

The Same Proportional Increase Buys Less at the Top

— Raising the compression ratio by **1.3333** gains **+0.0531** in efficiency starting from **6** but only **+0.0359** starting from **16** — the ratio is **1.480429**, matching the closed form $(r_2/r_1)^{\gamma-1}$ exactly — an efficiency of **0.70** needs a ratio of **20.2860** — [Paper 468]

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— *From thinking to syncing.* —

Abstract

Raising the compression ratio raises efficiency—so it is said. This paper shows that the same proportional increase buys **1.480429** times less at the top—and that the ratio has a closed form. No new theorem or law is claimed. First, a compression ratio of **8** gives a theoretical efficiency of **0.5647**, matching the standard figure (Section 2). Second, even a ratio of **20** gives only **0.6983** (Section 2). Third, this is the core of the paper. The same **1.3333** gains **+0.0531** from **6** but **+0.0359** from **16**, a ratio of **1.480429** (Section 3). Fourth, that ratio matches the closed form $(r_2/r_1)^{\gamma-1}$ exactly (Section 3). Fifth, an efficiency of **0.70** needs a ratio of **20.2860**, and even **1000** gives only **0.936904** (Section 3). Sixth, the separator is whether the effect is additive or a power (Section 4). **Keywords:** Otto cycle, compression ratio, thermal efficiency, heat capacity ratio, internal combustion engine.

Scope of this paper (scope note): No new theorem or law is claimed—the theoretical efficiency of the Otto cycle, the ratio of specific heats and adiabatic processes are all standard. No engine is designed—combustion, ignition timing, knock, emissions and materials are not the subject, and real design is bound tightly by them. No real efficiency is asserted—friction, heat loss, pumping loss and incomplete combustion are all excluded; real efficiency is far lower. The specific heat ratio is constant— $\gamma = 1.4$ is used, though it varies with temperature and composition. Other cycles are excluded—Diesel, Miller, Atkinson and Brayton are not the subject. Relation to earlier papers: Paper 467 showed that making a quantity additive can cost it its even spacing—there too, the form of the transformation set the effect. Paper 300 showed that common root and distinct root can be decided—“an additive improvement” and “a power-law improvement” are distinct roots by that criterion. Paper 458 showed that the lift coefficient acts only through a square root—there too, an exponent defeated intuition. What is added is tabulating the efficiency at six compression ratios, giving the ratio of gains at fixed proportional increase as 1.480429, confirming it against the closed form at two starting points, solving for the ratio giving 0.70 as 20.2860 and matching the closed form, and placing the separator at additive versus power.

1. Notation and the Question

Setting: The theoretical efficiency of the Otto cycle, with compression ratio r and specific heat ratio γ , is

$$\eta(r) = 1 - r^{1-\gamma}$$

Below, $\gamma = 1.4$.

Observation 1.1

Raising the compression ratio raises efficiency—so it is said. η is increasing in r —no lie in that part.

Observation 1.2

But there are two ways of raising it. Adding two is not multiplying by 1.33—and which sense of “the same increase” is meant changes the answer.

Question: Where does the effect fall off.

2. Even a Ratio of Twenty Gives Only 0.6983

Computation 2.1: Take the theoretical efficiency at six compression ratios.

Compression ratio	Theoretical efficiency
6	0.5116
8	0.5647
10	0.6019
12	0.6299
16	0.6701
20	0.6983

Observation 2.2

The **0.5647** at a ratio of 8 matches the standard figure. The textbook number for a petrol engine—an independent check.

Observation 2.3

Raising the ratio from 6 to 20, a factor of 3.33, takes the efficiency only from 0.5116 to 0.6983. A factor of 1.36—a small return on a large increase.

Observation 2.4

And at a ratio of 1 the efficiency is exactly 0.0000000000000000. No compression, no work—which confirms that the expression is set up correctly.

3. The Ratio Is 1.480429, Exactly the Closed Form

Computation 3.1: Apply the same 1.3333 increase at two starting points.

Operation	Gain in efficiency	Starting efficiency
$6 \rightarrow 8$	$+0.0531$	0.5116
$16 \rightarrow 21.33$	$+0.0359$	0.6701

Observation 3.2

The same proportion buys **1.480429** times less at the top. 0.0531 against 0.0359 —the same effort returning two thirds.

Observation 3.3

And the ratio matches the closed form $(r_2/r_1)^{\gamma-1}$ exactly. $(16/6)^{0.4} = 1.480429$ —to within 10^{-9} relative, computation and formula land on the same value.

Observation 3.4

The same formula holds from other starting points. The ratio for $9 \rightarrow 12$ against $30 \rightarrow 40$ matches $(40/12)^{0.4}$ —structure, not coincidence.

Observation 3.5

I first estimated this ratio at “over three” and was wrong. The computed value was 1.8845 (for $6 \rightarrow 8$ against $16 \rightarrow 20$)—rewriting the test as a closed form rather than fitting the threshold to the result is what made it meaningful.

Computation 3.6: Solve by bisection for the ratio giving an efficiency of 0.70.

20.2860 (matching the closed form $0.30^{1/(1-\gamma)}$).

Observation 3.7

A ratio of 100 gives 0.8415 and 1000 gives 0.936904. Never reaching 1 in finite compression—because $r^{1-\gamma}$ only approaches zero.

4. The Separator

The separator: is the effect additive, or a power.

Observation 4.1

Additive, and the same increase returns the same from anywhere. +2 is always +2 worth—improvements are easy to forecast.

Observation 4.2

A power, and the return depends on where one starts. The same proportion falls off by $(r_2/r_1)^{\gamma-1}$ —without saying where one is, the effect is undetermined.

Observation 4.3

These are distinct roots by the criterion of Paper 300. Both are called “the effect of raising the compression ratio”—one a property of the operation, one of the operation paired with a starting point.

Observation 4.4

This stands where Paper 458 stands. There, the lift coefficient acted on stall speed only through a square root—here the compression ratio acts only through the power $\gamma - 1 = 0.4$. In both, an exponent defeats intuition.

Observation 4.5

It overlaps Paper 467 as well. There, making a quantity additive cost it its even spacing—here taking logarithms gives addition but leaves the compression ratio unevenly spaced. The same structure.

Observation 4.6

So a proposal to “raise the compression ratio” should carry the present value. From 6 or from 16 differs by 1.48 in return—one phrase naming two different effects.

Observation 4.7

To be honest—these are theoretical values, not real efficiencies. Friction, heat loss, pumping loss and incomplete combustion are all excluded, and real efficiency is far lower—and at high compression a different wall, knock, stands in the way.

5. Roll Call and Closing

1. A ratio of 8 gives **0.5647** (Computation 2.1): **matching the standard.**
2. Even 20 gives only **0.6983** (Computation 2.1): **3.33 in, 1.36 out.**
3. A ratio of 1 gives exactly zero (Observation 2.4): **a check on the setup.**
4. $6 \rightarrow 8$ gains **+0.0531** (Computation 3.1): **the core.**
5. $16 \rightarrow 21.33$ gains **+0.0359** (Computation 3.1): **the same proportion.**
6. A ratio of **1.480429** (Observation 3.2): **two thirds of the return.**
7. Exactly the closed form (Observation 3.3): $(16/6)^{0.4}$.
8. **0.70** needs **20.2860** (Computation 3.6): **matching the closed form.**
9. The separator is additive or power (Observation 4.3): **distinct roots by Paper 300.**
10. Theoretical, not real (Observation 4.7): **knock is a different wall.**

Closing: Raising the compression ratio raises efficiency—so it is said— η is increasing in r and there is no lie in that. But there are two ways of raising it: adding two is not multiplying by 1.33, and which sense of “the same increase” is meant changes the answer. First, the room for improvement is smaller than it looks—a compression ratio of 8 gives a theoretical efficiency of **0.5647**, matching the textbook figure for a petrol engine. Raising the ratio from 6 to 20, a

factor of **3.33**, takes the efficiency only from 0.5116 to **0.6983**, a factor of 1.36: a small return on a large increase. And at a ratio of 1 the efficiency is exactly 0.000000000000000—no compression, no work, which confirms the expression. And the effect depends on where one starts—the same 1.3333 increase gains **+0.0531** from 6 but only **+0.0359** from 16, so the top buys **1.480429** times less: the same effort returning two thirds. And the ratio matches the closed form $(r_2/r_1)^{\gamma-1}$ exactly, $(16/6)^{0.4} = 1.480429$, to within 10^{-9} relative. The same formula holds from other starting points, $9 \rightarrow 12$ against $30 \rightarrow 40$ matching $(40/12)^{0.4}$: structure, not coincidence. I first estimated this ratio at “over three” and was wrong, the computed value being 1.8845. Rewriting the test as a closed form rather than fitting the threshold to the result is what made it meaningful. An efficiency of 0.70 needs a ratio of **20.2860**, matching $0.30^{1/(1-\gamma)}$, and a ratio of 100 gives 0.8415 while 1000 gives 0.936904: never reaching 1 in finite compression. One thing separates them—is the effect additive, or a power. Additive, and the same increase returns the same from anywhere, making improvements easy to forecast. A power, and the return depends on the starting point, falling off by $(r_2/r_1)^{\gamma-1}$: without saying where one is, the effect is undetermined. They are distinct roots by the criterion of Paper 300, one a property of the operation and one of the operation paired with a starting point. This stands where Paper 458 stands, where the lift coefficient acted only through a square root: here the compression ratio acts only through the power 0.4. It overlaps Paper 467 too, where making a quantity additive cost it its even spacing: the same structure. So a proposal to “raise the compression ratio” should carry the present value—from 6 or from 16 differs by 1.48 in return, one phrase naming two different effects. To be honest—these are theoretical values, not real efficiencies. Friction, heat loss, pumping loss and incomplete combustion are all excluded and real efficiency is far lower; and at high compression a different wall, knock, stands in the way.

References

1. Otto, N. A. (1877). *Verbesserungen an Gaskraftmaschinen*. Deutsches Reichspatent 532.
 2. Carnot, S. (1824). *Réflexions sur la Puissance Motrice du Feu*. Bachelier.
 3. Heywood, J. B. (1988). *Internal Combustion Engine Fundamentals*. McGraw-Hill.
 4. Yamagishi, Y. (2026). *The Three Hundredth Census: Whether Two Things Share a Root Is Decidable* [Paper 300]. Zenodo. 10.5281/zenodo.22171914
 5. Yamagishi, Y. (2026). *Raising the Lift Coefficient by Sixty Per Cent Lowers the Stall Speed by Twenty* [Paper 458]. Zenodo. 10.5281/zenodo.22733085
 6. Yamagishi, Y. (2026). *Making a Quantity Additive Can Cost It Its Even Spacing* [Paper 467]. Zenodo. 10.5281/zenodo.22733318
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