

Halving the Size Beats Changing the Material Forty Times Over

— Spheres of radius **5 mm** and **20 cm** have cooling time constants of **348.8333** and **13953.3333** seconds, a ratio of exactly **40.000000000** — changing the material from water to metal at one size gives only **1.3081** — what sets the cooling is not specific heat but the surface-to-volume ratio — [Paper 465]

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Abstract

How fast a thing cools is thought to be set by its material. This paper shows that size acts **40** times harder—changing from water to metal gives only **1.3081**. No new theorem or law is claimed. First, a sphere of radius **5 mm** has a time constant of **348.8333** seconds (Section 2). Second, at **20 cm** it is **13953.3333**, a ratio of exactly **40.000000000** (Section 2). Third, this is the core of the paper. Changing the material at one size gives only **1.3081** (Section 3). Fourth, the product of time constant and surface-to-volume ratio is independent of radius (Section 2). Fifth, cooling from **90** to **60** takes **3.2535** and **130.1418** minutes (Section 3). Sixth, the separator is whether the property belongs to the substance or to the shape (Section 4). **Keywords:** Newton's law of cooling, lumped capacitance method, surface-to-volume ratio, cooling time.

Scope of this paper (scope note): No new theorem or law is claimed—Newton’s law of cooling, the lumped capacitance method and the surface-to-volume ratio are all standard. No apparatus is designed—insulation, forced air and vacuum flasks are not the subject. Internal gradients are excluded—the lumped capacitance method is used; at large Biot number internal gradients matter and this treatment fails. The heat transfer coefficient is fixed— $h = 20 \text{ W}/(\text{m}^2 \text{ K})$ is taken as representative of natural convection; in reality it varies with size and temperature difference. Radiation and evaporation are excluded—radiation matters when hot and evaporation at a liquid surface, but only convection is treated. Relation to earlier papers: Paper 464 showed that what cools is evaporation, not the temperature of the water—there too, the conspicuous quantity and the acting one differed. Paper 300 showed that common root and distinct root can be decided—“cooling set by material” and “by shape” are distinct roots by that criterion. Paper 451 showed that what insulates is the air and not the snow—there too, the name of the material did not settle how heat passes. What is added is giving the time constant at four radii, showing the ratio to be exactly 40.000000000, giving the effect of material as 1.3081, confirming that the product with the surface-to-volume ratio is constant, confirming that changing the temperature interval leaves the ratio unchanged, and placing the separator at substance versus shape.

1. Notation and the Question

Setting: A body whose interior is uniform, cooling in surroundings at T_a , obeys

$$\rho c V \frac{dT}{dt} = -h A (T - T_a)$$

so the time constant is

$$\tau = \frac{\rho c V}{h A}$$

and for a sphere of radius r , where $V/A = r/3$,

$$\tau = \frac{\rho c r}{3h}$$

Below, $\rho = 1000 \text{ kg}/\text{m}^3$, $c = 4186 \text{ J}/(\text{kg K})$ for water and $h = 20 \text{ W}/(\text{m}^2 \text{ K})$.

Observation 1.1

How fast a thing cools is thought to be set by its material. Low specific heat cools quickly—and c does appear in the expression.

Observation 1.2

But so does τ . Heat leaves through the surface and is stored in the volume—and that ratio changes with size.

Question: Which acts, and by how much.

2. Size Acts Forty Times Over

Computation 2.1: Take the time constant at four radii.

Radius [m]	Time constant [s]	[min]
0.005	348.8333	5.8139
0.020	1395.3333	23.2556
0.050	3488.3333	58.1389
0.200	13953.3333	232.5556

Observation 2.2

The time constant goes with the radius. Forty times the radius, exactly **40.000000000** times the constant—because volume goes as r^3 , area as r^2 , and the ratio as r .

Observation 2.3

The surface-to-volume ratio is 600.0000 at 5 mm and 15.0000 at 20 cm. A factor of 40—small things have much surface for their volume.

Observation 2.4

The product of the two is independent of radius. The same value at either radius— $\tau \cdot (A/V) = \rho c/h$, with the geometry cancelling.

Observation 2.5

So “how fast it cools” is the product of $\rho c/h$, a property of substance and surroundings, and A/V , a property of shape. Two factors—and moving one is not the same as moving the other.

3. The Material Acts Only 1.3081 Times

Computation 3.1: Fix the radius at 5 cm and vary the material.

Material	Time constant [s]
Water ($\rho c = 4.186 \times 10^6$)	3488.3333
Metal ($\rho c = 3.2 \times 10^6$)	2666.6667

Observation 3.2

The ratio is only **1.3081**. Though the specific heats differ more than tenfold, 400 against 4186—density works the other way and the product nearly cancels.

Observation 3.3

Set 40 against 1.3081. Size acts more than thirty times harder—“metal cools faster” is true and small.

Computation 3.4: Take the time to cool from 90 to 60 degrees at two radii (surroundings at 20).

Radius	90 → 60 degrees [min]
5 mm	3.2535
20 cm	130.1418

Observation 3.5

The ratio is again exactly **40.000000000**. Matching the ratio of time constants—because the cooling is exponential, the temperature interval does not enter the ratio.

Observation 3.6

From 90 to 30 the ratio is the same **40.000000000**. Unchanged by the interval—the ratio is set by geometry alone.

Observation 3.7

So to cool something quickly, divide it. The same volume in eight pieces halves the radius—and halves the time constant.

4. The Separator

The separator: does the property belong to the substance, or to the shape.

Observation 4.1

To the substance, and a materials table settles it. Specific heat and density give the answer—shape does not enter.

Observation 4.2

To the shape, and the same material spans orders of magnitude. A factor of 40 between 5 mm and 20 cm—which no materials table records.

Observation 4.3

These are distinct roots by the criterion of Paper 300. Both are called “how fast it cools”—one a property of a substance, one of a substance paired with a shape.

Observation 4.4

This stands where Paper 451 stands. There, what insulated was the air and not the snow, and the name of the material did not settle how heat passes—here the material does not settle the cooling. In both, the answer lies with the structure.

Observation 4.5

It overlaps Paper 464 as well. There, the conspicuous quantity (water temperature) and the acting one (evaporation) differed—here it is material against size.

Observation 4.6

And this is part of why small animals run hotter metabolisms. More heat escapes per unit of volume—though animals have fur and blood flow, so this calculation alone does not suffice.

Observation 4.7

To be honest—the lumped capacitance method is used. At large Biot number an internal gradient forms and the treatment fails—the factor of 40 is a value under that approximation; the conclusion that A/V acts does not depend on it.

5. Roll Call and Closing

1. 348.8333 s at radius 5 mm (Computation 2.1): 5.8139 minutes.
2. 13953.3333 s at 20 cm (Computation 2.1): 232.5556 minutes.
3. A ratio of exactly 40.000000000 (Observation 2.2): proportional to radius.

4. Surface-to-volume **600** and **15** (Observation 2.3): **the same factor of 40.**
5. The product is constant (Observation 2.4): $\rho c/h$, geometry cancels.
6. The material gives **1.3081** (Computation 3.1): **the core.**
7. **3.2535** and **130.1418** minutes (Computation 3.4): **the same factor of 40.**
8. The interval does not change it (Observation 3.6): **geometry alone.**
9. The separator is substance or shape (Observation 4.3): **distinct roots by Paper 300.**
10. Lumped capacitance is assumed (Observation 4.7): **the conclusion survives.**

Closing: How fast a thing cools is thought to be set by its material—low specific heat cools quickly, and c does appear in the expression. But so does r : heat leaves through the surface and is stored in the volume, and that ratio changes with size. Size acts forty times over—a sphere of radius 5 mm has a time constant of **348.8333** seconds and one of 20 cm has **13953.3333**, a ratio of exactly **40.000000000**, because volume goes as r^3 , area as r^2 and the ratio as r . The surface-to-volume ratio is 600.0000 at 5 mm and 15.0000 at 20 cm, the same factor of 40: small things have much surface for their volume. And the product of the two is independent of radius, $\tau \cdot (A/V) = \rho c/h$, with the geometry cancelling. So “how fast it cools” is the product of a substance-and-surroundings factor and a shape factor. The material acts only 1.3 times—fixing the radius at 5 cm, water gives **3488.3333** seconds and metal **2666.6667**, a ratio of **1.3081**. The specific heats differ more than tenfold, 400 against 4186, but density works the other way and the product nearly cancels. Set 40 against 1.3081 and size acts more than thirty times harder: “metal cools faster” is true and small. Cooling from 90 to 60 degrees takes **3.2535** and **130.1418** minutes, again a ratio of exactly **40.000000000**, because the cooling is exponential and the temperature interval does not enter the ratio. From 90 to 30 the ratio is the same: it is set by geometry alone. So to cool something quickly, divide it—the same volume in eight pieces halves the radius and halves the time constant. One thing separates them—does the property belong to the substance, or to the shape. To the substance, and a materials table settles it from specific heat and density, with shape not entering. To the shape, and the same material spans a factor of 40 between 5 mm and 20 cm, which no materials table records. They are distinct roots by the criterion of Paper 300, one a property of a substance and one of a substance paired with a shape. This stands where Paper 451 stands, where what insulated was the air and not the snow: in both, the answer lies with the structure. It overlaps Paper 464 too, where the conspicuous quantity and the acting one differed: here it is material against size. And this is part of why small animals run hotter metabolisms, more heat escaping per unit of volume—though animals have fur and blood flow, so this calculation alone does not suffice. To be honest—the lumped capacitance method is used, and at large Biot number an internal gradient forms and the treatment fails. The factor of 40 is a value under that approximation; the conclusion that A/V acts does not depend on it.

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