

The Water Load Grows as the Square, and the Dam Only as the First Power

— Ten times the depth multiplies the resultant thrust by **100** and the overturning moment by **1000** — yet the required width only doubles — **18.6339** m at $H = 50$, **37.2678** at $H = 100$, with b/H constant at **0.3727** —

[Paper 454]

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— *From thinking to syncing.* —

Abstract

A deeper dam is thought to need a disproportionately thicker body. This paper shows that although the force grows as the square and the moment as the cube, the required width grows only as the first power—because the resisting side grows as the cube too. No new theorem or law is claimed. First, ten times the depth multiplies the thrust by **100.00000000** (Section 2). Second, the point of application is not at mid-depth but at $2H/3$ below the surface (Section 2). Third, this is the core of the paper. The overturning moment grows as the cube (**8.00000000** for twice the depth) while the required width grows only by **2.00000000** (Section 3). Fourth, the ratio b/H is constant at **0.3727**, independent of depth (Section 3). Fifth, **0.3727** is about half the practical rule of 0.7 to 0.8, the difference being the rectangular approximation (Section 4). Sixth, the separator is not which power grows, but whether both sides grow with the same power (Section 4). **Keywords:** hydrostatic pressure, gravity dam, overturning moment, centre of pressure.

Scope of this paper (scope note): No new theorem or law is claimed—hydrostatic pressure, the centre of pressure and the overturning balance of a gravity dam are all standard. No dam is designed—foundations, uplift, seismic loading and shape optimisation are not the subject, and real design always includes them. No factor of safety is used—only the exact balance point is examined; practice allows margin. The section is approximated as rectangular—real gravity dams are nearly triangular, which changes the coefficient. Sliding is excluded—only overturning is treated; sliding on the foundation is a separate condition. Arch dams are excluded—a form that carries thrust into the abutments lies outside this balance. Relation to earlier papers: Paper 453 showed that a floor on speed appears on ice—there too, looking at one side alone misled. Paper 300 showed that common root and distinct root can be decided—“how the load grows” and “how the required dimension grows” are distinct roots by that criterion. Paper 446 showed that a floor moves with the demand—there too, a ratio between two quantities took on a life of its own. What is added is giving the thrust and its point of application at three depths, checking the closed form independently by numerical integration, giving the cubic growth of the moment as 8.000000000, giving the linear growth of the width as 2.000000000, showing $b/H = 0.3727$ to be constant, and placing the separator at whether both sides share a power.

1. Notation and the Question

Setting: On a vertical wall of depth H , the resultant hydrostatic thrust per metre of width is

$$F = \frac{1}{2} \rho g H^2$$

acting at $2H/3$ below the surface, that is at $H/3$ above the base. The overturning moment about the toe is

$$M = F \cdot \frac{H}{3} = \frac{\rho g H^3}{6}$$

Approximating the body as a rectangle of density ρ_c , width b and height H , the resisting moment from its own weight is $\rho_c g b H \cdot b/2$. Below, $\rho = 1000$ and $\rho_c = 2400 \text{ kg/m}^3$.

Observation 1.1

A deeper dam is thought to need a disproportionately thicker body. The force grows as the square and the moment as the cube—so it appears.

Observation 1.2

But the resisting side also grows with depth. A taller body weighs more—what must be compared is how both sides grow.

Question: With what power of depth does the required width grow.

2. Force as the Square, Moment as the Cube

Computation 2.1: Take the thrust and its point of application at three depths.

Depth [m]	Thrust [N/m]	Application (below surface) [m]
10	4.9033×10^5	6.6667
50	1.2258×10^7	33.3333
100	4.9033×10^7	66.6667

Observation 2.2

Ten times the depth gives **100.000000000** times the thrust. Growth as the square—pressure goes with depth and so does area.

Computation 2.3: Check the closed form independently with a 200000-step numerical integration.

At $H = 50$ m, numerical 1.225831×10^7 against closed form 1.225831×10^7 ; the relative difference is below 10^{-9} .

Observation 2.4

The point of application is below mid-depth. **33.3333** m at $H = 50$, not 25—pressure is higher lower down, so the centroid is pulled downwards.

Observation 2.5

So the overturning moment grows as the cube. The thrust as the square and the lever arm as the first power — **2.0431×10^8 N m/m** at $H = 50$, and **8.000000000** times for twice the depth.

3. The Required Width Grows Only as the First Power

Computation 3.1: Take the required width from the overturning balance under the rectangular approximation.

Depth [m]	Required width [m]	b/H
50	18.6339	0.3727
100	37.2678	0.3727

Observation 3.2

Twice the depth needs **2.000000000** times the width. Though the moment went up eightfold—twice the width suffices.

Observation 3.3

Because the resisting moment grows as the cube too. The weight goes with bH and the arm with b , so the moment goes with b^2H —and with $b \propto H$ that is H^3 . Both sides share the power.

Observation 3.4

So b/H is constant with depth. **0.3727**—which is to say that dams come out geometrically similar.

Observation 3.5

Check the balance in reverse. At $H = 50$, water moment 2.043052×10^8 against weight moment 2.043052×10^8 —agreeing to within 10^{-9} relative.

Observation 3.6

The number 0.3727 is $\sqrt{\rho/(3\rho_c)}$. Set by the density ratio alone—depth and gravity both drop out.

4. The Separator

The separator: not which power grows, but whether both sides grow with the same power.

Observation 4.1

Seen on one side, a cube is alarming. A thousandfold for ten times the depth—it looks unmanageable.

Observation 4.2

If both sides share the power, the ratio does not move. The required dimension grows as the first power—scaling up similarly suffices.

Observation 4.3

These are distinct roots by the criterion of Paper 300. Both are called “how depth acts”—one the growth of a load, one the growth of a ratio of load to resistance.

Observation 4.4

The same shape as Paper 453. There, watching only the ceiling hid the floor—here, watching only the load hides the resistance. In both, a one-sided view produces the error.

Observation 4.5

0.3727 is smaller than the practical rule. Real gravity dams run to a base width of 0.7 to 0.8 times the height—because the rectangular approximation overstates the weight; a triangular section changes the coefficient.

Observation 4.6

To be honest—real design always includes uplift, seismic loading, sliding and the foundation. Every one of them pushes the width up—the 0.3727 given here is a floor, not a design value.

Observation 4.7

One last thing—the similarity holds because both sides come from the same physics. Water thrust and dead weight are both proportional to gravity—which is why g cancels, and the same 0.3727 would hold on the Moon.

5. Roll Call and Closing

1. Thrust grows as the square (Observation 2.2): **100.000000000** for tenfold.
2. Numerical matches closed form (Computation 2.3): **below 10^{-9} relative.**
3. Application at **$2H/3$** (Observation 2.4): **below mid-depth.**
4. Moment grows as the cube (Observation 2.5): **8.000000000** for double.
5. Width **18.6339 m** at $H = 50$ (Computation 3.1): **the core.**
6. **37.2678 m** at $H = 100$ (Computation 3.1): **a factor of 2.000000000.**
7. **$b/H = 0.3727$ constant** (Observation 3.4): **geometrically similar.**
8. It is $\sqrt{\rho/(3\rho_c)}$ (Observation 3.6): **the density ratio alone.**
9. The separator is a shared power (Observation 4.3): **distinct roots by Paper 300.**
10. **0.3727 is a floor, not a design** (Observation 4.6): **practice runs 0.7 to 0.8.**

Closing: A deeper dam is thought to need a disproportionately thicker body—the force grows as the square and the moment as the cube, so it appears. But the resisting side also grows with depth: a taller body weighs more, and what must be compared is how both sides grow. The load side does grow steeply—ten times the depth gives **100.000000000** times the thrust, because pressure goes with depth and so does area. A 200000-step numerical integration matches the closed form to within 10^{-9} relative. The point of application lies below mid-depth, at **33.3333 m** for $H = 50$, because pressure is higher lower down and the centroid is pulled downwards. So the overturning moment grows as the cube: 2.0431×10^8 N m/m at $H = 50$, and **8.000000000** times for twice the depth. Yet the required width grows only as the first power—**18.6339 m** at $H = 50$ and **37.2678** at $H = 100$, exactly **2.000000000** times. Because the resisting moment grows as the cube as well: the weight goes with bH and the arm with b , so the moment goes with b^2H , and with $b \propto H$ that is H^3 . Both sides share the power. So b/H is constant with depth at **0.3727**—dams come out geometrically similar. Checked in reverse, the water moment 2.043052×10^8 and the weight moment 2.043052×10^8 agree to within 10^{-9} relative. And 0.3727 is $\sqrt{\rho/(3\rho_c)}$: set by the density ratio alone, with depth and gravity both dropping out. One thing separates them—not which power grows, but whether both sides grow with the same power. Seen on one side, a cube is alarming: a thousandfold for ten times the depth looks unmanageable. If both sides share the power, the ratio does not move, the required

dimension grows as the first power, and scaling up similarly suffices. They are distinct roots by the criterion of Paper 300, one the growth of a load and one the growth of a ratio of load to resistance. It is the same shape as Paper 453, where watching only the ceiling hid the floor: in both, a one-sided view produces the error. 0.3727 is nonetheless smaller than the practical rule of 0.7 to 0.8, because the rectangular approximation overstates the weight; a triangular section changes the coefficient. To be honest—real design always includes uplift, seismic loading, sliding and the foundation, and every one of them pushes the width up. The 0.3727 given here is a floor, not a design value. One last thing—the similarity holds because both sides come from the same physics. Water thrust and dead weight are both proportional to gravity, which is why g cancels and the same 0.3727 would hold on the Moon.

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