

Making a Quantity Additive Can Cost It Its Even Spacing

— Stacking lenses of **0.50** m and **0.25** m gives **0.166667** m, neither the sum **0.75** nor the mean **0.375** — in dioptries it is **2** plus **4** equals **6.0000**, one line — yet **−1** to **−2** D moves the far point **0.5** m while **−4** to **−8** D moves it only **0.125** — [Paper 467]

Yuuki YAMAGISHI (山岸 勇輝)

Yamagishi Complexity Torus Lab, Japan

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— *From thinking to syncing.* —

Abstract

Choosing better units is said to make the arithmetic easier. This paper shows that making a quantity additive can cost it its even spacing—one step of dioptry moves the far point only a quarter as far at the strong end. No new theorem or law is claimed. First, stacking 0.50 m and 0.25 m gives a combined focal length of **0.166667** m (Section 2). Second, that is neither the sum **0.75** nor the mean **0.375**, and it is shorter than the shorter lens (Section 2). Third, this is the core of the paper. In dioptries it is $2.0000 + 4.0000 = \mathbf{6.0000}$ (Section 2). Fourth, but one step of dioptry moves the far point **4.000000000000** times less at the strong end (Section 3). Fifth, **−1** to **−2** D moves it 0.5 m; **−4** to **−8** D moves it **0.125** (Section 3). Sixth, the separator is whether additivity and even spacing can hold together (Section 4). **Keywords:** optical power, dioptry, myopia, spectacle lens, additivity.

Scope of this paper (scope note): No new theorem or law is claimed—the additivity of dioptric power, the combination of thin lenses and the far point of myopia are all standard. No prescription is given—correction of vision, choice of spectacles and clinical judgement are not the subject; follow professional diagnosis. The thin lens approximation is used—thickness and separation are set to zero, though both affect the combination in practice. Astigmatism and aberration are excluded—cylindrical power, chromatic and spherical aberration are not the subject. Accommodation is excluded—the range of the crystalline lens and the near point are not the subject; only the far point. Relation to earlier papers: Paper 466 showed that not everything can be tuned at once—there too, taking logarithms turned multiplication into addition. Paper 300 showed that common root and distinct root can be decided—“being additive” and “being evenly spaced” are distinct roots by that criterion. Paper 459 showed that a round trip multiplies the loss by 1.9—there too, the choice of quantity changed the shape of the arithmetic. What is added is giving the combined focal length as 0.166667 m and showing it is neither sum nor mean, confirming the additivity of dioptries by inversion, tabulating far points at four powers, showing the ratio of far-point movements to be exactly 4, and placing the separator at whether additivity and even spacing hold together.

1. Notation and the Question

Setting: Define the power of a lens of focal length f [m] as

$$D = \frac{1}{f} \text{ [m}^{-1} = \text{D]}$$

Thin lenses in contact combine by simple addition of powers,

$$D_{\text{tot}} = D_1 + D_2$$

A myopic eye cannot see beyond its far point x , and moving that to infinity requires a power of $-1/x$.

Observation 1.1

Choosing better units is said to make the arithmetic easier. Take reciprocals and multiplication becomes addition—the same idea as a logarithm.

Observation 1.2

But the eased quantity need not be evenly spaced. Being addable and having each step mean the same thing are different—that is what is examined here.

Question: What is gained and what is lost.

2. Focal Lengths Do Not Add; Powers Do

Computation 2.1: Take the combined focal length of 0.50 m and 0.25 m.

0.166667 m.

Observation 2.2

Neither the sum 0.75 nor the mean 0.375. Different from either naive combination—focal lengths mean nothing added or averaged.

Observation 2.3

And shorter than the shorter one, $0.25 \cdot 0.166667 < 0.25$ —two together are stronger than either alone.

Computation 2.4: Write the same combination in dioptries.

Lens	Focal length [m]	Power [D]
First	0.50	2.0000
Second	0.25	4.0000
Combined	0.166667	6.0000

Observation 2.5

$2.0000 + 4.0000 = 6.0000$, one line. And inverting returns $1/0.166667 = 6.000000000000$ exactly—this is why the diopetre exists as a unit.

Observation 2.6

So the choice of unit changes the shape of the arithmetic. One physics, two ways of writing it—addition on one side, a harmonic relation on the other.

3. But Dioptres Are Not Evenly Spaced

Computation 3.1: Take the far point at four powers.

Power [D]	Far point [m]
−1.0	1.0000
−2.0	0.5000
−4.0	0.2500
−8.0	0.1250

Observation 3.2

From −1 to −2 D the far point moves 0.5 m. From 1.0000 to 0.5000—a large change.

Observation 3.3

From −4 to −8 D it moves only 0.125 m. The same doubling—and the ratio is exactly **4.000000000000**.

Observation 3.4

So the scale of dioptres is not the scale of seeing. What is evenly ruled is the power, not the far point—the price of making it additive.

Observation 3.5

Only one of the two can be evenly spaced. Because $D = 1/x$ is a reciprocal relation—even spacing in D crowds x , and even spacing in x crowds D .

Observation 3.6

And the far point is exactly recoverable from the power. $-1/(-4.0000) = 0.250000000000$ —no information is lost; only the even spacing is.

4. The Separator

The separator: can additivity and even spacing hold together.

Observation 4.1

If they can, the change of unit costs nothing. Addition works and each step means the same—some quantities are like that.

Observation 4.2

If they cannot, one must be chosen. Power adds but is not evenly spaced—focal length can be evenly spaced but does not add.

Observation 4.3

These are distinct roots by the criterion of Paper 300. Both are called “the strength of a lens”—one an additive quantity, one a geometric distance.

Observation 4.4

This stands where Paper 466 stands. There, frequency ratios became additive through a logarithm, giving the evenly spaced scale of cents—here reciprocals gave addition and lost the spacing. The same manoeuvre, two outcomes.

Observation 4.5

The difference is whether the transform is a logarithm or a reciprocal. A logarithm turns multiplication into addition and makes ratios evenly spaced—a reciprocal turns a harmonic relation into addition but does not preserve the spacing of the original.

Observation 4.6

It overlaps Paper 459 as well. There, products of efficiencies became powers and the choice of quantity changed the arithmetic—here choosing power gives a sum. In both, the notation exposes the structure.

Observation 4.7

To be honest—the thin lens approximation is used. Thickness and separation are set to zero, though both affect the combination—so 6.0000 D is a value under that approximation; the conclusion that additivity and even spacing cannot hold together does not depend on it.

5. Roll Call and Closing

1. The combination is 0.166667 m (Computation 2.1): **neither sum nor mean.**
2. Shorter than the shorter (Observation 2.3): **0.166667 < 0.25.**
3. In dioptres, 6.0000 (Computation 2.4): **the core.**
4. Inversion returns it exactly (Observation 2.5): **6.000000000000.**
5. $-1 \rightarrow -2$ D moves 0.5 m (Computation 3.1): **a large change.**
6. $-4 \rightarrow -8$ D moves 0.125 m (Computation 3.1): **the same doubling.**
7. The ratio is 4.000000000000 (Observation 3.3): **not evenly spaced.**

8. No information is lost (Observation 3.6): **only the spacing.**
9. The separator is whether both hold (Observation 4.3): **distinct roots by Paper 300.**
10. Logarithm against reciprocal (Observation 4.5): **one preserves spacing.**

Closing: Choosing better units is said to make the arithmetic easier—take reciprocals and multiplication becomes addition, the same idea as a logarithm. But the eased quantity need not be evenly spaced: being addable and having each step mean the same thing are different. Focal lengths do not add—stacking 0.50 m and 0.25 m gives **0.166667** m, neither the sum 0.75 nor the mean 0.375. Focal lengths mean nothing added or averaged. And the result is shorter than the shorter lens, so two together are stronger than either alone. In dioptries it is $2.0000 + 4.0000 = 6.0000$, one line, and inverting returns 6.000000000000 exactly. This is why the dioptry exists as a unit. The choice of unit changes the shape of the arithmetic: one physics, two ways of writing it, addition on one side and a harmonic relation on the other. The price is the even spacing—powers of -1.0 , -2.0 , -4.0 and -8.0 D give far points of 1.0000, 0.5000, 0.2500 and **0.1250** m. From -1 to -2 D the far point moves 0.5 m, but from -4 to -8 D only 0.125: the same doubling, and a ratio of exactly **4.000000000000**. So the scale of dioptries is not the scale of seeing—what is evenly ruled is the power, not the far point, and that is the price of making it additive. Only one of the two can be evenly spaced, because $D = 1/x$ is reciprocal: even spacing in D crowds x , and even spacing in x crowds D . But the far point is exactly recoverable, $-1/(-4.0000) = 0.250000000000$, so no information is lost; only the even spacing is. One thing separates them—can additivity and even spacing hold together. If they can, the change of unit costs nothing: addition works and each step means the same. If they cannot, one must be chosen—power adds but is not evenly spaced, focal length can be evenly spaced but does not add. They are distinct roots by the criterion of Paper 300, one an additive quantity and one a geometric distance. This stands where Paper 466 stands, where frequency ratios became additive through a logarithm and gave the evenly spaced scale of cents: the same manoeuvre, two outcomes. The difference is whether the transform is a logarithm or a reciprocal—a logarithm makes ratios evenly spaced, a reciprocal does not preserve the spacing of the original. It overlaps Paper 459 too, where products of efficiencies became powers: in both, the notation exposes the structure. To be honest—the thin lens approximation is used, with thickness and separation set to zero though both affect the combination. So 6.0000 D is a value under that approximation; the conclusion that additivity and even spacing cannot hold together does not depend on it.

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Yuuki YAMAGISHI (山岸 勇輝)

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