

Reverse the Direction and the Outlet Overtakes Its Partner

— With inlets at **90** and **20°C** and **NTU = 2**, parallel flow leaves **54.3590** and **55.6410** and never crosses, while counterflow leaves **66.6667** and **43.3333** and does — parallel flow caps at **0.5000**, which counterflow reaches at **NTU = 0.9996** — [Paper 461]

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— *From thinking to syncing.* —

Abstract

A heat exchanger is thought to improve with more surface. This paper shows that in parallel flow, infinite surface still stops at an effectiveness of **0.5000**—while reversing the direction reaches it at **NTU = 0.9996**. No new theorem or law is claimed. First, at **NTU = 2** parallel flow gives **0.4908** and counterflow **0.6667** (Section 2). Second, parallel outlets are **54.3590** and **55.6410°C** and never cross (Section 2). Third, this is the core of the paper. Counterflow outlets are **66.6667** and **43.3333°C**: the cold side overtakes the hot by **23.3334** degrees (Section 2). Fourth, parallel effectiveness caps at **0.500000000** however large the **NTU** (Section 3). Fifth, counterflow reaches that cap at **NTU = 0.9996** (Section 3). Sixth, the separator is whether the surface is short or the direction is wrong (Section 4). **Keywords**: heat exchanger, counterflow, parallel flow, effectiveness-NTU method.

Scope of this paper (scope note): No new theorem or law is claimed—the effectiveness-NTU method and the difference between parallel and counterflow are entirely standard. No exchanger is designed—tube size, arrangement, pressure drop, fouling and materials are not the subject. The capacity rates are set equal—the ratio is fixed at 1; away from 1 the numbers change. Phase change is excluded—condensing and evaporating duties need separate treatment. Crossflow and multipass are excluded—real exchangers are often neither purely parallel nor purely counter. Relation to earlier papers: Paper 460 showed that the limit of heat is set by temperature and humidity together—there too, watching one quantity alone got the ordering wrong. Paper 300 showed that common root and distinct root can be decided—“the limit of parallel flow” and “of counterflow” are distinct roots by that criterion. Paper 459 showed that a round trip multiplies the loss by 1.9—there too, the arrangement set the quantity. What is added is giving both effectivenesses at **NTU = 2**, tabulating four outlet temperatures to show the crossing, giving the parallel cap as **0.500000000**, solving by bisection for the **NTU** at which counterflow reaches it, **0.9996**, and placing the separator at surface versus direction.

1. Notation and the Question

Setting: Let the effectiveness ε be the ratio of the heat actually transferred to the greatest heat that could be. With $NTU = UA/C_{\min}$ and capacity ratio $C_r = C_{\min}/C_{\max}$, parallel flow gives

$$\varepsilon_{\parallel} = \frac{1 - e^{-NTU(1+C_r)}}{1 + C_r}$$

and counterflow at $C_r = 1$ gives

$$\varepsilon_{\times} = \frac{NTU}{1 + NTU}$$

Below, $C_r = 1$, the hot inlet is 90°C and the cold inlet 20.

Observation 1.1

A heat exchanger is thought to improve with more surface. NTU is proportional to area—make it larger and it closes in, so the picture goes.

Observation 1.2

But in parallel flow the temperature difference runs out on the way. Both streams run the same way, so the temperatures meet at the outlet—and past that point, surface moves no heat.

Question: Where does it stop, and what happens on reversing the direction.

2. In Counterflow the Outlet Overtakes

Computation 2.1: Take the effectiveness and outlet temperatures at $NTU = 2$.

Arrangement	Effectiveness	Cold out [°C]	Hot out [°C]
Parallel	0.4908	54.3590	55.6410
Counter	0.6667	66.6667	43.3333

Observation 2.2

In parallel flow the cold outlet never passes the hot. $54.3590 < 55.6410$ —they meet at the same place, so overtaking would mean heat running backwards.

Observation 2.3

In counterflow it does. 66.6667 against 43.3333 , a gap of **23.3334** degrees—because the cooled stream leaves where the heating stream came in.

Observation 2.4

This does not violate the second law. At every cross-section the hot side is above the cold—what is being compared is opposite ends, not one section.

Observation 2.5

At the same NTU the effectiveness differs by $1.36.0.6667$ against 0.4908 —same area, same flows, only the plumbing reversed.

3. Parallel Flow Caps at 0.500000000

Computation 3.1: Take the parallel effectiveness as NTU grows without bound.

0.500000000 (computed at $NTU = 10^6$).

Observation 3.2

No amount of surface passes half. With equal capacity rates the two temperatures meet in the middle and stop— $NTU = 10^6$ still gives 0.5.

Computation 3.3: Solve by bisection for the NTU at which counterflow reaches that cap.

$NTU = \mathbf{0.9996}$ (inverting gives an effectiveness of 0.499900000).

Observation 3.4

What parallel flow needs infinity for, counterflow reaches at $NTU = 1$. At a finite area—and beyond it, counterflow keeps climbing.

Observation 3.5

Counterflow has no cap. 0.9804 at $NTU = 50$ —approaching 1. Its destination is a different one.

Observation 3.6

So “we need more surface” misdiagnoses parallel flow. What is short is not area but temperature difference—and until the direction changes, the investment does not return.

4. The Separator

The separator: is the surface short, or the direction wrong.

Observation 4.1

Short on surface, and adding it closes the gap. Raising NTU raises the effectiveness—monotone, and headed for 1.

Observation 4.2

Wrong direction, and adding it stops at 0.5. Monotone too, but headed elsewhere—both rise with surface, so the slope alone does not distinguish them.

Observation 4.3

These are distinct roots by the criterion of Paper 300. Both are called “the limit of the exchanger”—one a limit of size, one a limit imposed by the arrangement.

Observation 4.4

The same shape as Paper 460. There, watching air temperature alone got the ordering wrong—here, watching area alone gets the destination wrong. In both, a second component decides.

Observation 4.5

It overlaps Paper 459 as well. There, one device read 0.90 or 0.81 depending on the boundary—here one device reads 0.4908 or 0.6667 depending on the direction.

Observation 4.6

And reversing the direction is usually cheaper than adding surface. Sometimes it is only repiping—misdiagnose it and one buys the expensive remedy.

Observation 4.7

To be honest—the capacity ratio was fixed at 1. Away from 1, the parallel cap becomes $1/(1 + C_r)$ and rises—so 0.5 is a value under this condition; only the existence of a cap is unconditional.

5. Roll Call and Closing

1. 0.4908 parallel at $NTU = 2$ (Computation 2.1): **counter** gives 0.6667.
2. Parallel outlets 54.3590 and 55.6410 (Computation 2.1): **no crossing**.
3. Counter outlets 66.6667 and 43.3333 (Computation 2.1): **the core**.
4. An overtake of 23.3334 degrees (Observation 2.3): **opposite ends**.
5. No violation of the second law (Observation 2.4): **hot above cold everywhere**.
6. 1.36 at the same NTU (Observation 2.5): **only the direction differs**.
7. The parallel cap is 0.500000000 (Computation 3.1): **surface cannot pass it**.
8. Counter reaches it at $NTU = 0.9996$ (Computation 3.3): **at finite area**.
9. The separator is surface or direction (Observation 4.3): **distinct roots by Paper 300**.

10. Away from $C_r = 1$ the numbers move (Observation 4.7): the cap remains.

Closing: A heat exchanger is thought to improve with more surface—NTU is proportional to area, so making it larger should close the gap. But in parallel flow the temperature difference runs out on the way: both streams run the same direction, the temperatures meet at the outlet, and past that point surface moves no heat. Reverse the direction and the outlet overtakes its partner—at $NTU = 2$ parallel flow gives an effectiveness of **0.4908** with outlets at **54.3590** and **55.6410**°C, which never cross; they meet at the same place, so overtaking would mean heat running backwards. Counterflow gives **0.6667** with outlets at **66.6667** and **43.3333**, the cold side overtaking the hot by **23.3334** degrees, because the cooled stream leaves where the heating stream came in. This does not violate the second law: at every cross-section the hot side is above the cold, and what is being compared is opposite ends. At the same NTU the effectiveness differs by 1.36 with the same area and the same flows—only the plumbing reversed. And parallel flow has a cap that surface cannot pass— $NTU = 10^6$ still gives **0.500000000**, because with equal capacity rates the two temperatures meet in the middle and stop. Counterflow reaches that value at $NTU = 0.9996$: what parallel flow needs infinity for, it gets at finite area. And counterflow has no cap at all, giving 0.9804 at $NTU = 50$ and approaching 1. Its destination is a different one. So “we need more surface” misdiagnoses parallel flow—what is short is not area but temperature difference, and until the direction changes the investment does not return. One thing separates them—is the surface short, or the direction wrong. Short on surface, and adding it closes the gap: monotone, headed for 1. Wrong direction, and adding it stops at 0.5: monotone too, but headed elsewhere, so the slope alone does not distinguish them. They are distinct roots by the criterion of Paper 300, one a limit of size and one a limit imposed by the arrangement. It is the same shape as Paper 460, where watching air temperature alone got the ordering wrong: in both, a second component decides. It overlaps Paper 459 too, where one device read 0.90 or 0.81 depending on the boundary: here one device reads 0.4908 or 0.6667 depending on the direction. And reversing the direction is usually cheaper than adding surface—sometimes it is only repiping, and misdiagnosing it buys the expensive remedy. To be honest—the capacity ratio was fixed at 1. Away from 1 the parallel cap becomes $1/(1 + C_r)$ and rises, so 0.5 is a value under this condition; only the existence of a cap is unconditional.

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