

While the Wrong Input Is Being Added, the Limit Is Somewhere Else

— With sufficiencies of **N0.90**, **P0.50** and **K1.25** the yield is **0.500000** — adding **10** of **N** leaves it at **0.500000**, adding **10** of **P** takes it to **0.750000** — and once **P** is satisfied the limit moves to **N**, after which **P** does nothing — [Paper 477]

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— *From thinking to syncing.* —

Abstract

Supplying what is short is said to increase the result. This paper shows that supplying something short but not the smallest has exactly zero effect—and that satisfying it moves the limit to another element. No new theorem or law is claimed. First, the yield equals exactly the sufficiency of the scarcest element (Section 2). Second, adding to a non-minimal element changes it by exactly **0.0000000000000000** (Section 2). Third, this is the core of the paper. The same **10 kg** gives **0.000000** in **N** and **+0.250000** in **P** (Section 3). Fourth, satisfying **P** moves the limit to **N** (Section 3). Fifth, after the move, adding more **P** changes it by exactly zero (Section 3). Sixth, the separator is whether contributions add or a minimum decides (Section 4). **Keywords:** Liebig's law of the minimum, limiting factor, fertiliser, plant nutrients, Mitscherlich's law.

Scope of this paper (scope note): No new theorem or law is claimed—Liebig’s law of the minimum, nutrient sufficiency and the movement of the limiting factor are all standard. No fertilisation is prescribed—which fertiliser and how much, and how to survey soil, are not the subject; real decisions rest on professional guidance. The law of the minimum is taken strictly—yield is the minimum of the sufficiencies; in reality elements interact and the limit shifts smoothly. No requirement values are asserted—N100, P40 and K80 kg/ha are illustrative and vary greatly with crop and soil. Excess damage is excluded—harm from over-application and runoff into the environment are not the subject. Relation to earlier papers: Paper 476 showed that what breaks the concrete is the rust and not the iron lost—there too, looking in the wrong place saw nothing. Paper 300 showed that common root and distinct root can be decided—“a sum of contributions” and “a minimum” are distinct roots by that criterion. Paper 471 showed that what must be added is the scarce side—there a ratio was off; here the minimum lies elsewhere. This is a second instance of the same shape. What is added is giving the yield from three sufficiencies, showing that adding to a non-minimal element has exactly zero effect, giving the difference for the same 10 kg as 0.250000, showing that the limit moves, confirming that the effect is then exactly zero, and placing the separator at sum versus minimum.

1. Notation and the Question

Setting: Let element i have requirement N_i and supply h_i , with sufficiency h_i/N_i . Taking Liebig’s law of the minimum strictly, the relative yield is

$$Y = \min_i \frac{h_i}{N_i}$$

Below, the requirements are N100, P40 and K80 kg/ha, and the supplies are N90, P20 and K100.

Observation 1.1

Supplying what is short is said to increase the result. Both N and P are short—so either ought to help.

Observation 1.2

But under the law of the minimum only the smallest acts. The others are not the minimum—“short” and “limiting” are different.

Question: What happens on supplying a short element.

2. Short but Not Smallest Does Nothing

Computation 2.1: Take the three sufficiencies and the resulting yield.

Element	Requirement	Supply	Sufficiency
N	100	90	0.900000
P	40	20	0.500000
K	80	100	1.250000

Observation 2.2

The yield is **0.500000**, exactly the sufficiency of P. Neither the 0.90 of N nor the 1.25 of K—one smallest element decides the whole.

Observation 2.3

N is short as well. At 0.90, short by 10 kg—but it is not limiting.

Computation 2.4: Take the yield after adding 10 kg of N.

0.500000. The change is exactly **0.0000000000000000**.

Observation 2.5

The shortfall is exactly filled and nothing happens. Not approximately but identically zero—“what was short has been supplied”, and there is no effect.

Observation 2.6

K is in excess. A sufficiency of 1.25—the surplus contributing nothing at all.

3. The Same Ten Kilos Differ by 0.250000

Computation 3.1: Take the yield after adding 10 kg to N and to P.

Added	Yield	Change
Nothing	0.500000	—
10 to N	0.500000	0.000000
10 to P	0.750000	+0.250000

Observation 3.2

The same 10 kg differs by **0.250000**. Zero on one side, a twenty-five per cent gain on the other—where it goes is everything.

Computation 3.3: Take the yield and the limiting element after satisfying P to its requirement of 40.

The yield is **0.900000** and the limit moves to N.

Observation 3.4

The yield now equals exactly the sufficiency of N. **0.900000**—evidence that the minimum has changed hands.

Computation 3.5: From that state, add a further 20 kg of P.

The yield stays at **0.900000**. The change is exactly **0.0000000000000000**.

Observation 3.6

The element that worked a moment ago stops working. The same P, added the same way—because the limit has moved.

Observation 3.7

So “it worked before” is no guarantee that it works again. Reasoning from last season’s gain to this season’s dose fails—without measuring again, which element is smallest is unknown.

Observation 3.8

Satisfying all three gives exactly **1.0000000000000000**. Leave any one short and it stops there—a check that the expression is set up correctly.

4. The Separator

The separator: do contributions add, or does a minimum decide.

Observation 4.1

If they add, anywhere returns in proportion. Ten kilos is ten kilos wherever it goes—allocation need not be considered.

Observation 4.2

If a minimum decides, everywhere but one place gives zero. What goes to a non-minimal element is wholly wasted—allocation becomes everything.

Observation 4.3

These are distinct roots by the criterion of Paper 300. Both are called “the effect of an input”—one an additive quantity, one decided by a minimum.

Observation 4.4

A second instance of the shape of Paper 471. There a ratio was off and the scarce side had to be added—here the minimum lies elsewhere. In both, where the input goes decides the effect.

Observation 4.5

It overlaps Paper 476 as well. There, nothing was visible until one looked at the concrete rather than the iron—here nothing works until the input is redirected.

Observation 4.6

And the law of the minimum carries the movement of the limit within it. Satisfy one and the next becomes smallest—so improvement is a sequence of remedies each of which works only for a while.

Observation 4.7

To be honest—the law of the minimum was taken strictly. Elements interact in reality and the limit shifts smoothly—the exact zero belongs to the model; that input to a non-minimal element loses most of its value does not.

5. Roll Call and Closing

1. The yield is 0.500000 (Observation 2.2): **the sufficiency of P.**
2. N is short too (Observation 2.3): **but not limiting.**
3. Adding N gives 0.0000000000000000 (Computation 2.4): **exactly zero.**
4. K is in excess (Observation 2.6): **contributing nothing.**
5. Adding P gives +0.250000 (Computation 3.1): **the core.**
6. Satisfying P moves the limit to N (Computation 3.3): **yield 0.900000.**
7. P then does nothing (Computation 3.5): **exactly zero.**
8. Past success is no guarantee (Observation 3.7): **measure again.**
9. All satisfied gives 1.0000000000000000 (Observation 3.8): **a check on the setup.**

10. The separator is sum or minimum (Observation 4.3): distinct roots by Paper 300.

Closing: Supplying what is short is said to increase the result—both N and P are short, so either ought to help. But under the law of the minimum only the smallest acts, and “short” and “limiting” are different. Short but not smallest does nothing—with sufficiencies of N 0.900000, P 0.500000 and K 1.250000, the yield is **0.500000**, exactly the sufficiency of P: one smallest element decides the whole. N is short by 10 kg at 0.90 but is not limiting, and adding 10 kg of it leaves the yield at **0.500000** with a change of exactly **0.0000000000000000**. The shortfall is exactly filled and nothing happens—not approximately but identically zero. And K at 1.25 is in excess, its surplus contributing nothing at all. The same ten kilos differ by 0.250000—0.000000 into N and +**0.250000** into P: zero on one side, a twenty-five per cent gain on the other, so where it goes is everything. Satisfying P to its requirement gives a yield of **0.900000** and moves the limit to N, the yield now equalling exactly the sufficiency of N—evidence that the minimum has changed hands. Adding a further 20 kg of P from there leaves the yield at 0.900000 with a change of exactly **0.0000000000000000**: the element that worked a moment ago stops working, because the limit has moved. So “it worked before” is no guarantee that it works again—without measuring again, which element is smallest is unknown. Satisfying all three gives exactly **1.0000000000000000**, which checks the expression. One thing separates them—do contributions add, or does a minimum decide. If they add, anywhere returns in proportion and allocation need not be considered. If a minimum decides, everywhere but one place gives zero, what goes elsewhere is wholly wasted, and allocation becomes everything. They are distinct roots by the criterion of Paper 300, one an additive quantity and one decided by a minimum. This is a second instance of the shape of Paper 471, where a ratio was off and the scarce side had to be added: in both, where the input goes decides the effect. It overlaps Paper 476 too, where nothing was visible until one looked at the concrete rather than the iron: here nothing works until the input is redirected. And the law of the minimum carries the movement of the limit within it—satisfy one and the next becomes smallest, so improvement is a sequence of remedies each of which works only for a while. To be honest—the law of the minimum was taken strictly, whereas elements interact and the limit shifts smoothly. The exact zero belongs to the model; that input to a non-minimal element loses most of its value does not.

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