

Of $16\pi^2$, Only $\Omega_3 = 2\pi^2$ Is Geometry

— The $(2\pi)^4$ Is a Fourier Convention, and the 4π of $4\pi v$ Is Not a Solid Angle — [Paper 46] 3rd ed.

Yuuki YAMAGISHI (山岸 勇輝)

Yamagishi Complexity Torus Lab, Japan

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— *From thinking to syncing.* —

ABSTRACT

This paper derived that the $\frac{1}{16\pi^2}$ of one-loop corrections and the $\frac{1}{(16\pi^2)^2}$ of two-loop corrections are **the loop measure of four-dimensional momentum space**, and stated the general rule that an L -loop amplitude carries $(16\pi^2)^{-L}$. **Theorem 2.1 is correct and is not altered here:** $\int \frac{d^4 k_E}{(2\pi)^4} F = \frac{1}{16\pi^2} \int d^4 k^2 k^2 F$ (re-verified here). **What the third edition corrects is calling that $16\pi^2$ “a purely geometric consequence”. First, decomposing $16\pi^2$ produces three factors of three different standings:**

$$16\pi^2 = \underbrace{2}_{\text{Jacobian}} \times \underbrace{(2\pi)^4}_{\text{Fourier convention}} / \underbrace{\Omega_3 = 2\pi^2}_{\text{geometry}}$$

(verified here symbolically). Ω_3 is the surface area of the unit three-sphere, and it alone is geometry. The $(2\pi)^4$ comes from where one parks the 2π 's of the Fourier transform, and the 2 is the Jacobian of $k^3 dk = \frac{1}{2} u du$. Second, the proof that it is a convention is that changing the convention changes the number. Replacing $\int d^4 k / (2\pi)^4$ by $\int d^4 k / (2\pi)^2$ turns the same combination into 4, and $\int d^4 k$ turns it into $1/\pi^2$ (verified here). The physical amplitude does not change; the number $16\pi^2$ does. Hence “containing no dynamical assumption” is correct while “a purely geometric consequence” is not — containing no dynamics and being geometry are two different things (Paper 95). Third, the isomorphism of Corollary 2.2 is an agreement of numbers whose two sides are composed entirely differently. The left side $2(2\pi)^4/\Omega_3$ is convention $\times$ geometry $\times$ Jacobian; the right side $(4\pi) \times (4\pi)$ is convention $\times$ convention (the patch constant may be any $a > 0$; Paper 116). The second edition's Note 2.3 was right to limit the isomorphism to “an interpretation”. The third edition supplies the reason: one side is contaminated by a Fourier convention. Fourth, the 4π of $\Delta \lesssim 4\pi v$ is not a solid angle. It is $\sqrt{16\pi^2}$, the square root of the loop factor, which as above is not geometry. The numbers are correct — $4\pi v = 3091 \text{ GeV} = 3.09 \text{ TeV}$, and $1262 \text{ GeV} = 1.26 \text{ TeV}$ once the top-Yukawa $\sqrt{6}$ is included (verified here). What is corrected is only the phrase “a bound expressed as the solid angle” (the same shape as Paper 45, which called an area a solid angle). The cutoff circularity retracted in the second edition was rightly retracted and is not reinstated here. The explicit statement of the LHC tension and the discipline of giving no interpretation to 209/2880 are also kept. **Scope:** structural interpretation and self-audit of established standard facts (Layer 2). No ultraviolet completion of quantum gravity is offered, and the hierarchy problem is not solved.

Standing of this paper (discipline): the physics used here is **entirely standard** — the four-dimensional Euclidean momentum integral, $\Omega_3 = 2\pi^2$, the quadratic divergence of the Higgs self-energy, Goroff--Sagnotti (1986), naive dimensional analysis (Manohar--Georgi 1984). **Not one new theorem is proved.** The third edition's contribution is **separating the composition of $16\pi^2$** : (i) decomposing it into **convention** $\times$ **geometry** $\times$ **Jacobian**, (ii) showing the convention part by **changing the Fourier convention**, (iii) making explicit that the isomorphism of Corollary 2.2 is **an agreement between two differently composed numbers**, and (iv) judging that the 4π of $4\pi v$ is **not a solid angle**. **The most important restraints:** “contains no dynamics” is not reread as “is geometry” (Paper 95). $16\pi^2$ and the patch's $(4\pi)^2$ are not treated as the same geometric quantity (strengthening Note 2.3). 4π is **not called a solid angle** (Paper 45). **No claim is made that a cutoff renders the divergences finite** (retracted in the second edition as circular). **No claim is made to have solved the hierarchy problem** (the naturalness bound is under observational tension). **No geometric interpretation is given to 209/2880** (it comes from the combinatorics of cubic Weyl contractions, a different origin). **No ultraviolet completion of quantum gravity is offered.** Layer-3 lenses are quarantined under a non-scientific label.

1. Introduction: The Theorem Is Correct; What Is Corrected Is Its Name

The second edition set two universal coefficients side by side — the quadratic divergence of the Higgs self-energy, $\Delta m_H^2 = \frac{\Lambda_{UV}^2}{16\pi^2} [\lambda_H - 6y_t^2 + \dots]$, and the Goroff--Sagnotti two-loop divergence $\Gamma_{\text{div}}^{(2)} \propto \frac{1}{(16\pi^2)^2}$ — and showed that both are instances of the L -loop rule $(16\pi^2)^{-L}$.

That identification is correct, and the third edition does not alter it.

One thing is corrected: the second edition called $\frac{1}{16\pi^2}$ “a purely geometric consequence containing no dynamical assumption whatsoever”. The first half is right. The second is not.

2. Theorem 2.1 (Unchanged)

Theorem 2.1 (The loop measure)

The four-dimensional Euclidean momentum integral reduces to

$$\int \frac{d^4 k_E}{(2\pi)^4} F(k^2) = \frac{\Omega_3}{(2\pi)^4} \int_0^\infty k^3 dk F = \frac{1}{8\pi^2} \int_0^\infty k^3 dk F = \frac{1}{16\pi^2} \int_0^\infty dk^2 k^2 F,$$

where $\Omega_3 = \frac{2\pi^2}{\Gamma(2)} = 2\pi^2$ is the surface area of the unit three-sphere and the last equality uses $k^3 dk = \frac{1}{2} u du$ with $u = k^2$.

Re-verified here symbolically. That an L -loop amplitude carries $(16\pi^2)^{-L}$ (the second edition's Theorem 3.1) follows by applying this to each integral independently, and stands unchanged.

3. Decomposing $16\pi^2$ Produces Three Factors of Different Standing

Theorem 3.1 (The composition of $16\pi^2$)

$$16\pi^2 = 2 \times \frac{(2\pi)^4}{\Omega_3}$$

(verified here). The three factors on the right have three different standings.

Factor	Value	Standing
$(2\pi)^4$	$16\pi^4$	Convention (where the Fourier 2π 's are parked)
Ω_3	$2\pi^2$	Geometry (surface area of the unit three-sphere)
2	2	Jacobian ($k^3 dk = \frac{1}{2} u du$)

Observation 3.1 (Only Ω_3 is geometry)

$\Omega_3 = 2\pi^2$ is a quantity the unit three-sphere itself possesses. That is geometry. The $(2\pi)^4$ is not. It comes from the way one writes $\int d^4k/(2\pi)^4$. Whether the Fourier 2π 's go entirely on the momentum side, are split between the two, or are omitted — the physics is the same in every case, and only the written number changes. The 2 is not either. It is the Jacobian of trading the radial variable k for k^2 , and attends the choice of which variable to write in.

4. The Proof That It Is a Convention: Change the Convention and the Number Changes

Fourier convention	$2(2\pi)^n/\Omega_3$
$\int \frac{d^4k}{(2\pi)^4}$ (the QFT standard)	$16\pi^2$
$\int \frac{d^4k}{(2\pi)^2}$ (symmetric)	4
$\int d^4k$ (no 2π)	$1/\pi^2$

Verified here.

Observation 4.1 (The physics does not move; the number does)

The physical amplitude is the same in every convention, since the propagators and vertices pick up compensating 2π 's. What changes is only whether the number $16\pi^2$ gets written down. What changes its answer when the scale changes is a convention, not a fact (Paper 95). This is the same judgement Paper 45 passed on $R = 2$. Hence “containing no dynamical assumption” is correct, and “a purely geometric consequence” is not. Containing no dynamics and being geometry are two different things. A convention, too, contains no dynamics.

5. The Isomorphism of Corollary 2.2: The Agreement Is Numerical, the Composition Is Not

The second edition wrote

$$16\pi^2 = \underbrace{\frac{(2\pi)^4}{\Omega_3} \times 2}_{\text{inverse momentum-space measure}} = \underbrace{(4\pi) \times (4\pi)}_{\text{Yamagishi double-torus boundary}}.$$

Observation 5.1 (The two compositions)

Left side	Convention $\times$ geometry $\times$ Jacobian
Right side	Convention $\times$ convention (any $a > 0$ will do; Paper 116)

The numbers agree. The compositions are entirely different. The second edition's Note 2.3 was right — “what this paper asserts strictly is Theorem 2.1 alone, and the isomorphism with the Yamagishi geometry is presented as an interpretive correspondence”. The third edition supplies the reason for that limitation: because the left side is contaminated by a Fourier convention, even finding a transformation between the two would not make it a correspondence between geometries. The isomorphism is therefore weak even as an interpretation. This edition records it as **an agreement of numbers** and no more.

6. The 4π of $4\pi v$ Is Not a Solid Angle

Established Fact 6.1 (The naturalness bound (numbers unchanged))

From $\frac{\Lambda^2}{16\pi^2} \lesssim v^2$ one gets $\Lambda \lesssim \sqrt{16\pi^2} v = 4\pi v$. With $v = 246$ GeV this is **3091 GeV = 3.09 TeV**. Including the top-Yukawa term $-6y_t^2$ gives $4\pi v/\sqrt{6} = \mathbf{1262\ GeV = 1.26\ TeV}$ (verified here).

Observation 6.1 (Only the name is corrected)

The second edition called this “a testable bound expressed as the solid angle 4π ”. The number, the testability, and the note of observational tension are all correct. What is corrected is the word “solid angle”. The 4π here is $\sqrt{16\pi^2}$, the square root of the loop factor, which as shown above is **convention $\times$ geometry $\times$ Jacobian**. It is not a solid angle. This is the same shape Paper 45 found inside itself — there, an area was called a solid angle; here, the square root of a loop factor is. The correct name is “the 4π counting of naive dimensional analysis” (Manohar--Georgi 1984, already in the second edition's bibliography). The bound itself does not change at all.

7. What the Second Edition Rightly Retracted (Not Reinstated Here)

Established Fact 7.1 (Retraction of the cutoff circularity)

The second edition retracted the first edition's voxel-cutoff argument — introducing $\Lambda_{\text{voxel}} = (16\pi^2)^{-1/2}$ by hand to render the divergence finite — as **circular**, since **the conclusion (finiteness) is already contained in the assumption (a finite cutoff)**. The content of Goroff--Sagnotti, perturbative non-renormalisability, is not dissolved by introducing a cutoff.

That retraction is sound and is not reinstated here. The other disciplines the second edition kept are also unchanged — the explicit statement that **the LHC has seen no new physics to the TeV scale and the naturalness bound is already under observational tension**, the refusal to give 209/2880 a geometric interpretation (it comes from the combinatorics of cubic Weyl contractions, a different origin from $16\pi^2$), and the placement of general relativity as an effective field theory.

Observation 7.1 (On the reading “each loop consumes a geometric capacity”)

The second edition's discussion read the $(16\pi^2)^{-L}$ suppression as “**each loop consuming one double-torus boundary capacity**”. **The suppression itself is correct** — the size $16\pi^2 \approx 158$ quantitatively accounts for why one-loop corrections typically land at the 10^{-2} level. **This is standard loop counting and is not altered. But the reading “consumes a geometric capacity” presupposes that $16\pi^2$ is geometry.** As shown above, only Ω_3 is. **The reading is therefore removed from the body as Layer 3.**

8. The Scope, Restated

Item	What it is
$\int \frac{d^4 k_E}{(2\pi)^4} F = \frac{1}{16\pi^2} \int dk^2 k^2 F$	Fact (Theorem 2.1; unchanged)
L loops carry $(16\pi^2)^{-L}$	Fact (Theorem 2.1 applied L times)
$\Omega_3 = 2\pi^2$	Geometry
$(2\pi)^4$	Fourier convention (change it and the number moves)
The 2 of $k^3 dk = \frac{1}{2} u du$	Jacobian
That $16\pi^2$ is “purely geometric”	Retracted (a product of three standings)
The isomorphism $16\pi^2 = (4\pi)^2$	An agreement of numbers (weak even as interpretation)
$\Lambda \lesssim 4\pi v = 3.09 \text{ TeV}$	Fact (naive dimensional analysis; under tension)
That its 4π is a solid angle	Retracted (square root of the loop factor)
A cutoff rendering the divergences finite	Retracted (circular; second edition)
A geometric interpretation of 209/2880	Not given (different origin)
An ultraviolet completion of quantum gravity	Not given

Of the twelve rows, four are settled facts. The rest are conventions, a Jacobian, retractions, or things not given.

9. Conclusion

This paper was looking for the identity of $16\pi^2$. **What it found is genuine:** the four-dimensional Euclidean momentum integral does reduce to $\frac{1}{16\pi^2} \int dk^2 k^2$.

But that $16\pi^2$ was a product of three standings. The surface area of the unit three-sphere, which is **geometry**; where one parks the Fourier 2π 's, which is **a convention**; and the **Jacobian** of a change of variable.

Change the convention and the same combination becomes **4**, or $1/\pi^2$. **The physics does not move a millimetre.** What moves is the number we write down.

Not everything free of dynamics is geometry.

A convention, too, is free of dynamics.

What is not claimed (restraint): “contains no dynamics” is not reread as “is geometry” (Paper 95) / $16\pi^2$ and the patch's $(4\pi)^2$ are not treated as the same geometric quantity (different compositions; Paper 116) / 4π is not called a solid angle (the square root of a loop factor; Paper 45) / **no claim that a cutoff renders the divergences finite** (circular; retracted in the second edition) / **no claim to have solved the hierarchy problem** (the bound is under observational tension) / **no geometric interpretation of 209/2880** (different origin) / **no ultraviolet completion of quantum gravity** / **the reading “each loop consumes a geometric capacity” is not placed in the body** (Layer 3, quarantined) / no new mathematics and no prediction (Layer 2).

Revision Record

Third edition (August 2026): Theorem 2.1 (the loop measure) and Theorem 3.1 (the L -loop rule) are unchanged (re-verified symbolically). The cutoff circularity retracted in the second edition is not reinstated. (i) Calling $\frac{1}{16\pi^2}$ “a purely geometric consequence” was retracted: $16\pi^2 = 2 \times (2\pi)^4 / \Omega_3$, a product of three different standings — $(2\pi)^4$ a Fourier convention, $\Omega_3 = 2\pi^2$ geometry, and 2 a Jacobian. Only Ω_3 is geometry. (ii) The convention part was demonstrated by changing the Fourier convention: $\int d^4k / (2\pi)^2$ gives 4 and $\int d^4k$ gives $1/\pi^2$ (verified here). The physical amplitude does not move; only the written number does (Paper 95). “Contains no dynamical assumption” is correct; “is geometry” is not. (iii) For Corollary 2.2, it was made explicit that the left side is convention $\times$ geometry $\times$ Jacobian and the right side convention $\times$ convention, so the compositions differ entirely. The second edition's Note 2.3 (the isomorphism is an interpretation) was correct, and this edition supplies its reason and reduces the isomorphism to a record of numerical agreement. (iv) The phrase “a bound expressed as the solid angle 4π ” was retracted for $\Lambda \lesssim 4\pi v$: that 4π is $\sqrt{16\pi^2}$, the square root of the loop factor, not a solid angle. The correct name is the 4π counting of naive dimensional analysis (Manohar--Georgi 1984). The numbers (3091 GeV; 1262 GeV with $\sqrt{6}$) and the note of observational tension are unchanged. (v) The reading “each loop consumes a geometric capacity” was removed from the body as Layer 3 (the suppression itself is kept as standard loop counting).

On the numbering: this paper merges the first editions of Paper 46 and 47. The doubled label “46/47” is dropped, since the content is one paper: the number is now simply 46. Number 47 is left permanently vacant as a record of that merge, and is not reassigned.

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