls, what happens to the admissible band.

2. On Dry Road There Is No Floor

Computation 2.1: Take the speed that balances without any friction.

$v = 13.2861$ m/s, that is **47.8299** km/h.

Observation 2.2

This is the design speed of the curve. At it, the tyres need exert no lateral force at all—the superelevation alone balances.

Computation 2.3: Take the admissible band on dry road ($f = 0.15$).

Surface	Floor [km/h]	Ceiling [km/h]
Dry ($f = 0.15$)	0.0000	89.8869

Observation 2.4

The floor is zero. Because $e = 0.06$ is below $f = 0.15$ —even at rest nothing slides.

Observation 2.5

So on dry road “slower is safer” holds. With no floor, slowing always adds margin—and that experience is carried straight onto ice.

3. On Ice the Floor Rises to 33.79 km/h

Computation 3.1: Take the admissible band on ice ($f = 0.03$).

Surface	Floor [km/h]	Ceiling [km/h]	Width [km/h]
Dry ($f = 0.15$)	0.0000	89.8869	89.8869
Iced ($f = 0.03$)	33.7904	58.6322	24.8417

Observation 3.2

The floor has risen from zero to 33.79 km/h. Because $f = 0.03$ has fallen below $e = 0.06$ —friction can no longer hold against the bank.

Observation 3.3

The band narrows to **24.84** km/h. From 89.89 when dry—a factor of 3.62.

Observation 3.4

At 30 km/h one is below the floor. Slowing down carefully lands one on the sliding side—“slower is safer” inverts here.

Observation 3.5

The boundary is $f = e = 0.06$. Exactly there, the floor returns to zero—whether friction exceeds the superelevation decides whether the phenomenon exists.

Observation 3.6

And the ceiling has fallen too, from 89.89 to 58.63. Squeezed from both sides—too fast and too slow are both dangerous.

4. The Separator

The separator: is there only a ceiling, or a band on both sides.

Observation 4.1

With only a ceiling, slowing is always the safe direction. A monotone relation—no judgement is required.

Observation 4.2

With a band, slowing too far fails on the other side. The best point is interior—without knowing which side one is on, the direction to move is undetermined.

Observation 4.3

These are distinct roots by the criterion of Paper 300. Both are called “the safe speed range”—one a constraint from one side, one from both.

Observation 4.4

The same shape as Paper 447. There, thickening the sludge too far cost effective performance—here, slowing too far slides one down. In both, the improving direction had a limit.

Observation 4.5

It overlaps Paper 452 as well. There, a zero difference removed the phenomenon entirely—here, friction above the superelevation removes the floor entirely. Both exist only on one side of a boundary.

Observation 4.6

So on an iced curve with strong superelevation, staying near the design speed is closest to balance. 47.83 km/h is the point that uses no friction at all—though this is a statement about force balance; real driving calls for judgement, stopping included.

Observation 4.7

To be honest—a point mass has been used. Centre of gravity, load transfer, rollover and tyre behaviour are not included—the conclusion that a floor appears is unchanged, but the real limit arrives sooner than this.

5. Roll Call and Closing

1. The balance speed is **47.8299 km/h** (Computation 2.1): **no friction used.**
2. The dry ceiling is **89.8869** (Computation 2.3): **ample margin.**
3. The dry floor is zero (Observation 2.4): **because $e < f$.**
4. The iced floor is **33.7904** (Computation 3.1): **the core.**
5. The iced ceiling is **58.6322** (Computation 3.1): **the top falls too.**
6. The band is **24.8417** (Computation 3.1): **a factor of 3.62.**
7. **30 km/h** is below the floor (Observation 3.4): **caution inverts.**
8. The boundary is **$f = e = 0.06$** (Observation 3.5): **there the floor vanishes.**
9. The separator is one side or two (Observation 4.3): **distinct roots by Paper 300.**
10. A point mass was used (Observation 4.7): **the real limit is sooner.**

Closing: On ice, going slowly is taken to be safe—lower speed means less centrifugal force, and looking at the ceiling alone that is right. But the curve is banked, and standing still the vehicle slides down that bank towards the inside. What holds it is friction. On dry road that hold is ample—a curve of radius 300 m with 6 per cent superelevation balances without friction at **47.8299 km/h**, where the tyres need exert no lateral force at all. With friction 0.15 the admissible band runs from **0** to **89.8869 km/h** and the floor is zero, because $e = 0.06$ is below $f = 0.15$ and even at rest nothing slides. So on dry road “slower is safer” holds—and that experience is carried straight onto ice. On ice the floor rises—with friction 0.03 the band runs from **33.7904** to **58.6322 km/h**, a width of only **24.8417**: a factor of 3.62 narrower than dry. Because $f = 0.03$ has fallen below $e = 0.06$, friction can no longer hold against the bank. So at 30 km/h one is below the floor—slowing down carefully lands one on the sliding side, and “slower is safer” inverts here. The boundary is **$f = e = 0.06$** , where the floor returns to zero: whether friction exceeds the superelevation decides whether the phenomenon exists at all. And the ceiling has fallen too, from 89.89 to 58.63, so the vehicle is squeezed from both sides. One thing separates them—is there only a ceiling, or a band on both sides. With only a ceiling, slowing is always the safe direction, and a monotone relation requires no judgement. With a band, slowing too far fails on the other side; the best point is interior, and without knowing which side one is on the direction to move is undetermined. They are distinct roots by the criterion of Paper 300, one a constraint from one side and one from both. It is the same shape as Paper 447, where thickening the sludge too far cost effective

performance: in both, the improving direction had a limit. It overlaps Paper 452 too, where a zero difference removed the phenomenon entirely: both exist only on one side of a boundary. So on an iced curve with strong superelevation, staying near the design speed is closest to balance, 47.83 km/h being the point that uses no friction at all—though this is a statement about force balance, and real driving calls for judgement, stopping included. To be honest—a point mass has been used, with centre of gravity, load transfer, rollover and tyre behaviour left out. The conclusion that a floor appears is unchanged, but the real limit arrives sooner than this.

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