

How Dry You Want It Decides Which Stage Is Rate-Limiting

— To a moisture content of **0.10** the falling-rate period is **2.9193** hours, shorter than the constant-rate **8.0000**; to **0.03** it is **10.4053** and longer — the crossing lies at **0.039506** — removing the same **0.001** near **0.023** costs **3.85** times what it costs near **0.030** — [Paper 455]

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— *From thinking to syncing.* —

Abstract

Drying falls into two periods, and the second is said to be the long one. This paper shows that which is longer switches with how dry one wants it—and the switch lies at a moisture content of **0.039506**. No new theorem or law is claimed. First, the constant-rate period (0.60 to 0.20) is **8.0000** hours regardless of the target (Section 2). Second, the falling-rate period moves from **2.9193** to **16.1993** hours with the target (Section 2). Third, this is the core of the paper. The crossing lies at **0.039506**, and which period is longer flips across it (Section 3). Fourth, removing the same **0.001** costs **1.4597** hours near **0.023** against **0.3793** near **0.030** (Section 3). Fifth, the equilibrium moisture content itself is never reached in finite time (Section 3). Sixth, the separator is whether the rate-limiting step is fixed or moves with the demand (Section 4). **Keywords:** drying, constant-rate period, falling-rate period, equilibrium moisture content, rate-limiting step.

Scope of this paper (scope note): No new theorem or law is claimed—the distinction between constant-rate and falling-rate drying, the equilibrium moisture content and exponential approach are all standard. No dryer is designed—air flow, temperature, circulation and heat recovery are not the subject. No material is specified—nothing here is particular to timber, food or sludge; anything describable by a moisture content is covered. The numbers are a model—the constant rate of 0.05 per hour, the critical content 0.20 and the equilibrium content 0.02 are chosen, not measured. Quality is excluded—cracking, discolouration and flavour change from over-drying are not the subject. Relation to earlier papers: Paper 454 showed that a cubic load still needs only a linear width—there too, no answer came from one side alone. Paper 300 showed that common root and distinct root can be decided—“drying limited by the constant-rate period” and “by the falling-rate period” are distinct roots by that criterion. Paper 446 showed that a floor moves with the demand—there too, moving the demand moved the limit; this is a third instance of the same shape. Paper 445 showed that the waiting is for diffusion, not reaction—there too, which of two limits decided what to do. What is added is giving the falling-rate period at five targets, giving the crossing in closed form as 0.039506, confirming the flip on both sides of it, giving the time ratio for the same increment as 3.85, and placing the separator at a fixed versus a demand-dependent limit.

1. Notation and the Question

Setting: Let W be the moisture content on a dry basis. While the surface is wet, evaporation limits and the content falls at a constant rate R .

$$\frac{dW}{dt} = -R \quad (W \geq W_c)$$

Below the critical content W_c , internal diffusion limits and the content approaches the equilibrium content W_e exponentially.

$$\frac{dW}{dt} = -R \frac{W - W_e}{W_c - W_e} \quad (W < W_c)$$

Below, $W_0 = 0.60$, $W_c = 0.20$, $W_e = 0.02$ and $R = 0.05$ per hour.

Observation 1.1

Drying falls into two periods, and the second is said to be the long one. What follows the drying of the surface takes longer—a familiar experience.

Observation 1.2

But “the second” has no length until one says where it ends. The first ends when W_c is reached—the second ends when we stop.

Question: How dry must one go before the second becomes the longer.

2. The First Period Is Fixed at Eight Hours; the Second Moves

Computation 2.1: Take the length of the constant-rate period.

$$(0.60 - 0.20)/0.05 = 8.0000 \text{ hours.}$$

Observation 2.2

It does not depend on the target. However dry one intends to go, the first 8 hours are the same—a fixed quantity.

Computation 2.3: Take the falling-rate period at five targets.

Target content	Falling-rate [h]	Constant-rate [h]
0.100	2.9193	8.0000
0.050	6.4503	8.0000
0.030	10.4053	8.0000
0.025	12.9007	8.0000
0.022	16.1993	8.0000

Observation 2.4

From 2.9193 to 16.1993, a factor of 5.55. Just from lowering the target from 0.100 to 0.022—a difference of only 0.078 in the water removed.

Observation 2.5

And at 0.100 the first period is longer, at 0.030 the second. $2.9193 < 8.0000$ against $10.4053 > 8.0000$ —which stage limits has switched with the target.

3. The Crossing Lies at 0.039506

Computation 3.1: Take the content at which the two periods are exactly equal, in closed form.

$$W_x = W_e + (W_c - W_e) e^{-Kt_{\text{const}}}, \quad K = \frac{R}{W_c - W_e}$$

$W_x = 0.039506$.

Observation 3.2

Inverting, the falling-rate period to this content is exactly 8.0000 hours. To within 10^{-9} relative—the closed form really does solve the balance.

Observation 3.3

On either side of the crossing, the order reverses. At 1.05 times W_x the first period wins, at 0.95 times the second—a boundary confirmed from both sides.

Observation 3.4

0.039506 falls between the targets 0.05 and 0.03. Practical drying targets straddle it—so the switch is not an academic one.

Observation 3.5

Removing the same amount of water takes longer the drier it is. 0.3793 hours from 0.030 to 0.029, **1.4597** from 0.023 to 0.022—a factor of **3.85**.

Observation 3.6

The equilibrium content itself is never reached in finite time. A residual of 1.554×10^{-13} at 100 hours and 1.342×10^{-25} at 200—always falling, never zero.

Observation 3.7

On a computer, though, the residual vanishes from the sum at 142 hours. Adding 10^{-18} to 0.02 changes nothing in floating point—that is a limit of precision, not of physics. I wrote the test wrongly here once.

4. The Separator

The separator: is the rate-limiting step fixed, or does it move with the demand.

Observation 4.1

If fixed, there is one thing to do. Blow air if evaporation limits, make it thinner if diffusion does—one chooses between them.

Observation 4.2

If it moves with the demand, nothing can be chosen without a target. Air for 0.10, thickness for 0.03—the same material in the same machine gives a different answer.

Observation 4.3

These are distinct roots by the criterion of Paper 300. Both are called “what limits the drying”—one a property of material and machine, one of those paired with a target.

Observation 4.4

A third instance of the shape of Paper 446 and Paper 453. There, raising the recovery raised the floor, and losing friction raised the floor on speed—here, lowering the target switches what limits. In all three, the limit was a function of the demand.

Observation 4.5

It overlaps Paper 445 as well. There, the waiting was for diffusion rather than reaction—here, which one is being waited for changes with the target. Identifying the limiting step is itself conditional work.

Observation 4.6

So when “shortening the drying time” is discussed, print the target. An improvement aimed at 0.10 does nothing at 0.03—because the limit has moved elsewhere.

Observation 4.7

To be honest—the numbers are a model. R , W_c and W_e are chosen, not measured—the location 0.039506 differs by material. What does not differ is that a crossing exists.

5. Roll Call and Closing

1. The constant-rate period is 8.0000 h (Computation 2.1): **target-independent.**
2. The falling-rate runs 2.9193 to 16.1993 (Computation 2.3): **a factor of 5.55.**
3. At 0.100 the first is longer (Observation 2.5): **2.9193 < 8.0000.**
4. At 0.030 the second is longer (Observation 2.5): **10.4053 > 8.0000.**
5. The crossing is 0.039506 (Computation 3.1): **the core.**
6. The order reverses on both sides (Observation 3.3): **a boundary confirmed.**
7. Practical targets straddle it (Observation 3.4): **not academic.**
8. 3.85 times for the same 0.001 (Observation 3.5): **drier is slower.**
9. Equilibrium is never reached (Observation 3.6): **the residual stays positive.**

10. The separator is fixed or demand-dependent (Observation 4.3): distinct roots by Paper 300.

Closing: Drying falls into two periods, and the second is said to be the long one—what follows the drying of the surface takes longer, and that experience is familiar. But “the second” has no length until one says where it ends: the first ends when the critical content is reached, the second ends when we stop. The first period is fixed— $(0.60 - 0.20)/0.05 = 8.0000$ hours, the same however dry one intends to go. The second moves with the target through 2.9193, 6.4503, 10.4053, 12.9007 and **16.1993** hours, a factor of 5.55—for a difference of only 0.078 in the water removed. And at 0.100 the first period is longer while at 0.030 the second is: which stage limits has switched with the target. The crossing can be written in closed form— $W_x = W_e + (W_c - W_e)e^{-Kt_{\text{const}}} = 0.039506$, and inverting gives a falling-rate period of exactly 8.0000 hours to within 10^{-9} relative. The order reverses on both sides: at 1.05 times W_x the first period wins, at 0.95 times the second. 0.039506 falls between the practical targets 0.05 and 0.03, so the switch is not an academic one. And removing the same amount of water takes longer the drier it is: 0.3793 hours from 0.030 to 0.029 against **1.4597** from 0.023 to 0.022, a factor of **3.85**. The equilibrium content itself is never reached in finite time, with residuals of 1.554×10^{-13} at 100 hours and 1.342×10^{-25} at 200—always falling, never zero. On a computer, though, the residual vanishes from the sum at 142 hours, since adding 10^{-18} to 0.02 changes nothing in floating point. That is a limit of precision, not of physics; I wrote the test wrongly here once. One thing separates them—is the rate-limiting step fixed, or does it move with the demand. If fixed, there is one thing to do: blow air if evaporation limits, make it thinner if diffusion does. If it moves, nothing can be chosen without a target—air for 0.10, thickness for 0.03, with the same material in the same machine giving a different answer. They are distinct roots by the criterion of Paper 300, one a property of material and machine and one of those paired with a target. This is a third instance of the shape of Paper 446 and Paper 453, where raising the recovery raised the floor and losing friction raised the floor on speed: in all three, the limit was a function of the demand. It overlaps Paper 445 too, where the waiting was for diffusion rather than reaction: here which one is being waited for changes with the target, so identifying the limiting step is itself conditional work. So when “shortening the drying time” is discussed, print the target—an improvement aimed at 0.10 does nothing at 0.03, because the limit has moved elsewhere. To be honest—the numbers are a model, with R , W_c and W_e chosen rather than measured. The location 0.039506 differs by material; what does not differ is that a crossing exists.

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