

What Breaks It Is Not the Iron Lost but the Rust That Takes Its Place

— Losing **0.1** mm from a bar of radius **8** mm turns **4.995132** mm² of iron into **19.980529** mm² of rust at a volume ratio of **4**, swelling the radius by **0.292768** mm — cracking **30** mm of cover takes a loss of only **0.001000** mm — and at a volume ratio of **1** the swelling is exactly zero — [Paper 476]

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— *From thinking to syncing.* —

Abstract

Corroding reinforcement is thought to weaken by growing thinner. This paper shows that what breaks the concrete is not the thinning but the swelling—a loss of **0.001000** mm cracks the cover. No new theorem or law is claimed. First, a loss of **0.1** mm removes **4.995132** mm² of iron per unit length (Section 2). Second, at a volume ratio of **4** that becomes **19.980529** mm² of rust, a gain of **14.985397** (Section 2). Third, this is the core of the paper. Cracking **30** mm of cover takes a loss of only **0.001000** mm (Section 3). Fourth, at a volume ratio of **1** the swelling is exactly **0.000000000000** mm (Section 2). Fifth, at a ratio of **2.0** the required loss is **0.003001** mm and at **6.4** it is **0.000556** (Section 3). Sixth, the separator is whether the section shrinks or the volume grows (Section 4). **Keywords:** corrosion of reinforcing steel, expansion of rust, cracking of concrete, concrete cover, Pilling-Bedworth ratio.

Scope of this paper (scope note): No new theorem or law is claimed—the volume expansion of iron corrosion products and the cracking of concrete cover are all standard. No structure is assessed—capacity, remaining life and the need for repair are not the subject; real judgement rests on professional survey. Corrosion rate is excluded—how many years 0.001 mm takes is not the subject, nor are carbonation, chlorides and electrochemistry. The mechanics are simplified—strain is taken as swelling divided by cover thickness; in reality the stress field is three-dimensional and the hoop tension governs. The numbers are representative—volume ratios of 2 to 6.4 and a tensile failure strain of 100 microstrain are representative, not measured. Relation to earlier papers: Paper 475 showed that nine tenths of what is added never reaches the interface—there too, the same act was useful or wasteful depending on the aim. Paper 300 showed that common root and distinct root can be decided—“loss of section” and “growth of volume” are distinct roots by that criterion. Paper 454 showed that a cubic load still needs only a linear width—there too, geometry settled the answer. What is added is tabulating the volume ratio of four corrosion products, giving the iron lost and the rust formed, giving the radial swelling as 0.292768 mm, confirming exactly zero swelling at a ratio of 1, solving for the loss that cracks the cover as 0.001000 mm and inverting, and placing the separator at section versus volume.

1. Notation and the Question

Setting: When a bar of radius r loses a thickness δ , the iron lost per unit length is

$$V_{\text{lost}} = \pi(r^2 - (r - \delta)^2)$$

With a corrosion product of volume ratio κ , the rust occupies κV_{lost} . Taking the rust to accumulate outward, the outer radius R follows from

$$\pi(R^2 - (r - \delta)^2) = \kappa V_{\text{lost}}$$

and the radial swelling is $R - r$. Below, $r = 8$ mm, the cover is 30 mm and the tensile failure strain of concrete is 100 microstrain.

Observation 1.1

Corroding reinforcement is thought to weaken by growing thinner. Less section, less capacity—a natural argument.

Observation 1.2

But the rust does not disappear. It stays in the same place, occupying more—and pushes what surrounds it.

Question: Which arrives first.

2. Rust Swells Fourfold

Computation 2.1: Tabulate the volume ratio of four corrosion products relative to iron.

Product	Volume ratio
Fe_3O_4	2.0000
Fe_2O_3	2.2000
$\text{Fe}(\text{OH})_3$	4.0000
Hydrated hydroxide	6.4000

Observation 2.2

Every one exceeds 1.From 2.0 to 6.4—iron always swells when it rusts.

Computation 2.3: Take the volumes for a bar of radius 8 mm losing 0.1 mm at a ratio of 4.

Quantity	Value [mm ²]
Iron lost	4.995132
Rust formed	19.980529
Increase	14.985397

Observation 2.4

The increase is three times the iron lost. $\kappa - 1 = 3$ —matching the closed form exactly.

Computation 2.5: Convert that increase into a radial swelling.

0.292768 mm.

Observation 2.6

Larger than the 0.1 mm that was lost. By a factor of 2.93—far from growing thinner, the outside is growing.

Observation 2.7

At a volume ratio of 1 the swelling is exactly 0.000000000000 mm. Rust of the same volume as the iron leaves the radius unchanged—a boundary, and a check on the expression.

3. Cracking Takes a Loss of 0.001000 mm

Computation 3.1: Solve by bisection for the loss at which swelling over cover reaches 100 microstrain.

0.001000 mm (inverting returns a strain of 1.000000×10^{-4}).

Observation 3.2

One micrometre. Invisible—a loss of section of only 0.025 per cent.

Observation 3.3

At that point the capacity has hardly fallen. 0.025 per cent of section gone—and the concrete is already cracked.

Computation 3.4: Vary the volume ratio and take the loss that cracks the cover.

Volume ratio	Loss to crack [mm]
2.0	0.003001
4.0	0.001000
6.4	0.000556

Observation 3.5

The larger the ratio, the less loss is needed. A factor of 5.40 between 2.0 and 6.4—which product forms decides the life.

Observation 3.6

And once cracked, water and oxygen enter more easily. Corrosion accelerates and swells further—a positive feedback.

Observation 3.7

So inspection should look at cracks in the concrete, not thinning of the bar. The thinning cannot be measured until the end—the cracks appear far sooner.

4. The Separator

The separator: does the section shrink, or does the volume grow.

Observation 4.1

If the section shrinks, weakening follows the amount lost. 0.025 per cent gives 0.025 per cent—it proceeds slowly.

Observation 4.2

If the volume grows, what it pushes against breaks. The cover thickness sets the ceiling—and it arrives far sooner.

Observation 4.3

These are distinct roots by the criterion of Paper 300. Both are called “deterioration by corrosion of the reinforcement”—one a quantity of iron, one of iron paired with the cover.

Observation 4.4

This stands where Paper 454 stands. There, no answer came without looking at both load and resistance—here both the lost side and the gained side must be seen. In both, one side alone misleads.

Observation 4.5

It overlaps Paper 475 as well. There, the same act was useful or wasteful depending on the aim—here the same corrosion is trivial for capacity and fatal for the cover.

Observation 4.6

And thickening the cover raises the loss required. Because strain is swelling over cover—so the cover is not only a barrier against corrosion but room to absorb the swelling.

Observation 4.7

To be honest—the mechanics were simplified. Strain was taken as swelling over cover, whereas the real field is three-dimensional and governed by hoop tension—0.001000 mm depends on that; that swelling occurs whenever the volume ratio exceeds 1 follows from geometry alone.

5. Roll Call and Closing

1. Volume ratios of 2.0 to 6.4 (Computation 2.1): all exceed 1.
2. 4.995132 mm² of iron lost (Computation 2.3): for 0.1 mm.
3. 19.980529 mm² of rust (Computation 2.3): a gain of 14.985397.
4. The gain is three times the loss (Observation 2.4): exactly $\kappa - 1$.

5. A radial swelling of **0.292768 mm** (Computation 2.5): **2.93 times the loss.**
6. Exactly zero at a ratio of **1** (Observation 2.7): **a boundary confirmed.**
7. Cracking at **0.001000 mm** (Computation 3.1): **the core.**
8. A section loss of **0.025 %** (Observation 3.2): **capacity nearly intact.**
9. A factor of **5.40** with the ratio (Observation 3.5): **the product sets the life.**
10. The separator is section or volume (Observation 4.3): **distinct roots by Paper 300.**

Closing: Corroding reinforcement is thought to weaken by growing thinner—less section, less capacity, a natural argument. But the rust does not disappear: it stays in the same place occupying more, and pushes what surrounds it. Rust swells—corrosion products run from 2.0 to 6.4 times the volume of the iron, all exceeding 1: iron always swells when it rusts. A bar of radius 8 mm losing 0.1 mm turns **4.995132 mm²** of iron into **19.980529 mm²** of rust at a ratio of 4, a gain of **14.985397**, which is three times the iron lost and matches $\kappa - 1$ exactly. The radial swelling is **0.292768 mm**, **2.93 times the 0.1 mm lost**: far from growing thinner, the outside is growing. At a volume ratio of 1 the swelling is exactly **0.000000000000 mm**, since rust of the same volume leaves the radius unchanged—a boundary and a check on the expression. And the cracking arrives astonishingly early—cracking 30 mm of cover takes a loss of only **0.001000 mm**, one micrometre, with inversion returning a strain of 1.000000×10^{-4} . That is a loss of section of only 0.025 per cent, at which point the capacity has hardly fallen—and the concrete is already cracked. Varying the volume ratio, the required loss is **0.003001 mm** at 2.0 and **0.000556** at 6.4, a factor of 5.40: which product forms decides the life. And once cracked, water and oxygen enter more easily, corrosion accelerates and it swells further—a positive feedback. So inspection should look at cracks in the concrete and not thinning of the bar: the thinning cannot be measured until the end, while the cracks appear far sooner. One thing separates them—does the section shrink, or does the volume grow. If the section shrinks, weakening follows the amount lost and proceeds slowly. If the volume grows, what it pushes against breaks, the cover thickness sets the ceiling, and it arrives far sooner. They are distinct roots by the criterion of Paper 300, one a quantity of iron and one of iron paired with the cover. This stands where Paper 454 stands, where no answer came without looking at both load and resistance: in both, one side alone misleads. It overlaps Paper 475 too, where the same act was useful or wasteful depending on the aim: here the same corrosion is trivial for capacity and fatal for the cover. And thickening the cover raises the loss required, since strain is swelling over cover—so the cover is not only a barrier against corrosion but room to absorb the swelling. To be honest—the mechanics were simplified, with strain taken as swelling over cover where the real field is three-dimensional and governed by hoop tension. 0.001000 mm depends on that; that swelling occurs whenever the volume ratio exceeds 1 follows from geometry alone.

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