

On Nights When Distant Sounds Carry, the Sky Is Warm

— The speed of sound is **331.3000** m/s at 0°C and **349.0194** at 30 — under a normal atmosphere a ray leaving at 80 degrees bends up to **77.5204**, but under an inversion it bends down to **83.3000** — past the critical angle of **82.5593** degrees it reflects back to the ground — [Paper 474]

Yuuki YAMAGISHI (山岸 勇輝)

Yamagishi Complexity Torus Lab, Japan

September 2026

— *From thinking to syncing.* —

Abstract

Sound is thought to carry because the night is quiet. This paper shows that the temperature aloft is bending the sound back—under a normal atmosphere it escapes upward; under an inversion it returns. The sign reverses. No new theorem or law is claimed. First, the speed of sound at 0°C is the standard **331.3000000** m/s (Section 2). Second, it runs from 325.1790 at −10 to **349.0194** at 30 (Section 2). Third, this is the core of the paper. A ray leaving at 80 degrees becomes **77.5204** under a normal atmosphere and **83.3000** under an inversion (Section 3). Fourth, that the bending reverses is shown by the sign of the departure from 80 degrees (Section 3). Fifth, beyond the critical angle of **82.5593** degrees no refracted solution exists and the ray reflects totally (Section 3). Sixth, the separator is whether this is a question of attenuation or of refraction (Section 4). **Keywords:** sound refraction, speed of sound, temperature inversion, Snell's law, sound propagation.

Scope of this paper (scope note): No new theorem or law is claimed—the temperature dependence of the speed of sound, Snell’s law and atmospheric refraction of sound are all standard.No weather is forecast—when and where inversions form is not the subject.No noise is assessed—how loud it seems and whether a limit is met are not the subject.Wind is excluded—wind gradients also bend rays, in practice as much as temperature or more; only temperature is treated.Two layers only—ground and aloft; the real atmosphere varies continuously and rays curve.Absorption and ground effect are excluded—atmospheric absorption and reflection or absorption at the ground are not the subject.Relation to earlier papers: Paper 473 showed that what separates is the gel and not the field—there too, the named side was not the working side.Paper 300 showed that common root and distinct root can be decided —“sound diminishing” and “sound bending” are distinct roots by that criterion.Paper 456 showed that doubling the distance does not halve the noise—it treated geometric spreading alone and excluded refraction; this paper takes the outside of that.What is added is giving the speed at four temperatures and matching known values, giving the refracted angle under a normal atmosphere and an inversion, showing the reversal by sign, giving the critical angle as 82.5593 degrees and confirming that no solution exists beyond it, and placing the separator at attenuation versus refraction.

1. Notation and the Question

Setting: In dry air the speed of sound at Celsius temperature T is

$$c(T) = 331.3 \sqrt{1 + \frac{T}{273.15}}$$

and a ray crossing between layers of different speed obeys Snell’s law,

$$\frac{\sin \theta_1}{c_1} = \frac{\sin \theta_2}{c_2}$$

with θ measured from the vertical.Below, the ground is at 20°C, the normal air aloft at 15, and the inversion aloft at 25.

Observation 1.1

Sound is thought to carry because the night is quiet.Background noise falls after dark—so distant sounds are heard, runs the account.

Observation 1.2

But the speed of sound depends on temperature. Warmer is faster—and where the speed varies with place, rays bend.

Question: Which way, and by how much.

2. Forty Degrees Moves the Speed by 23.84 m/s

Computation 2.1: Take the speed at four temperatures.

Temperature [°C]	Speed [m/s]
−10	325.1790
0	331.3000
15	340.2751
30	349.0194

Observation 2.2

At 0°C it is the standard **331.3000000** m/s. Trivially, by construction—but the 340.2751 at 15 also matches the textbook “about 340”, which is an independent check.

Observation 2.3

Forty degrees moves it by 23.84 m/s. Seven point three per cent—large, spoken of as a refractive index.

Observation 2.4

And temperature varies with height. Normally colder going up; on a clear night the ground cools and it reverses—to sound, the atmosphere is a medium with a gradient of refractive index.

3. Normally Upward, Under an Inversion Downward

Computation 3.1: Take the angle aloft for a ray leaving the 20°C ground at 80 degrees.

Aloft	Speed [m/s]	Refracted [deg]	From 80
Normal (15°C)	340.2751	77.5204	−2.4796
Inversion (25°C)	346.1292	83.3000	+3.3000

Observation 3.2

Under a normal atmosphere the angle shrinks.From 80 to 77.5204 degrees—towards the vertical, that is, upward.

Observation 3.3

Under an inversion it grows.From 80 to 83.3000 degrees—towards the horizontal, and eventually back down.

Observation 3.4

The negative product of the departures shows the reversal.−2.4796 and +3.3000—same air, same sound, only the temperature aloft differing.

Computation 3.5: Take the critical angle for total reflection under the inversion.

82.5593 degrees.

Observation 3.6

Beyond it no refracted solution exists.At 83.5593 degrees the sine exceeds 1—the ray cannot escape upward and reflects totally back to the ground.

Observation 3.7

So an inversion becomes a waveguide holding sound at the ground. What should have escaped comes back—which is what a night of far-carrying sound really is.

4. The Separator

The separator: is this a question of attenuation, or of refraction.

Observation 4.1

Attenuation, and it is set by background noise and distance. Quiet means audible—a natural account of night.

Observation 4.2

Refraction, and it is set by the temperature aloft. The same quiet gives nothing if the air above is cold—the condition is in the sky.

Observation 4.3

These are distinct roots by the criterion of Paper 300. Both are called “it carries well”—one a ratio against noise, one a property of the ray path.

Observation 4.4

This is the side Paper 456 excluded. There, geometric spreading alone was treated and refraction was set outside by the scope note—this paper takes the outside. The distance-decay formula holds only for straight rays.

Observation 4.5

It overlaps Paper 473 as well. There, what separated was the gel and not the field—here what carries the sound is the temperature aloft and not the quiet. In both, the conspicuous side is not the working one.

Observation 4.6

And an inversion forms on a clear, still night when the ground radiates its heat away. “A quiet night” and “a night with an inversion” are the same night—so the two accounts blend and are hard to tell apart.

Observation 4.7

To be honest—wind is not included. Wind gradients also bend rays, in practice as much as temperature or more—82.5593 degrees is a temperature-only figure, and downwind the critical angle is shallower still.

5. Roll Call and Closing

1. **331.3000000 m/s at 0°C** (Observation 2.2): **the standard value.**
2. **340.2751 at 15** (Computation 2.1): **matching the textbook.**
3. **23.84 m/s over forty degrees** (Observation 2.3): **seven per cent.**
4. **77.5204 degrees normally** (Computation 3.1): **bending upward.**
5. **83.3000 under an inversion** (Computation 3.1): **the core.**
6. **A negative product** (Observation 3.4): **the reversal.**
7. **Critical angle 82.5593 degrees** (Computation 3.5): **total reflection.**
8. **No solution beyond it** (Observation 3.6): **the sine exceeds 1.**
9. **The separator is attenuation or refraction** (Observation 4.3): **distinct roots by Paper 300.**
10. **The side Paper 456 excluded** (Observation 4.4): **it assumed straight rays.**

Closing: Sound is thought to carry because the night is quiet—background noise falls after dark, so distant sounds are heard. But the speed of sound depends on temperature, warmer being faster, and where the speed varies with place, rays bend. Temperature moves the speed considerably—**331.3000000 m/s at 0°C**, **340.2751 at 15** matching the textbook “about 340”, and from **325.1790 at –10** to **349.0194 at 30**: **23.84 m/s over forty degrees**, seven point three per cent. And temperature varies with height—normally colder going up, reversing on a clear night when the

ground cools. To sound, the atmosphere is a medium with a gradient of refractive index. The bending reverses with the temperature aloft—a ray leaving the 20°C ground at 80 degrees becomes **77.5204** under a normal atmosphere and **83.3000** under an inversion. Normally the angle shrinks towards the vertical and the ray goes up; under an inversion it grows towards the horizontal and comes back down. The departures are -2.4796 and $+3.3000$, and their negative product shows the reversal: same air, same sound, only the temperature aloft differing. The critical angle for total reflection under the inversion is **82.5593** degrees, and beyond it no refracted solution exists—at **83.5593** degrees the sine exceeds 1, so the ray cannot escape and reflects totally back to the ground. So an inversion becomes a waveguide holding sound at the ground, and what should have escaped comes back. That is what a night of far-carrying sound really is. One thing separates them—is this a question of attenuation, or of refraction. Attenuation, and it is set by background noise and distance: quiet means audible, a natural account of night. Refraction, and it is set by the temperature aloft: the same quiet gives nothing if the air above is cold, because the condition is in the sky. They are distinct roots by the criterion of Paper 300, one a ratio against noise and one a property of the ray path. This is the side Paper 456 excluded, where geometric spreading alone was treated: the distance-decay formula holds only for straight rays. It overlaps Paper 473 too, where what separated was the gel and not the field: in both, the conspicuous side is not the working one. And an inversion forms on a clear, still night when the ground radiates its heat away, so “a quiet night” and “a night with an inversion” are the same night, and the two accounts blend. To be honest—wind is not included. Wind gradients also bend rays, in practice as much as temperature or more; **82.5593** degrees is a temperature-only figure, and downwind the critical angle is shallower still.

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Yuuki YAMAGISHI (山岸 勇輝)

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