

Two Different States Carry Exactly the Same Load

— In the linear model, a flow of **3000** vehicles per hour occurs at densities **41.4590** and **108.5410** per km, at speeds of **72.3607** and **27.6393** km/h — a factor of **2.6180** — and the maximum **3750** occurs at half the free-flow speed — [Paper 462]

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— *From thinking to syncing.* —

Abstract

A road is thought to carry more the faster it runs. This paper shows that two states carry the same flow at speeds differing by **2.6180**—and the maximum flow occurs at half the free-flow speed. No new theorem or law is claimed. First, the maximum flow is **3750.000000000** per hour at a density of **75** per km (Section 2). Second, the speed there is exactly half the free-flow value, **50.000000000000** km/h (Section 2). Third, this is the core of the paper. A flow of 3000 occurs at densities **41.4590** and **108.5410**, at **72.3607** against **27.6393** km/h (Section 3). Fourth, densities of 20 and 130 both give **1733.3333** per hour (Section 2). Fifth, the maximum matches the closed form $v_f k_j / 4$ exactly (Section 2). Sixth, the separator is whether flow is set by speed or by the product of speed and density (Section 4). **Keywords:** traffic flow, fundamental diagram, traffic capacity, Greenshields model, congestion.

Scope of this paper (scope note): No new theorem or law is claimed—the fundamental diagram of traffic flow and the Greenshields linear model are entirely standard. No road is designed—lanes, merges, signals and tolls are not the subject. No driving is discussed—headway, speed choice and driver behaviour are not the subject. The linear model is a model—a speed falling linearly with density is too simple; measured diagrams are more complex and unstable near the peak. Congestion onset is not explained—where jams form and how shockwaves travel are not the subject; only steady states are treated. Relation to earlier papers: Paper 461 showed that reversing the direction lets the outlet overtake—there too, one device became a different quantity with its arrangement. Paper 300 showed that common root and distinct root can be decided—“a fast road” and “a road that carries much” are distinct roots by that criterion. Paper 447 showed that the optimum is interior—there too, looking at one side alone misled; this is a second instance. What is added is tabulating speed and flow at five densities, giving the maximum as **3750.000000000** and matching the closed form, solving for the two densities at a given flow, **41.4590** and **108.5410**, giving the speed ratio as **2.6180**, and placing the separator at speed versus the product.

1. Notation and the Question

Setting: Let the density k be vehicles per unit length and the flow q be vehicles past a section per unit time, so by definition

$$q = kv$$

Taking the speed to fall linearly with density (Greenshields),

$$v(k) = v_f \left(1 - \frac{k}{k_j}\right)$$

Below, the free-flow speed is $v_f = 100$ km/h and the jam density $k_j = 150$ per km.

Observation 1.1

A road is thought to carry more the faster it runs. Raise the limit and the traffic flows better—natural enough as an intuition.

Observation 1.2

But what is carried is speed times number. Running faster means longer gaps—so the number falls.

Question: Where does the road carry most.

2. The Maximum Occurs at Half the Free-Flow Speed

Computation 2.1: Take speed and flow at five densities.

Density [/km]	Speed [km/h]	Flow [/h]
20	86.6667	1733.3333
50	66.6667	3333.3333
75	50.0000	3750.0000
100	33.3333	3333.3333
130	13.3333	1733.3333

Observation 2.2

The maximum is **3750.0000000000** per hour at a density of 75. Matching the closed form $v_f k_j / 4$ exactly—the peak sits in the same place by formula and by computation.

Observation 2.3

The speed there is **50.000000000000** km/h, exactly half the free-flow value. The state that carries most is not the fastest state—the maximum arrives at half speed.

Observation 2.4

Densities of 20 and 130 both give 1733.3333. At 86.6667 and 13.3333 km/h, a factor of 6.5—and the same amount carried.

Observation 2.5

Densities of 50 and 100 likewise both give 3333.3333. Arranged symmetrically about the peak—watching flow alone does not say which side one is on.

3. Two States Carry the Same Flow

Computation 3.1: Solve the quadratic for the densities that give 3000 per hour.

State	Density [/km]	Speed [km/h]
Uncongested	41.4590	72.3607
Congested	108.5410	27.6393

Observation 3.2

Both give 3000.000000 per hour. Confirmed by inversion—the same amount carried at speeds differing by **2.6180**.

Observation 3.3

Counted at a section, the two cannot be told apart. A device recording only vehicles past a point sees the same number—without density or speed, the side is unknown.

Observation 3.4

And the remedy reverses. On the uncongested side more inflow raises the flow; on the congested side only less inflow does—one observation, two opposite actions.

Observation 3.5

On the congested side, more inflow lowers the flow. Pack beyond 108.5410 and q falls—which is the direction in which ramp metering works.

Observation 3.6

The peak of 3750 is only 1.25 times 3000. At best a 25 per cent gain—while on the congested side, left alone, it can fall to 1733.3333.

4. The Separator

The separator: is flow set by speed, or by the product of speed and density.

Observation 4.1

Set by speed, and faster is better. A monotone relation—yielding the prescription of a higher limit.

Observation 4.2

Set by the product, and the peak is interior. Speed and density move against each other—maximising either alone does not maximise the product.

Observation 4.3

These are distinct roots by the criterion of Paper 300. Both are called “a good road”—one a property of speed, one of speed paired with density.

Observation 4.4

The same shape as Paper 447. There, thickening the sludge too far cost effective performance—here, packing the cars too tightly costs flow. In both, the increasing direction has a peak.

Observation 4.5

It overlaps Paper 461 as well. There, one device became a different quantity with its arrangement—here one flow corresponds to two states. In both, the observation alone does not fix the state.

Observation 4.6

So flow records should carry a density or a speed. “3000 per hour” alone does not say whether the road is comfortable or jammed—one number naming opposite states.

Observation 4.7

To be honest—the linear model is too simple. Measured diagrams scatter near the peak and become unstable—the values 3750 and 50 km/h depend on the model; the existence of a peak and of two states at one flow does not.

5. Roll Call and Closing

1. Maximum flow 3750.000000000 (Observation 2.2): matching the closed form.
2. At 50.000000000000 km/h (Observation 2.3): half the free-flow speed.
3. 20 and 130 both give 1733.3333 (Observation 2.4): speeds differ by 6.5.
4. 50 and 100 both give 3333.3333 (Observation 2.5): symmetric about the peak.
5. 3000 occurs at 41.4590 and 108.5410 (Computation 3.1): the core.
6. Speeds differ by 2.6180 (Observation 3.2): carrying the same amount.
7. A section cannot tell them apart (Observation 3.3): density or speed is needed.
8. The remedy reverses (Observation 3.4): more inflow, or less.
9. The separator is speed or product (Observation 4.3): distinct roots by Paper 300.
10. Print density or speed with flow (Observation 4.6): one number, opposite states.

Closing: A road is thought to carry more the faster it runs—raise the limit and the traffic flows better, natural enough as an intuition. But what is carried is speed times number, and running faster means longer gaps, so the number falls. The maximum sits in the middle—the greatest flow is 3750.000000000 per hour at a density of 75 per km, matching the closed form $v_f k_j / 4$ exactly. The speed there is 50.000000000000 km/h, exactly half the free-flow value: the state that carries most is not the fastest state. Densities of 20 and 130 both give 1733.3333 per hour at speeds differing by 6.5, and densities of 50 and 100 both give 3333.3333, arranged symmetrically about the peak. So two states carry the same flow—3000 per hour occurs at densities 41.4590 and 108.5410, at 72.3607 against 27.6393 km/h, a factor of 2.6180. Counted at a section the two cannot be told apart: a device recording only vehicles past a point sees the same number, and without density or speed the side is unknown. And the remedy reverses—on the uncongested side more inflow raises the flow, on the congested side only less inflow does. One observation, two opposite actions. On the congested side more inflow lowers the flow, which is the direction in which ramp metering works. The peak of 3750 is only 1.25 times 3000, at best a 25 per cent gain, while on the congested side, left alone, it can fall to 1733.3333. One thing separates them—is flow set by speed, or by the product of speed and density. Set by speed, faster is better, and the monotone relation yields the prescription of a higher limit. Set by the product, the peak is interior: speed and density move against each other, so maximising either alone does not

maximise the product. They are distinct roots by the criterion of Paper 300, one a property of speed and one of speed paired with density. It is the same shape as Paper 447, where thickening the sludge too far cost effective performance: in both, the increasing direction has a peak. It overlaps Paper 461 too, where one device became a different quantity with its arrangement: in both, the observation alone does not fix the state. So flow records should carry a density or a speed—“3000 per hour” alone does not say whether the road is comfortable or jammed, one number naming opposite states. To be honest—the linear model is too simple, and measured diagrams scatter near the peak and become unstable. The values 3750 and 50 km/h depend on the model; the existence of a peak and of two states at one flow does not.

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